

DPP

DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : Maths
DPP No. : 1

Topic :- Binomial Theorem

1

(b)

We have, $(1+x)^{15} = C_0 + C_1x + C_2x^2 + \dots + C_{15}x^{15}$

$$\Rightarrow \frac{(1+x)^{15} - 1}{x} = C_1 + C_2x + \dots + C_{15}x^{14}$$

On differentiating both sides w.r.t. x , we get

$$\frac{x \cdot 15(1+x)^{14} - (1+x)^{15} + 1}{x^2} = C_2 + 2C_3x + \dots + 14C_{15}x^{13}$$

On putting $x = 1$, we get

$$C_2 + 2C_3 + \dots + 14C_{15} = 15 \cdot 2^{14} - 2^{15} + 1$$

$$= 13 \cdot 2^{14} + 1$$

2

(b)

It is given that

${}^{2n}C_1, {}^{2n}C_2$ and ${}^{2n}C_3$ are A.P.

$$\therefore 2 \cdot {}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$$

$$2 \cdot \frac{(2n)!}{(2n-2)!2!} = \frac{(2n)!}{(2n-1)!} + \frac{(2n)!}{(2n-3)!3!}$$

$$\Rightarrow 2 \frac{(2n)(2n-1)}{2} = 2n + \frac{(2n)(2n-1)(2n-2)}{3!}$$

$$\Rightarrow 6(2n-1) = 6 + (2n-1)(2n-2)$$

$$\Rightarrow 12n-6 = 6+4n^2-6n+2$$

$$\Rightarrow 4n^2-18n+14=0 \Rightarrow 2n^2-9n+7=0$$

3

(d)

$$\frac{1+2x}{(1-2x)^2} = (1+2x)(1-2x)^{-2}$$

$$= (1+2x) \left(1 + \frac{2}{1!}(2x) + \frac{2 \cdot 3}{2!}(2x)^2 + \dots + \frac{2 \cdot 3 \dots r}{(r-1)!}(2x)^{r-1} + \frac{2 \cdot 3 \cdot 4 \dots (r+1)(2x)^r}{r!} \right)$$

$$\text{The coefficient of } x^r = 2 \frac{r!}{(r-1)!} 2^{r-1} + \frac{(r-1)!}{r!} 2^r$$

$$= r2^r + (r+1)2^r = 2^r(2r+1)$$

4

(c)

$$\text{Given, } A = {}^{30}C_0 \cdot {}^{30}C_{10} - {}^{30}C_1 \cdot {}^{30}C_{11} + {}^{30}C_2 \cdot {}^{30}C_{12} + \dots + {}^{30}C_{20} \cdot {}^{30}C_{30}$$

$$= \text{coefficient of } x^{20} \text{ in } (1+x)^{30}(1-x)^{30}$$

$$= \text{coefficient of } x^{20} \text{ in } (1+x^2)^{30}$$

$$= \text{coefficient of } x^{20} \text{ in } \sum_{r=0}^{30} (-1)^r {}^{30}C_r (x^2)^r$$

$$= (-1)^{10} \cdot {}^{30}C_{10} \{ \text{for coefficient of } x^{20}, \text{ let } r = 10 \}$$

$$= {}^{30}C_{10}$$

5

(a)

We have,

$$a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \dots + a_{2n}x^{2n} = (1-x+x^2)^n$$



Putting $x = 1$ and -1 , we get

$$(a_0 + a_2 + a_4 + \dots) + (a_1 + a_3 + a_5 + \dots) = 1 \quad \dots (i)$$

And,

$$(a_0 + a_2 + a_4 + \dots) - (a_1 + a_3 + a_5 + \dots) = 3^n \quad \dots (ii)$$

Adding (i) and (ii), we get

$$a_0 + a_2 + a_4 + \dots = \frac{3^n + 1}{2}$$

6

(a)

We know that,

$$(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$$

On integrating both sides, from 0 to 1, we get

$$\left[\frac{(1+x)^{n+1}}{n+1} \right]_0^1 = \left[C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + \dots + \frac{C_nx^{n+1}}{n+1} \right]_0^1$$

$$\Rightarrow \frac{2^{n+1} - 1}{n+1} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n+1}$$

7

(a)

7th term from the beginning in the expansion of $(2^{1/3} + \frac{1}{3^{1/3}})^x$ is given by

$$T_7 = {}^x C_6 (2^{1/3})^{x-6} \left(\frac{1}{3^{1/3}} \right)^6$$

7th term from the end in the expansion of $(2^{1/3} + \frac{1}{3^{1/3}})^x$ is the $(x + 1 - 7 + 1)^{\text{th}} = (x - 5)^{\text{th}}$ term from the beginning. Therefore,

$$T_{x-5} = {}^x C_{x-6} (2^{1/3})^6 \left(\frac{1}{3^{1/3}} \right)^{x-6}$$

We have,

$$\frac{T_7}{T_{x-5}} = \frac{1}{6}$$

$$\Rightarrow 6T_7 = T_{x-5}$$

$$\Rightarrow 6 {}^x C_6 2^{\frac{x-6}{3}} 3^{-2} = {}^x C_{x-6} 2^2 3^{-\frac{(x-6)}{3}}$$

$$\Rightarrow 2^{\frac{x-9}{3}} = 3^{-\frac{(x-9)}{3}}$$

$$\Rightarrow 6^{\frac{x-9}{3}} = 1 \Rightarrow x - 9 = 0 \Rightarrow x = 9$$

9

(b)

$$\therefore T_{r+1} = {}^{10} C_r (x \sin^{-1} \alpha)^{10-r} \left(\frac{\cos^{-1} \alpha}{x} \right)^r$$

$$= {}^{10} C_r (\sin^{-1} \alpha)^{10-r} (\cos^{-1} \alpha)^r x^{10-2r}$$

\therefore For the term independent of x ,

$$10 - 2r = 0 \Rightarrow r = 5$$

$$T_{5+1} = {}^{10} C_5 (\sin^{-1} \alpha)^5 (\cos^{-1} \alpha)^5$$

$$= {}^{10} C_5 (\sin^{-1} \alpha \cos^{-1} \alpha)^5$$

Let $f(\alpha) = \sin^{-1} \alpha \cos^{-1} \alpha$

$$= \sin^{-1} \alpha \left(\frac{\pi}{2} - \sin^{-1} \alpha \right)$$

Put $\sin^{-1} \alpha = t$

$$\therefore f(\alpha) = t \left(\frac{\pi}{2} - t \right)$$

$$= - \left\{ t^2 - \frac{\pi}{2} t \right\}$$

$$= - \left\{ \left(t - \frac{\pi}{4} \right)^2 - \frac{\pi^2}{16} \right\}$$



$$= \frac{\pi^2}{16} - \left(t - \frac{\pi}{4}\right)^2$$

$$\therefore f(\alpha) = \frac{\pi^2}{16} - \left(\sin^{-1} \alpha - \frac{\pi}{4}\right)^2$$

Maximum value of $f(\alpha)$ is $\frac{\pi^2}{16}$, when $\sin^{-1} \alpha = \frac{\pi}{4}$

Also, $-1 \leq \alpha \leq 1$

$$\therefore -\frac{\pi}{2} \leq \sin^{-1} \alpha \leq \frac{\pi}{2}$$

$$\text{Minimum value } f(\alpha) = \frac{\pi^2}{16} - \left(-\frac{\pi}{2} - \frac{\pi}{4}\right)^2 = -\frac{\pi^2}{2}$$

$$\therefore \text{Range is } \left[{}^{10}C_5 \left(-\frac{\pi^2}{2}\right)^5, {}^{10}C_5 \left(\frac{\pi^2}{16}\right)^5 \right]$$

$$\text{ie, } \left[-\frac{{}^{10}C_5 \pi^{10}}{2^5}, \frac{{}^{10}C_5 \pi^{10}}{2^{20}} \right]$$

10

(c)

$$\text{Let } A = \binom{30}{0} \binom{30}{10} - \binom{30}{1} \binom{30}{11} x$$

$$+ \binom{30}{2} \binom{30}{12} - \dots + \binom{30}{20} \binom{30}{30}$$

$$\text{or } A = {}^{30}C_0 \cdot {}^{30}C_{10} - {}^{30}C_1 \cdot {}^{30}C_{11}$$

$$+ {}^{30}C_2 \cdot {}^{30}C_{12} - \dots + {}^{30}C_{20} \cdot {}^{30}C_{30}$$

$$= \text{coefficient of } x^{20} \text{ in } (1+x)^{30} (1-x)^{30}$$

$$= \text{coefficient of } x^{20} \text{ in } (1-x^2)^{30}$$

$$= \text{coefficient of } x^{20} \text{ in } \sum_{r=0}^{30} (-1)^r {}^{30}C_r (x^2)^r$$

$$= (-1)^{10} {}^{30}C_{10} \text{ (for coefficient of } x^{20}, \text{ let } r = 10)$$

$$= {}^{30}C_{10}$$

11

(b)

The r th term of $(a+2n)^n$ is ${}^n C_{r-1} (a)^{n-r+1} (2x)^{r-1}$

$$= \frac{n!}{(n-r+1)!(r-1)!} a^{n-r+1} (2x)^{r-1}$$

$$= \frac{n(n-1)\dots(n-r+2)}{(r-1)!} a^{n-r+1} (2x)^{r-1}$$

12

(a)

We have, $(1+t^2)^{12} (1+t^{12}) (1+t^{24})$

$$= (1 + {}^{12}C_1 t^2 + {}^{12}C_2 t^4 + \dots + {}^{12}C_6 t^{12} + \dots + {}^{12}C_{12} t^{24} + \dots) (1 + t^{12} + t^{24} + t^{36})$$

\therefore Coefficient of t^{24} in $(1+t^2)^{12} (1+t^{12}) (1+t^{24})$

$$= {}^{12}C_6 + {}^{12}C_{12} + 1 = {}^{12}C_6 + 2$$

13

(c)

We have,

$$\frac{(1+x)^2}{(1-x)^3} = (x^{2v} + 2x + 1)(1-x)^{-3}$$

$$\Rightarrow \frac{(1+x)^2}{(1-x)^3} = x^2(1-x)^{-3} + 2x(1-x)^{-3} + (1-x)^{-3}$$

$$\therefore \text{Coeff. of } x^n \text{ in } \frac{(1+x)^2}{(1-x)^3}$$

$$= \text{Coeff. of } x^n \text{ in } x^2(1-x)^{-3} + \text{Coeff. of } x^n \text{ in } 2x(1-x)^{-3} + \text{Coeff. of } x^n \text{ in } (1-x)^{-3}$$

$$= \text{Coeff. of } x^{n-2} \text{ in } (1-x)^{-3} + 2 \cdot \text{Coeff. of } x^{n-1} \text{ in } (1-x)^{-3} + \text{Coeff. of } x^n \text{ in } (1-x)^{-3}$$

$$= {}^{n-2+3-1}C_{3-1} + 2 \cdot {}^{n-1+3-1}C_{3-1} + {}^{n+3-1}C_{3-1}$$



$$\begin{aligned}
 &= {}^n C_2 + 2 \cdot {}^{n+1} C_2 + {}^{n+2} C_2 \\
 &= \frac{n(n-1)}{2} + 2 \frac{(n+1)n}{2} + (n+2) \frac{(n+1)}{2} \\
 &= \frac{1}{2} (n^2 - n + 2^2 + 2n + n^2 + 3n + 2) = 2n^2 + 2n + 1
 \end{aligned}$$

14

(c)

On substituting $x = 1$ in $(1 + x - 3x^2)^{3148}$, then sum of coefficient
 $= (1 + 1 - 3)^{3148} = (-1)^{3148} = 1$

15

(c)

$$\begin{aligned}
 &a {}^n C_0 - (a+d) {}^n C_1 + (a+2d) {}^n C_2 - (a+3d) {}^n C_3 + \dots + (-1)^n (a+nd) {}^n C_n \\
 &= \sum_{r=0}^n (a+rd) (-1)^r {}^n C_r \\
 &= a \sum_{r=0}^n (-1)^r {}^n C_r - dn \sum_{r=1}^{n-1} {}^{n-1} C_{r-1} (-1)^{r-1} \\
 &= a \times 0 - dn \times 0 = 0
 \end{aligned}$$

16

(b)

We have,

$$\begin{aligned}
 &\frac{18^3 + 7^3 + 3 \cdot 18 \cdot 7 \cdot 25}{3^6 + 6 \cdot 243 \cdot 2 + 15 \cdot 81 \cdot 4 + 20 \cdot 27 \cdot 8 + 15 \cdot 9 \cdot 16 + 6 \cdot 3 \cdot 32 + 64} \\
 &= \frac{18^3 + 7^3 + 3 \cdot 18 \cdot 7 \cdot (18+7)}{(3+2)^6} = \frac{{}^6 C_0 3^6 + {}^6 C_1 3^5 \cdot 2^1 + {}^6 C_2 3^4 \cdot 2^2 + {}^6 C_3 3^3 \cdot 2^3 + {}^6 C_4 3^2 \cdot 2^4 + {}^6 C_5 3^1 \cdot 2^5 + {}^6 C_6 3^0 \cdot 2^6}{(3+2)^6} = \frac{5^6}{5^6} = 1
 \end{aligned}$$

17

(b)

We have,

$$\begin{aligned}
 &(1+x^2)^5 (1+x)^4 \\
 &= ({}^5 C_0 + {}^5 C_1 x^2 + {}^5 C_2 x^4 + \dots) \\
 &\times ({}^4 C_0 + {}^4 C_1 x + {}^4 C_2 x^2 + {}^4 C_3 x^3 + {}^4 C_4 x^4) \\
 &\therefore \text{Coefficient of } x^5 \text{ in } \{(1+x^2)^5 (1+x)^4\} \\
 &= {}^5 C_2 \times {}^5 C_1 + {}^4 C_3 \times {}^5 C_1 = 60
 \end{aligned}$$

18

(d)

$$\begin{aligned}
 &({}^n C_0)^2 + ({}^n C_1)^2 + ({}^n C_2)^2 + \dots + ({}^n C_5)^2 \\
 &= ({}^5 C_0)^2 + ({}^5 C_1)^2 + ({}^5 C_2)^2 + ({}^5 C_3)^2 + ({}^5 C_4)^2 + ({}^5 C_5)^2 \\
 &= 1 + 25 + 100 + 100 + 25 + 1 = 252
 \end{aligned}$$

19

(c)

Let

$$S = (1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \dots + 1000x^{999}(1+x) + 1001x^{1000} \dots (i)$$

$$\therefore \frac{x}{1+x} S = x(1+x)^{999} + 2x^2(1+x)^{998} + \dots + 1000x^{1000} + \frac{1001x^{1001}}{1+x} \dots (ii)$$

Subtracting (ii) from (i), we get

$$\begin{aligned}
 \left(1 - \frac{x}{x+1}\right) S &= (1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \dots + x^{1000} - \frac{1001x^{1001}}{1+x} \\
 \Rightarrow S &= (1+x)^{1001} + x(1+x)^{1000} + x^2(1+x)^{999} + \dots + x^{1000}(1+x) - 1001x^{1001}
 \end{aligned}$$

$$\Rightarrow S = (1+x)^{1001} \frac{\left\{1 - \left(\frac{x}{1+x}\right)^{1001}\right\}}{\left\{1 - \frac{x}{1+x}\right\}} - 1001x^{1001}$$

$$\Rightarrow S = (1+x)^{1002} \left\{1 - \left(\frac{x}{1+x}\right)^{1001}\right\} - 1001x^{1001}$$

$$\Rightarrow S = (1+x)^{1002} - x^{1001}(1+x) - 1001x^{1001}$$



20 \therefore Coefficient of x^{50} in S is ${}^{1002}C_{50}$
(d)

$$\begin{aligned} \frac{1}{(x-1)^2(x-2)} &= \frac{1}{-2(1-x)^2\left(1-\frac{x}{2}\right)} \\ &= -\frac{1}{2}\left[(1-x)^{-2}\left(1-\frac{x}{2}\right)^{-1}\right] \\ &= -\frac{1}{2}\left[(1+2x+\dots)\left(1+\frac{x}{2}+\dots\right)\right] \end{aligned}$$

\therefore Coefficient of constant term is $-\frac{1}{2}$.

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	D	C	A	A	A	C	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	C	C	C	B	B	D	C	D