DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :1

Topic :-SEQUENCES AND SERIES

1 **(b)**

If p, q, r, s are in A.P., then in an A.P. or a G.P. or an H.P. $a_1, a_2, a_3, \ldots$, the terms a_p, a_q, a_r are in A.P., G.P. or H.P. respectively

2 **(c)**

$$T_n = \frac{\frac{n(n+1)}{2.2}}{1^3+2^3+3^3+\ldots+n^3}$$
$$= \frac{\frac{n(n+1)}{4}}{\left(\frac{n(n+1)}{2}\right)^2} = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$
$$\therefore T_n = \Sigma\left(\frac{1}{n} - \frac{1}{n+1}\right)$$
$$= \left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \ldots + \left(\frac{1}{n}-\frac{1}{n+1}\right)$$
$$= 1 - \frac{1}{n+1} = \frac{n}{n+1}$$

3 **(a)**

Since, $a_1, a_2, a_3, \ldots, a_n$ are in AP.
Then, $a_2 - a_1 = a_3 - a_2 = \ldots = a_n - a_{n-1} = d$
Where d is the common difference of the give AP
Also, $a_n = a_1 + (n-1)d$
Then, by rationalizing each term
$$\frac{1}{\sqrt{a_2}+\sqrt{a_1}} + \frac{1}{\sqrt{a_3}+\sqrt{a_2}} + \ldots + \frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}}$$
$$= \frac{\sqrt{a_2}-\sqrt{a_1}}{a_2-a_1} + \frac{\sqrt{a_3}-\sqrt{a_2}}{a_3-a_2} + \ldots + \frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{a_n-a_{n-1}}$$
$$= \frac{1}{d}\left(\sqrt{a_2} - \sqrt{a_1} + \sqrt{a_3} - \sqrt{a_2} + \ldots + \sqrt{a_n} - \sqrt{a_{n-1}}\right)$$
$$= \frac{1}{d}\left(\sqrt{a_n} - \sqrt{a_1}\right) \times \frac{\sqrt{a_n}+\sqrt{a_1}}{\sqrt{a_n}+\sqrt{a_1}}$$
$$= \frac{1}{d}\left(\frac{a_n - a_1}{\sqrt{a_n}+\sqrt{a_1}}\right) = \frac{1}{d}\left(\frac{(n-1)d}{\sqrt{a_n}+\sqrt{a_1}}\right) = \frac{n-1}{\sqrt{a_n}+\sqrt{a_1}}$$

4 **(a)**

We have,
$$\frac{1}{x^2} + \frac{1}{2x^4} + \frac{1}{3x^6} + \cdots \text{ad. inf.}$$
$$= -\log_e\left(1 - \frac{1}{x^2}\right)$$

$= -\log_e\left(1-\frac{2}{y+1}\right) \quad \left[\because y = 2x^2 - 1 \;\therefore x^2 = \frac{y+1}{2}\right]$

$= -\log_e\left(\frac{y-1}{y+1}\right) = \log_e\left(\frac{y+1}{y-1}\right)$

5 **(c)**

Let the number of sides of the polygon be n.Then, the sum of interior angles of the polygon

$= (2n-4)\frac{\pi}{4} = (n-2)\pi$

Since, the angles are in AP and $a = 120°, d = 5$

Therefore, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \frac{n}{2}[2 \times 120 + (n-1)5] = (n-2)180$

$\Rightarrow n^2 - 25n + 144 = 0$

$\Rightarrow (n-9)(n-16) = 0$

$\Rightarrow n = 9, 16$

Take $n = 16$

$T_{16} = a + 15d = 120° + 15(5°) = 195°$, which is impossible, an interior angle cannot be greater than 180°.

Hence, $n = 9$

6 **(d)**

We have,

$\log_x(4.x^{\log_5 x} + 5) = 2\log_5 x$

$\Rightarrow \log_x(4.x^{\log_5 x} + 5) = \log_5 x^2$

$\Rightarrow 4.x^{\log_5 x} + 5 = x^{\log_5 x^2}$

$\Rightarrow 4.x^{\log_5 x} + 5 = x^{2\log_5 x}$

$\Rightarrow 4y + 5 = y^2$, where $y = x^{\log_5 x}$

$\Rightarrow y^2 - 4y - 5 = 0$

$\Rightarrow y = 5, -1$

$\Rightarrow x^{\log_5 x} = 5 \qquad [\because y \neq -1]$

$\Rightarrow \log_5 x = \log_x 5$

$\Rightarrow (\log_5 x)^2 = 1 \Rightarrow \log_5 x = \pm 1 \Rightarrow x = 5, 5^{-1}$

7 **(d)**

Since, $x = 1 + a + a^2 + \ldots\infty$

$\Rightarrow \;x = \frac{1}{1-a} \quad \Rightarrow \quad a = \frac{x-1}{x}$

Similarly, $b = \frac{y-1}{y}$ and $c = \frac{z-1}{z}$

Since, a, b, c are in AP.

$\therefore b = \frac{a+c}{2}$

$\Rightarrow \frac{y-1}{y} = \frac{\frac{x-1}{x} + \frac{z-1}{z}}{2}$

$\Rightarrow 2xz(y-1) = y[z(x-1) + x(z-1)]$

$\Rightarrow 2xz = xy + yz$

8 **(a)**

We have,

$x^{(3/2)(\log_2 x - 3)} = 2^{-3}$

$\Rightarrow \frac{3}{2}(\log_2 x - 3) = \log_x 2^{-3}$

$\Rightarrow \frac{3}{2}(\log_2 x - 3) = -3\log_x 2$

$\Rightarrow \frac{1}{2}(\log_2 x - 3) = -\frac{1}{\log_2 x}$
$\Rightarrow (\log_2 x)^2 - 3(\log_2 x) + 2 = 0$
$\Rightarrow (\log_2 x - 1)(\log_2 x - 2) = 0$
$\Rightarrow \log_2 x = 1, 2 \Rightarrow x = 2, 2^2$

11 **(b)**

Since, a, b, c are in AP.
$\Rightarrow 2b = a + c$, then straight line $ax + by + c = 0$ will pass through$(1, -2)$ because it satisfies condition $a - 2b + c = 0$ or $2b = a + c$.

12 **(a)**

We have,
$$\frac{e^x}{1-x} = B_0 + B_1 x + B_2 x^2 + \cdots + B_n x^n + \cdots$$
$$\Rightarrow \sum_{r=0}^{\infty} \frac{x^r}{r!} = (B_0 + B_1 x + B_2 x^2 + \cdots + B_n x^n + \cdots)(1 - x)$$
On equating the coefficients of x^n on both sides, we get
$$\frac{1}{n!} = B_n - B_{n-1}$$

13 **(b)**

We have,
$$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$$
Applying componendo and dividendo rule, we get
$$\frac{2a}{2bx} = \frac{2b}{2cx} = \frac{2c}{2dx}$$
$$\Rightarrow \frac{a}{b} = \frac{b}{c} = \frac{c}{d}$$
$\Rightarrow b^2 = ac$ and $c^2 = bd$
$\Rightarrow a, b, c$ and b, c, d are in GP, therefore a, b, c, d are in GP.

14 **(c)**

We have,
$$\sum_{r=1}^{n} \frac{1}{(2r-1)^2} = \frac{\pi^2}{8}$$
$$\Rightarrow \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$
Let $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{5^2} + -\text{to } \infty = x$
$$\Rightarrow \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right) + \frac{1}{2^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right) = x$$
$$\Rightarrow \frac{\pi^2}{8} + \frac{x}{4} = x \Rightarrow x = \frac{\pi^2}{6}$$

15 **(c)**

It will take 10yr for Jairam to pay off Rs 10000 in 10 yearly installments.
$\therefore$ He pays 10% annual interest on remaining amount.
$\therefore$ Money given in the first year
$= 1000 + \frac{10000 \times 10}{100} = 1000 + 1000$
$=$ Rs 2000
Money given in second year
$= 1000 +$ interest of $(10000 - 1000)$
$= 1000 + \frac{9000 \times 10}{100} = 100 + 900 =$ Rs 1900

Similarly, money paid in third year = Rs 1800 etc.
So, money given by Jairam in 10 yr will be Rs 2000, Rs 1900, Rs 1800, Rs 1700 …
Which is in arithmetic progression, whose first term
$a = 2000$ and $d = -100$
Total money given in 10 yr
$= \frac{10}{2}[2(2000) + (10-1)(-100)] = \text{Rs } 15500$
Therefore, total money given by Jairam
$= 5000 + 15500 = \text{Rs } 20500$

16 **(a)**

We have,
a, b, c are in A.P. ...(i)
$\Rightarrow \frac{a}{abc}, \frac{b}{abc}, \frac{c}{abc}$ are in A.P. $\Rightarrow \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in A.P. ...(ii)
From (i) and (ii), we obtain
$a + \frac{1}{bc}, b + \frac{1}{ca}, c + \frac{1}{ab}$ are in A. P.

17 **(d)**

We observe that the successive differences of the terms of the sequence 12,28,50,78, … are in A.P. So, let its n^{th} term be
$t_n = an^2 + bn + c,$
Putting $n = 1,2,3$, we get
$t_1 = a + b + c \Rightarrow a + b + c = 12$
$t_2 = 4a + 2b + c \Rightarrow 4a + 2b + c = 28$
$t_3 = 9a + 3b + c \Rightarrow 9a + 3b + c = 50$
Solving these equations, we get
$a = 3, b = 7, c = 2$
$\therefore t_n = 3n^2 + 7n + 2$
Hence,
$$\frac{12}{2!} + \frac{28}{3!} + \frac{50}{4!} + \frac{78}{5!} + \cdots$$
$$= \sum_{n=1}^{\infty} \frac{3n^2 + 7n + 2}{(n+1)!}$$
$$= \sum_{n=2}^{\infty} \frac{3(n-1)^2 + 7(n-1) + 2}{n!}$$
$$= \sum_{n=2}^{\infty} \frac{3n^2 + n - 2}{n!}$$
$$= 3\sum_{n=2}^{\infty} \frac{n^2}{n!} + \sum_{n=2}^{\infty} \frac{n}{n!} - 2\sum_{n=2}^{\infty} \frac{1}{n!}$$
$= 2(2e-1) + (e-1) - 2(e-2) = 5e$

18 **(b)**

We have,
$$\frac{x^2}{2} + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \frac{4}{5}x^5 + \cdots$$
$$= \sum_{n=1}^{\infty} \frac{n}{n+1} x^{n+1}$$
$$= \sum_{n=1}^{\infty} \frac{n+1-1}{n+1} x^{n+1}$$

$$= \sum_{n=1}^{\infty}\left(1-\frac{1}{n+1}\right)x^{n+1}$$

$$= \sum_{n=1}^{\infty} x^{n+1} - \overset{\infty}{(n = 1}\frac{x^{n+1}}{n+1}$$

$$= \frac{x^2}{1-x} + x - \sum_{n=0}^{\infty}\frac{x^{n+1}}{n+1}$$

$$= \frac{x^2}{1-x} + x + \log(1-x) = \frac{x}{1-x} + \log(1-x)$$

19 **(c)**

$$\left(\frac{1}{3}\right)^2 + \frac{1}{3}\left(\frac{1}{3}\right)^4 + \frac{1}{5}\left(\frac{1}{3}\right)^6 + \ldots$$

$$= \frac{1}{3}\left[\left(\frac{1}{3}\right) + \frac{1}{3}\left(\frac{1}{3}\right)^3 + \frac{1}{5}\left(\frac{1}{3}\right)^5 + \ldots\right]$$

$$= \frac{1}{3}\cdot\frac{1}{2}\log\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right) \quad \left[\because \frac{1}{2}\log\left(\frac{1+x}{1-x}\right) = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\right]$$

$$= \frac{1}{6}\log_e 2$$

20 **(a)**

Since, $T_m = \frac{1}{n} \Rightarrow a + (m-1)d$ ...(i)

and $T_n = \frac{1}{m} = a + (n-1)d$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a = \frac{1}{mn}$ and $d = \frac{1}{mn}$

$\therefore\ a - d = \frac{1}{mn} - \frac{1}{mn} = 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10

A.	B	C	A	A	C	D	D	A	D	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	B	C	C	A	D	B	C	A