

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. : 1

Topic :- CONIC SECTION

1 (d)

The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts an intercept of length $2\sqrt{f^2 - c}$ on y-axis.

For the circle $x^2 + y^2 + 4x - 7y + 12 = 0$, we have
 $g = 2, f = -7/2$ and $c = 12$

$$\therefore y - \text{intercept} = 2\sqrt{f^2 - c} = 2\sqrt{\frac{49}{4} - 12} = 1$$

2 (a)

$$\therefore \text{Eccentricity of ellipse} = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$$

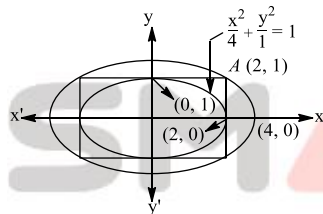
$$\therefore \text{Eccentricity of hyperbola} = 2$$

$$\Rightarrow \sqrt{1 + \frac{b^2}{64}} \Rightarrow 2$$

$$\Rightarrow 4 = 1 + \frac{b^2}{64} \Rightarrow 192 = b^2$$

3 (a)

Let the equation of the required ellipse be $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$
But the ellipse passes through the point (2,1)



$$\Rightarrow \frac{1}{4} + \frac{1}{b^2} = 1$$

$$\Rightarrow \frac{1}{b^2} = \frac{3}{4} \Rightarrow b^2 = \frac{4}{3}$$

Hence, equation is

$$\frac{x^2}{16} + \frac{3y^2}{4} = 1$$

$$\Rightarrow x^2 + 12y^2 = 16$$

4 (b)

We have,

$$x = 2t + 1, y = t^2 + 2$$

$$\Rightarrow y = \left(\frac{x-1}{2}\right)^2 + 2$$

$$\Rightarrow (x-1)^2 = 4(y-2)$$

The equation of the directrix of this parabola is

$$y - 2 = -1 \text{ or, } y = 1 \quad [\text{Using } y = -a]$$



5 (c)

Given equation can be rewritten as

$$y^2 = \frac{4k}{4} \left(x - \frac{8}{k} \right)$$

The standard equation of parabola is

$$Y^2 = 4AX, \text{ where } A = \frac{k}{4}$$

\therefore Equation of directrix is $X + \frac{k}{4} = 0$

$$\Rightarrow x - \frac{8}{k} + \frac{k}{4} = 0$$

But the given equation of directrix is $x - 1 = 0$

Since, both equations are same

$$\therefore \frac{8}{k} - \frac{k}{4} = 1$$

$$\Rightarrow 32 - k^2 = 4k \Rightarrow k = -8, 4$$

6 (d)

The equation of the ellipse is

$$3(x+1)^2 + 4(y-1)^2 = 12 \text{ or, } \frac{(x+1)^2}{2^2} + \frac{(y-1)^2}{(\sqrt{3})^2} = 1$$

The equations of its major and minor axes are $y - 1 = 0$ and $x + 1 = 0$ respectively

7 (a)

Let mid point of the chord be (h, k) , then equation of the chords be

$$\frac{hx^2}{a^2} + \frac{ky^2}{b^2} - 1 = \frac{h^2}{a^2} + \frac{k^2}{b^2} - 1$$

$$\Rightarrow y = -\frac{b^2}{a^2} \cdot \frac{h}{k} x + \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} \right) \frac{b^2}{k} \dots(i)$$

Since, line (i) is touching the circle $x^2 + y^2 = c^2$

$$\therefore \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} \right) \frac{b^4}{k^2} = c^2 \left(1 + \frac{b^4 h^2}{a^4 k^2} \right)$$

Hence, locus is $(b^2 x^2 + a^2 y^2)^2 = c^2 (b^4 x^2 + a^4 y^2)$

8 (c)

Given curve is $y^2 = 4x \dots(i)$

Let the equation of line be $y = mx + c$

Since, $\frac{dy}{dx} = m = 1$ and above line is passing through the point $(0, 1)$

$$1 = 1(0) + c \Rightarrow c = 1$$

$$y = x + 1 \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$x = 1 \text{ and } y = 2$$

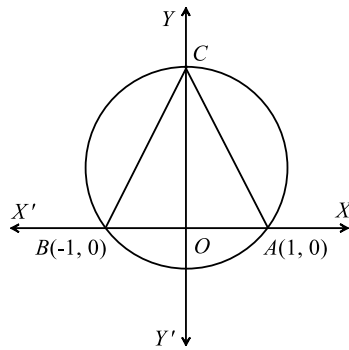
This shows that line touch the curve at one point. So, length of intercept is zero.

9 (c)

We have, $AB = 2$

Since ΔABC is equilateral. Therefore,

$$AC = BC = 2 \text{ and } OC = \frac{\sqrt{3}}{2} (\text{Side}) = \sqrt{3}$$



Thus, the coordinates of C are $(0, \sqrt{3})$

Let the circumcircle of ΔABC be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

It passes through $(1, 0)$, $(-1, 0)$ and $(0, \sqrt{3})$

$$\therefore 1 + 2g + c = 0, 1 - 2g + c = 0 \text{ and } 3 + 2\sqrt{3}f + c = 0$$

Solving these three equations, we get

$$g = 0, c = -1 \text{ and } f = -\frac{1}{\sqrt{3}}$$

Thus, the equation of the circumcircle is

$$x^2 + y^2 - \frac{2}{\sqrt{3}}y - 1 = 0$$

10 (c)

The coordinates of P be (h, k)

Let the equation of a tangent from $P(h, k)$ to the circle

$$x^2 + y^2 = a^2 \text{ be } y = mx + a\sqrt{1 + m^2}$$

Since $P(h, k)$ lies on $y = mx + a\sqrt{1 + m^2}$

$$\therefore k = mh + a\sqrt{1 + m^2}$$

$$\Rightarrow (k - mh)^2 = a^2(1 + m^2)$$

$$\Rightarrow m^2(h^2 - a^2) - 2mkh + k^2 - a^2 = 0$$

This is a quadric in m . Let the two roots be m_1 and m_2 . Then,

$$m_1 + m_2 = \frac{2hk}{h^2 - a^2}$$

But, $\tan \alpha = m_1, \tan \beta = m_2$ and it is given that

$$\cot \alpha + \cot \beta = 0$$

$$\Rightarrow \frac{1}{m_1} + \frac{1}{m_2} = 0 \Rightarrow m_1 + m_2 = 0 \Rightarrow \frac{2hk}{k^2 - a^2} = 0 \Rightarrow hk = 0$$

Hence, the locus of (h, k) is $xy = 0$

11 (b)

We have,

$$x = 2 + t^2, y = 2t + 1$$

$$\Rightarrow x - 2 = t^2 \text{ and } y - 1 = 2t$$

$$\Rightarrow (y - 1)^2 = 4t^2 \text{ and } x - 2 = t^2$$

$$\Rightarrow (y - 1)^2 = 4(x - 2),$$

Which is a parabola with vertex at $(2, 1)$

12 (b)

Given equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a < b)$$

It is a vertical ellipse with foci $(0, \pm be)$

Equation of any tangent line to the above ellipse is

$$y = mx + \sqrt{a^2m^2 + b^2}$$

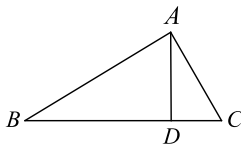


∴ Required product

$$\begin{aligned}
 &= \left| \frac{-be + \sqrt{a^2m^2 + b^2}}{\sqrt{m^2 + 1}} \right| \left| \frac{be + \sqrt{a^2m^2 + b^2}}{\sqrt{m^2 + 1}} \right| \\
 &= \left| \frac{a^2m^2 + b^2 - b^2e^2}{m^2 + 1} \right| \\
 &= \left| \frac{a^2m^2 + b^2(1 - e^2)}{m^2 + 1} \right| \\
 &= \left| \frac{a^2m^2 + a^2}{m^2 + 1} \right| \quad [\because a^2 = b^2(1 - e^2)] \\
 &= a^2
 \end{aligned}$$

13 (b)

Since, $\angle ADB = \angle ADC = 90^\circ$, circle on AB and AC as diameters pass through D and therefore the altitude AD is the common chord. Similarly, the other two common chords are the other two altitudes and hence they concur at the orthocenter



14 (b)

Given equation of ellipse can be rewritten as

$$\frac{(x-2)^2}{25} + \frac{(y+3)^2}{16} = 1 \Rightarrow \frac{X^2}{25} + \frac{Y^2}{16} = 1$$

Where $X = x - 2, Y = y + 3$

Here, $a > b$

$$\therefore e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$$

∴ Focus $(\pm ae, 0) = (\pm 3, 0)$

$$\Rightarrow x - 2 = \pm 3, y + 3 = 0$$

$$\Rightarrow x = 5, -1, y = -3$$

∴ Foci are $(-1, -3)$ and $(5, -3)$

Distance between $(2, -3)$ and $(-1, -3)$

$$= \sqrt{(2+1)^2 + (-3+3)^2} = 3$$

and distance between $(2, -3)$ and $(5, -3)$

$$= \sqrt{(2-5)^2 + (-3+3)^2} = 3$$

Hence, sum of the distance of point $(2, -3)$ from the foci

$$= 3 + 3 = 6$$

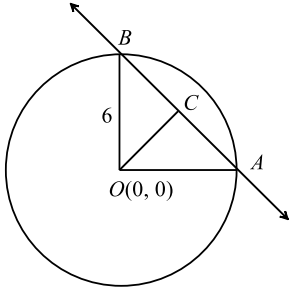
15 (c)

We have,

OC = Length of the perpendicular from $(0,0)$ on the line $3x + 4y - 15 = 0$

$$\Rightarrow OC = \frac{15}{\sqrt{3^2 + 4^2}} = 3$$

$$\therefore AB = 2AC = 2\sqrt{OA^2 - OC^2} = 2\sqrt{36 - 9} = 6\sqrt{3}$$



16 (a)

We know that the normal at $(at_1^2, 2at_1)$ meets the parabola at $(at_2^2, 2at_2)$, if $t_2 = -t_1 - \frac{2}{t_1}$

Here, the normal is drawn at (x_1, x_1)

$$\therefore at_1^2 = 2at_1 \Rightarrow t_1 = 2 \Rightarrow t_2 = -2 - \frac{2}{2} = -3$$

The coordinates of the end points of the normal chord are $P(4a, 4a)$ and $Q(9a, -6a)$

Clearly, PQ makes a right angle at the focus $(a, 0)$

17 (a)

The equation of the family of circles touching $2x - y - 1 = 0$ at $(3, 5)$ is

$$(x - 3)^2 + (y - 5)^2 + \lambda(2x - y - 1) = 0 \quad \dots(i)$$

It has its centre $(-\lambda + 3, \frac{\lambda + 10}{2})$ on the line $x + y = 5$

$$\therefore -\lambda + 3 + \frac{\lambda + 10}{2} = 5 \Rightarrow \lambda = 6$$

Putting $\lambda = 6$ in (i), we get

$$x^2 + y^2 + 6x - 16y + 28 = 0$$

As the equation of the required circle

18 (d)

Given that equation of parabola is $y^2 = 9x$

On comparing with $y^2 = 4ax$, we get $a = \frac{9}{4}$

Now, equation of tangent to the parabola $y^2 = 9x$ is

$$y = mx + \frac{9/4}{m} \quad \dots(i)$$

If this tangent passing through the point $(4, 10)$, then

$$10 = 4m + \frac{9}{4m}$$

$$\Rightarrow 16m^2 - 40m + 9 = 0$$

$$\Rightarrow (4m - 9)(4m - 1) = 0$$

$$\Rightarrow m = \frac{1}{4}, \frac{9}{4}$$

On putting the values of m in Eq. (i)

$$4y = x + 36 \text{ and } 4y = 9x + 4$$

$$\Rightarrow x - 4y + 36 = 0 \text{ and } 9x - 4y + 4 = 0$$

19 (a)

$$\text{Required length} = y\text{-intercept} = 2\sqrt{\frac{9}{4}} - 2 = 1$$

20 (b)

Given equation is $xy = a$

On differentiating, we get

$$x \frac{dy}{dx} + y = 0$$



$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(a,1)} = -\frac{1}{a}$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	D	A	A	B	C	D	A	C	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	B	B	C	A	A	D	A	B

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COACHING