DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :1

Topic :-LIMITS & DERIVATIVES

1 **(c)**

We have, $\lim_{\theta \to \frac{\pi}{2}} \frac{\frac{\pi}{2}-\theta}{\cot\theta} = \lim_{\theta \to \frac{\pi}{2}} \frac{-1}{-\text{cosec}^2\,\theta}$

$= \lim_{\theta \to \frac{\pi}{2}} \sin^2\theta = 1$

2 **(a)**

$\lim_{x\to 0} \frac{a^x - b^x}{e^x - 1} = \lim_{x\to 0} \frac{a^x - b^x}{x} . \frac{x}{a^x - 1}$

$= \left[\lim_{x\to 0}\left(\frac{a^x - 1}{x}\right) - \lim_{x\to 0}\left(\frac{b^x - 1}{x}\right)\right] . \lim_{x\to 0}\frac{x}{e^x - 1}$

$= (\log_e a - \log_e b) . \lim_{x\to 0} \frac{1}{\frac{e^x - 1}{x}}$

$= \log_e\left(\frac{a}{b}\right)$

3 **(b)**

$\lim_{x\to -\infty} \frac{2x - 1}{\sqrt{x^2 + 2x + 1}} = \lim_{y\to\infty} \frac{-2x - \frac{1}{y}}{\sqrt{1 - \frac{2}{y} + \frac{1}{y^2}}}$

[put $x = -y \;\; \therefore \;\; x \to -\infty \; ie, y \to \infty$]

$= -\frac{2}{1} = -2$

4 **(a)**

$\lim_{x\to 2} = \frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x - 2}$

$= \lim_{x\to 2} \frac{(1+\sqrt{2+x} - 3)}{(x-2)(\sqrt{1+\sqrt{2+x}} + \sqrt{3})}$

$= \lim_{x\to 2} \frac{x - 2}{(x-2)\left(\sqrt{1+\sqrt{2+x}} + \sqrt{3}\right)(\sqrt{2+x} + 2)}$

$= \lim_{x\to 2} \frac{1}{(\sqrt{1+\sqrt{2+x}} + \sqrt{3})(\sqrt{2+x} + 2)}$

$= \frac{1}{(\sqrt{1+2} + \sqrt{3})(\sqrt{2+2} + 2)} = \frac{1}{8\sqrt{3}}$

5 **(d)**

We have,

$\lim_{x\to 0}\left\{\frac{1}{x\sqrt[3]{8+x}} - \frac{1}{2x}\right\} \quad [\infty - \infty \text{ form}]$

$$= \lim_{x\to 0}\frac{1}{2x}\left\{\left(1+\frac{x}{8}\right)^{-1/3}-1\right\}$$

$$= \frac{1}{16}\lim_{x\to 0}\frac{\left(1+\frac{x}{8}\right)^{-1/3}-1^{-1/3}}{\left(1+\frac{x}{8}\right)-1}$$

$$= \frac{1}{16}\lim_{y\to 1}\frac{y^{-1/3}-1^{-1/3}}{y-10}, \text{where } y = 1+\frac{x}{8}$$

$$= \frac{1}{16}\times\frac{-1}{3}(1)^{-1/3-1} = -\frac{1}{48}$$

6 **(a)**

We have,

$$\lim_{x\to 0}\frac{a^x-b^x}{x}$$

$$= \lim_{x\to 0}\left\{\left(\frac{a^x-1}{x}\right)-\left(\frac{b^x-1}{x}\right)\right\} = \log(a)-\log(b) = \log\left(\frac{a}{b}\right)$$

7 **(b)**

We have,

$$\lim_{x\to 1}\frac{\sum_{r=1}^{n} x^r - n}{x-1} = \lim_{x\to 1}\frac{x-1}{x-1}+\frac{x^2-1^2}{x-1}+\frac{x^3-1^3}{x-1}+\cdots+\frac{x^n-1^n}{x-1}$$

$$= 1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

8 **(b)**

We have,

$$\lim_{x\to 1}(\log_4 5x)^{\log_x 5} = \lim_{x\to 1}(\log_5 5+\log_5 x)^{\log_x 5}$$

$$= \lim_{x\to 1}(1+\log_5 x)^{\frac{1}{\log_5 x}} = e^{\lim_{x\to 1}\log_5 x\cdot\frac{1}{\log_x 5}} = e^1 = e$$

9 **(a)**

We have,

$$\lim_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x - x} = \lim_{x\to 0}\frac{e^x\{e^{\tan x - x}-1\}}{\tan x - x}$$

$$= \lim_{x\to 0}e^x\times\lim_{x\to 0}\frac{e^{\tan x - x}-1}{\tan x - x}e^0\times 1 = 1$$

10 **(a)**

We have,

$$\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x$$

$$= \lim_{x\to\infty}\left(1+\frac{2x-1}{x^2-4x+2}\right)^x = e^{\lim_{x\to\infty}\frac{(2x-1)x}{x^2-4x+2}} = e^2$$

11 **(c)**

$$\lim_{x\to 0}\frac{\log(x+a)-\log a}{x}+k\lim_{x\to e}\frac{\log x - 1}{x-e} = 1$$

Using L' Hospital's rule

$$\lim_{x\to 0}\frac{\frac{1}{x+a}}{1}+k\lim_{x\to 0}\frac{\frac{1}{x}}{1} = 1$$

$$\Rightarrow \quad \frac{1}{a}+\frac{k}{e} = 1$$

$$\Rightarrow \quad k = e\left(1-\frac{1}{a}\right)$$

12 **(b)**

We have,

$$\lim_{x\to\infty}\frac{\sin x}{x}=\lim_{y\to 0} y\sin\left(\frac{1}{y}\right)=0$$

13 **(a)**

$$\lim_{x\to 0} x\log\sin x=\lim_{x\to 0}\frac{\log\sin x}{1/x}\left(\frac{0}{0}\text{form}\right)$$

$$=\lim_{x\to 0}\frac{\frac{1}{\sin x}\cos x}{\frac{-1}{x^2}}=\lim_{x\to 0}-\frac{x^2}{\tan x} \quad \text{[by L'Hospital's rule]}$$

$$=\lim_{x\to 0}\frac{-2x}{\sec^2 x} \quad \text{[by L'Hospital's rule]}$$

$$=0$$

14 **(c)**

$$\lim_{x\to 0}\frac{d}{dx}\int\left(\frac{1-\cos x}{x^2}\right)dx=\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x/2}{4.x^2/4}$$

$$=\frac{1}{2}\lim_{x\to 0}\left(\frac{\sin x/2}{x/2}\right)^2=\frac{1}{2}$$

15 **(a)**

We have,

$$\lim_{x\to 0}\frac{\left(\int_y^a e^{\sin^2 t}dt-\int_{x+y}^a e^{\sin^2 t}\,dt\right)}{x}$$

$$=\lim_{x\to 0}\frac{\left(\int_y^a e^{\sin^2 t}dt+\int_a^{x+y} e^{\sin^2 t}\,dt\right)}{x}$$

$$=\lim_{x\to 0}\frac{\int_y^{x+y} e^{\sin^2 t}\,dt}{x}$$

$$=\lim_{x\to 0}\frac{\frac{d}{dx}(x+y)e^{\sin^2(x+y)}-0}{1} \quad \text{[Using L' Hospital's Rule]}$$

$$=\lim_{x\to 0}1.e^{\sin^2(x+y)}=e^{\sin^2 y}$$

16 **(c)**

We have,

$$\lim_{x\to 0}\frac{2f(x)-3f(2x)+f(4x)}{x^2}\left[\text{Form}\frac{0}{0}\right]$$

$$=\lim_{x\to 0}\frac{2f'(x)-6f'(2x)+4f'(4x)}{2x} \quad \text{[By L' Hospital's Rule]}$$

$$=\lim_{x\to 0}\frac{2f'(x)-3f'(2x)+2f'(4x)}{x}\left[\text{Form}\frac{0}{0}\right]$$

$$=\lim_{x\to 0}\frac{2f''(x)-6f''(2x)+8f''(4x)}{1} \quad \text{[By L' Hospital's Rule]}$$

$$=f''(0)-6f''(0)+8f''(0)=3f''(0)=3\times 4=12$$

17 **(d)**

Given, $\lim_{x\to 0}\frac{\{(a-n)nx-\tan x\}\sin nx}{x^2}=0$

$$\Rightarrow \quad \lim_{x\to 0}\left((a-n)n-\frac{\tan x}{x}\right).\frac{\sin nx}{x}=0$$

$$\Rightarrow \quad [\{a-n\}n-1]n=0$$

$$\Rightarrow \quad a=n+\frac{1}{n}$$

18 **(c)**

We have,

$$\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$$

$$\Rightarrow \lim_{x\to 0} \frac{x\left\{1+a\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots\right)\right\}-b\left\{x-\frac{x^3}{3!}+\frac{x^5}{5!}\ldots\right\}}{x^3}=1$$

$$\Rightarrow \lim_{x\to 0} \frac{(1+a-b)+x^2\left(\frac{b}{3!}-\frac{a}{2!}\right)+x^4\left(\frac{a}{4!}-\frac{b}{5!}\right)+\cdots}{x^2}=1 \quad \ldots\text{(i)}$$

If $1+a-b\neq 0$, then LHS $\to\infty$ as $x\to 0$ which RHS $=1$

$\therefore 1+a-b=0$

From (i), we have

$$\lim_{x\to 0} \frac{x^2\left(\frac{b}{3!}-\frac{a}{2!}\right)+x^4\left(\frac{a}{4!}-\frac{b}{5!}\right)+\cdots}{x^2}=1$$

$$\therefore \frac{b}{3!}-\frac{a}{2!}=1 \Rightarrow b-3a=6$$

Solving $1+a-b=0$ and $b-3a=6$, we get $a=-\frac{5}{2}, b=-\frac{3}{2}$

19 **(a)**

$$\lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x=\lim_{x\to\infty}\left(1+\frac{2x-1}{x^2-4x+2}\right)^x$$

$$=e^{\lim_{x\to\infty}\left(\frac{x(2x-1)}{x^2-4x+2}\right)}=e^2$$

20 **(d)**

$$\lim_{x\to 0}\frac{(1-\cos 2x)}{x^2}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x}{x^2}$$

$$=2\lim_{x\to 0}\left(\frac{\sin x}{x^2}\right)^2=2$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	B	A	D	A	B	B	A	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	A	C	A	C	D	C	A	D