DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : Maths
DPP No. : 2

Topic :-Binominal Theorem

1 **(c)**

Let $S = 1 + \frac{2\cdot1}{3\cdot2} + \frac{2\cdot5}{3\cdot6}\left(\frac{1}{2}\right)^2 + \frac{2\cdot5\cdot8}{3\cdot6\cdot9}\left(\frac{1}{2}\right)^3 + \ldots$

$= 1 + \frac{\frac{2}{3}}{1}\left(\frac{1}{2}\right) + \frac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)}{2!}\left(\frac{1}{2}\right)^2 + \frac{\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)\left(\frac{8}{3}\right)}{3!}\left(\frac{1}{2}\right)^3 + \ldots$

$= \left(1 - \frac{1}{2}\right)^{-2/3} = \left(\frac{1}{2}\right)^{-2/3} = 2^{2/3} = 4^{1/3}$

$\left[\because (1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \ldots\right]$

2 **(a)**

rth term in the expansion of $\left(3x - \frac{2}{x^2}\right)^{15}$ is

$T_r = {}^{15}C_{r-1}(3x)^{15-r+1}\left(\frac{-2}{x^2}\right)^{r-1}$

$= {}^{15}C_{r-1}(3)^{15-r+1}(-2)^{r-1}(x)^{15-3r+3}$

For the term independent of x, put

$15 - 3r + 3 = 0 \Longrightarrow r = 6$

3 **(a)**

We have, $\sum_{r=0}^{n}\sum_{s=0}^{n}(r+s)(C_r + C_s)$

$= \sum_{r=0}^{n}\sum_{s=0}^{n}(rC_r + rC_s + sC_r + sC_s)$

$= \sum_{r=0}^{n}\left[\sum_{s=0}^{n} rC_r + r\sum_{s=0}^{n} C_s + C_s\sum_{s=0}^{n} s + \sum_{s=0}^{n} sC_s\right]$

$= \sum_{r=0}^{n}\left[(n+1)r\cdot C_r + r2^n + \frac{n(n+1)}{2}C_r + n\cdot 2^{n-1}\right]$

$= (n+1)n\cdot 2^{n-1} + (2^n)\frac{n(n+1)}{2} + \frac{n(n+1)}{2}2^n + n2^{n-1}(n+1)$

$= n(n+1)2^n + n(n+1)2^n$

$= 2n(n+1)2^n$ ...(i)

Also, $\sum_{r=0}^{n}\sum_{s=0}^{n}(r+s)(C_r + C_s)$

$= \sum_{r=0}^{n} 4rC_r + 2\sum_{0\le r<s\le n}\sum (r+s)(C_r + C_s)$

$\therefore\ 2n(n+1)2^n = 4n\cdot 2^{n-1} + 2\sum_{0\le r<s\le n}\sum (r+s)(C_r + C_s)$

$$\Rightarrow \sum_{0\le r<s\le n}\sum (r+s)(C_r+C_s) = n^2\cdot 2^n$$

5 **(a)**

We know,

$(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_rx^r + \ldots$ ...(i)

and $\left(1+\frac{1}{x}\right)^n = C_0 + C_1\frac{1}{x} + C_2\frac{1}{x^2} + \ldots + C_r\frac{1}{x^r}$

$+C_{r+1}\frac{1}{x^{r+1}} + C_{r+2}\frac{1}{x^{r+2}}\ldots C_n\frac{1}{x^n}$ ...(ii)

On multiplying Eqs. (i) and (ii), equation coefficient of x^r in $\frac{1}{x^n}(1+x)^{2n}$ or the coefficient of x^{n+r} in $(1+x)^{2n}$, we get the value of required expression which is

$${}^{2n}C_{n+r} = \frac{(2n)!}{(n-r)!\,(n+r)!}$$

7 **(b)**

In $(x+a)^{100} + (x-a)^{100}$ n is even

$\therefore$ Total number of terms $= \frac{n}{2} + 1 = \frac{100}{2} + 1 = 51$

8 **(b)**

Given polynomial is

$(x-1)(x-2)(x-3)\ldots(x-19)(x-20)$

$= x^{20} - (1+2+3+\cdots+19+20)x^{19}$

$+(1\times 2 + 2\times 3 + \cdots + 19\times 20)x^{18}$

$-\cdots + (1\times 2\times 3\times 4\times \ldots \times 19\times 20)$

$\therefore$ Coefficient of $x^{19} = -(1+2+3+\ldots+19+20)$

$= -\left[\frac{20}{2}(1+20)\right]$

$= -10\times 21 = -210$

9 **(c)**

We know that,

${}^{15}C_0 + {}^{15}C_1 + {}^{15}C_2 + \cdots + {}^{15}C_{15} = 2^{15}$

$\Longrightarrow 2({}^{15}C_8 + {}^{15}C_9 + \cdots + {}^{15}C_{15})2^{15}$ $[\because {}^nC_r = {}^nC_{n-r}]$

$\Longrightarrow {}^{15}C_8 + {}^{15}C_9 + \cdots + {}^{15}C_{15} = 2^{14}$

10 **(c)**

The number of terms in the expansion of $(a+b+c)^n$

$$= \frac{(n+1)(n+2)}{2}$$

11 **(c)**

We have,

$T_{r+1} = {}^5C_r(y^2)^{5-r}\left(\frac{c}{y}\right)^r = {}^5C_r y^{10-3r}\, c^r$

This will contain y, if $10 - 3r = 1 \Rightarrow r = 3$

$\therefore$ Coefficient of $y = {}^5C_3\, c^3 = 10\, c^3$

12 **(b)**

$\because$ $(0.99)^{15} = (1-0.01)^{15}$

$= 1 - {}^{15}C_1(0.01) + {}^{15}C_2(0.01)^2 - {}^{15}C_3(0.01)^3 + \ldots$

We want to answer correct upto 4 decimal places and as such, we have left further expansion.

$= 1 - 15(0.01) + \frac{15 \cdot 14}{1 \cdot 2}(0.0001) - \frac{15 \cdot 14 \cdot 13}{1 \cdot 2 \cdot 3}(0.000001) + \ldots$

$= 1 - 0.15 + 0.0105 - 0.000455 + \ldots$

$= 1.0105 - 0.150455$

$= 0.8601$

13 **(b)**

By hypothesis, $2^n = 4096 = 2^{12} \Rightarrow n = 12$

Since, n is even, hence greatest coefficient

$= {}^nC_{n/2} = {}^{12}C_6 = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = 924$

14 **(c)**

Given that, ${}^nC_{r-1} = {}^nC_{r+1}$

$\Rightarrow \frac{n!}{(n-r+1)(n-r)(n-r-1)!\,(r-1)!}$

$= \frac{n!}{(n-r-1)!\,(r+1)(r)(r-1)!}$

$\Rightarrow r^2 + r = n^2 - nr + n - nr + r^2 - r$

$\Rightarrow n^2 - 2nr - 2r + n = 0$

$\Rightarrow (n-2r)(n+1) = 0 \Rightarrow r = \frac{n}{2}$

15 **(d)**

It is given that

${}^nC_1\, x^{n-1} a^1 = 240$ ... (i)

${}^nC_2\, x^{n-2}\, a^2 = 720$ ... (ii)

${}^nC_3\, x^{n-3}\, a^3 = 1080$ ... (iii)

From (i), (ii) and (iii)

$\frac{({}^nC_2)^2\, x^{2n-4} a^4}{{}^nC_1\, {}^nC_3\, x^{2n-4}\, a^4} = \frac{720 \times 720}{240 \times 1080}$

$\Rightarrow \frac{6\, n^2 (n-1)^2}{4\, n^2 (n-1)(n-2)} = 2$

$\Rightarrow \frac{3\,(n-1)}{2\,(n-2)} = 2$

$\Rightarrow 3n - 3 = 4n - 8 \Rightarrow n = 5$

16 **(d)**

$\frac{1}{81^n}(1 - 10 \cdot {}^{2n}C_1 + 10^2 \cdot {}^{2n}C_2 - 10^3 \cdot {}^{2n}C_3 + \cdots + 10^{2n})$

$= \frac{1}{(81)^n}(1-10)^{2n} = 1$

17 **(b)**

We have,

$(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_{2n} x^{2n}$

On differentiating both sides, we get

$n(1 - 1 + 1)^{n-1}(1 + 2x) = a_1 + 2a_2 x + 3a_3 x^2$
$+ \ldots + 2n a_{2n} x^{2n-1}$

On putting $x = -1$ we get

$n(1-1+1)^{n-1}(1-2)=a_1-2a_2+3a_3-\ldots-2na_{2n}$
$\Rightarrow a_1-2a_2+3a_3-\ldots-2na_{2n}=-n$

18 **(a)**

Since $(n+1)^{\text{th}}$ term is the middle term in the expansion of $(1+x)^{2n}$
∴ Coefficient of the middle term

$={}^{2n}C_n=\frac{(2n)!}{n!n!}$

$=\frac{(1\cdot3\cdot5\ldots(2n-1)(2\cdot4\cdot6\ldots(2n-2)(2n))}{n!n!}$

$=\frac{1\cdot3\cdot5\ldots.(2n-1)2^n n!}{n!n!}=\frac{1\cdot3\cdot5\ldots(2n-1)2^n}{n!}$

19 **(a)**

We have,

$(1+x)^{10}\left(1+\frac{1}{x}\right)^{12}=\frac{(1+x)^{22}}{x^{12}}$

∴ Constant term in $(1+x)^{10}\left(1+\frac{1}{x}\right)^{12}$

$=$ Coefficient of x^{12} in $(1+x)^{22}$

$={}^{22}C_{12}={}^{22}C_{10}$

20 **(b)**

Given, $a_n=na_{n-1}$

For $n=2$

$a_2=2a_1=2 \quad (\because a_1=1\text{given})$

$a_3=3a_2=3(2)=6$

$a_4=4(a_3)=4(6)=24$

$a_5=5(a_4)=5(24)=120$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	A	C	A	D	B	B	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	B	C	D	D	B	A	A	B