CLASS : XIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :2

Topic :-SEQUENCES AND SERIES

1   **(c)**

Given, sum

$$= (x + 2)^{n-1} \left\{ 1 + \left(\frac{x+1}{x+2}\right) + \left(\frac{x+1}{x+2}\right)^2 + \ldots + \left(\frac{x+1}{x+2}\right)^{n-1} \right\}$$

$$= (x + 2)^{n-1} \left\{ \frac{1 - \left(\frac{x+1}{x+2}\right)^n}{1 - \left(\frac{x+1}{x+2}\right)} \right\}$$

$$= \frac{(x + 2)^{n-1}\{(x+2)^n - (x+1)^n\} \cdot (x+2)}{(x+2)^n}$$

$$= (x + 2)^n - (x + 1)^n$$

2   **(a)**

Given $b^2 = ac$, $x = \frac{a+b}{2}$ and $y = \frac{b+c}{2}$

$$\therefore \frac{1}{x} + \frac{1}{y} = \frac{2}{a+b} + \frac{2}{b+c}$$

$$= \frac{2(2b + a + c)}{ab + b^2 + bc + ac}$$

$$= \frac{2(2b + a + c)}{ab + 2b^2 + bc}$$

$$= \frac{2(2b + a + c)}{b(2b + a + c)}$$

$$= \frac{2}{b}$$

3   **(c)**

$\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots$

$1 \times \sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} + \ldots$

$= \sqrt{2}(1 + 2 + 3 + 4 + \ldots \text{upto 24 terms})$

$$= \sqrt{2} \times \frac{24 \times 25}{2} = 300\sqrt{2} \quad \left[\because \Sigma n = \frac{n(n = 1)}{2} \right]$$

4   **(c)**

Let $S = 1^3 + 2^3 + 3^3 + \ldots + 15^3$

$$= \sum_{n=1}^{15} n^3 = \left(\frac{15(15 + 1)}{2}\right)^2$$

$$= 14400$$

6   **(c)**

Since, $\Sigma n = \left(\frac{1}{5}\right) \Sigma n^2$

$$\Rightarrow \frac{n(n + 1)}{2} = \frac{1}{5} \frac{n(n + 1)(2n + 1)}{6}$$

$\Rightarrow 2n + 1 = 15 \Rightarrow n = 7$

7 (a)

We know that, $\dfrac{e^x + e^{-x}}{2} = 1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \dfrac{x^6}{6!} + \ldots$

Put $x = \dfrac{1}{2}$, we get

$\dfrac{e^{1/2} + e^{-1/2}}{2} = 1 + \left(\dfrac{1}{2}\right)^2 \dfrac{1}{2!} + \left(\dfrac{1}{2}\right)^4 \dfrac{1}{4!} + \ldots$

$\Rightarrow \dfrac{e + 1}{2\sqrt{e}} = 1 + \dfrac{1}{4.2!} + \dfrac{1}{16.4!} + \ldots \infty$

8 (b)

Let the three distinct real numbers in G.P. be a, ar, ar^2, where $r \neq \pm 1$

It is given that

$a^2 + a^2 r^2 + a^2 r^4 = S^2$

And,

$a + ar + ar^2 = \alpha S$

$\therefore \dfrac{a^2(1 + r + r^2)^2}{a^2(1 + r^2 + r^4)} = \dfrac{\alpha^2 S^2}{S^2}$

$\Rightarrow \dfrac{(1 + r + r^2)^2}{(r^2 + r + 1)(r^2 - r + 1)} = \alpha^2$

$\Rightarrow \dfrac{r^2 + r + 1}{r^2 - r + 1} = \alpha^2$

$\Rightarrow r^2(\alpha^2 - 1) - r(\alpha^2 + 1) + (\alpha^2 - 1) = 0$

$\Rightarrow (\alpha^2 + 1)^2 - 4(\alpha^2 - 1)^2 \geq 0$ [$\because r$ is real]

$\Rightarrow (3\alpha^2 - 1)(\alpha^2 - 3) \leq 0 \Rightarrow \dfrac{1}{3} \leq \alpha^2 \leq 3$

But, $\alpha^2 = 3$ for $r = 1$ and $\alpha^2 = \dfrac{1}{3}$ for $r = -1$

$\therefore \dfrac{1}{3} < \alpha^2 < 3$ i.e. $\alpha^2 \in (1/3, 3)$

9 (c)

Using, $S_\infty = \dfrac{a}{1-r} + \dfrac{dr}{(1-r)^2}$

Here, $a = 1$, $r = \dfrac{1}{5}$, $d = 3$

$\therefore S_\infty = \dfrac{1}{1 - \frac{1}{5}} + \dfrac{3 \times \frac{1}{5}}{\left(1 - \frac{1}{5}\right)^2}$

$= \dfrac{5}{4} + \dfrac{3}{5 \times \frac{16}{25}}$

$= \dfrac{5}{4} + \dfrac{15}{16} = \dfrac{35}{16}$

10 (b)

Since a, b, c are in HP.

$\therefore b = \dfrac{2ac}{a + c}$

$\Rightarrow \dfrac{b}{a} = \dfrac{2c}{a + c}$

$\Rightarrow \dfrac{b + a}{b - a} = \dfrac{3c + a}{c - a}$ [Applying componendo dividendo]

Again,

$b = \dfrac{2ac}{a + c}$

$$\Rightarrow \frac{b}{c} = \frac{2a}{a+c}$$

$$\Rightarrow \frac{b+c}{b-c} = \frac{3a+c}{a-c}$$

$$\therefore \frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{3c+a}{c-a} + \frac{3a+c}{a-c} = 2$$

11 **(a)**

Here, $T_n = 1 + 2 + 2^2 + 2^3 + \ldots + 2^n$

$$= \frac{1(2^{n+1} - 1)}{2-1} = 2^{n+1} - 1$$

$$\therefore \Sigma T_n = \Sigma(2^{n+1} - 1) = \Sigma 2^{n+1} - \Sigma 1$$

$$= (2^2 + 2^3 + \ldots + 2^{n+1}) - n$$

$$= 2(2 + 2^2 + \ldots + 2^n) - n$$

$$= \frac{4(2^n - 1)}{2-1} - n = 2^{n+2} - n - 4$$

12 **(b)**

$$\frac{3}{4.8} - \frac{3.5}{4.8.12} + \frac{3.5.7}{4.8.12.16} - \ldots + \frac{3}{4} - \frac{3}{4}$$

$$= 1 - \frac{1}{4} + \frac{1.3}{2.4.4} - \frac{1.3.5}{4.4.2.4.3} + \ldots - \frac{3}{4}$$

$$= \left[1 + \frac{1}{1!}\left(-\frac{1}{4}\right) + \frac{1(1+2)}{2!}\left(-\frac{1}{4}\right)^2 + \frac{1(1+2)(1+4)}{3!}\left(-\frac{1}{4}\right)^3 + \ldots \right] - \frac{3}{4}$$

$$= \left(1 - \frac{1}{4}\right)^{-1/2} - \frac{3}{4} = \sqrt{\frac{2}{3}} - \frac{3}{4}$$

13 **(c)**

$$\therefore 1 - \log 2 + \frac{(\log 2)^2}{2!} - \frac{(\log 2)^3}{3!} + \cdots = e^{-\log 2}$$

$$= e^{\log 2^{-1}} = \frac{1}{2}$$

14 **(c)**

Let S_n and S' be the sums of n terms of two AP's and T_{11} and T'_{11} be the respective 11th term, then

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}(2a' + (n-1)d')} = \frac{7n+1}{4n+27} \quad \text{(given)}$$

$$\Rightarrow \frac{a + \frac{(n-1)}{2}d}{a' + \frac{(n-1)}{2}d'} = \frac{7n+1}{4n+27}$$

Now put, $n = 21$, we get

$$\frac{a + 10d}{a' + 10d'} = \frac{T_{11}}{T'_{11}} = \frac{148}{111}$$

$$= \frac{4}{3}$$

15 **(a)**

$$2.\overline{357} = 2 + 0.357 + 0.000357 + \ldots$$

$$\Rightarrow 2.\overline{357} = 2 + \frac{357}{10^3} + \frac{357}{10^6} + \ldots$$

$$\Rightarrow 2.\overline{357} = 2 + \frac{\frac{357}{10^3}}{1 - \frac{1}{10^3}} = 2 + \frac{357}{999} = \frac{2355}{999}$$

16 (c)

Here, $T_n = \frac{1+2+3+\dots+n}{n!} = \frac{n(n+1)}{2(n)!} \left[\because \sum n = \frac{n(n+1)}{2} \right]$

$= \frac{(n+1)}{2(n-1)!} = \frac{1}{2(n-2)!} + \frac{1}{(n-1)!}$

$T_1 = 0 + \frac{1}{1}$

$T_2 = \frac{1}{2} \cdot \frac{1}{1} + \frac{1}{1!}$

$T_3 = \frac{1}{2} \cdot \frac{1}{1} + \frac{1}{2!}$

$T_4 = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{3!}$

$\quad \vdots \quad \vdots$
$\quad \vdots \quad \vdots$

$\therefore S = \Sigma T_n$

$= \frac{1}{2}\left(1 + \frac{1}{1!} + \frac{1}{2!} + \dots\right) + \left(1 + \frac{1}{1!} + \frac{1}{2!} + \dots\right)$

$= \frac{1}{2}e + e = \frac{3}{2}e$

17 (b)

Let $S = 3 + 7 + 13 + 21 + \dots + T_n$

$\Rightarrow T_n = n^2 + n + 1$

Let $T_r = \cot^{-1}(r^2 + r + 1)$

$= \tan^{-1}(r+1) - \tan^{-1} r$

Put $r = 1, 2, \dots, n$

$T_1 = \tan^{-1} 2 - \tan^{-1} 1$

$T_2 = \tan^{-1} 3 - \tan^{-1} 2$

$\quad \dots \dots \dots$
$\quad \dots \dots \dots$

$T_n = \tan^{-1}(n+1) - \tan^{-1} n$

On adding all these, we get

$T_1 + T_2 + \dots + T_n = \tan^{-1}(n+1) - \tan^{-1} 1 = \tan^{-1}\left(\frac{n}{n+2}\right) = \cot^{-1}\left(\frac{n+2}{n}\right)$

19 (a)

Let $x^{18} = y^{21} = z^{28} = k$

Then,

$18 \log x = 21 \log y = 28 \log z = \log k$

$\Rightarrow \log_y x = \frac{21}{18}, \log_z y = \frac{28}{21}, \log_x z = \frac{18}{28}$

$\Rightarrow 3 \log_y x = \frac{7}{2}, 3 \log_z y = 4, 7 \log_x z = \frac{9}{2}$

$\Rightarrow 3, 3 \log_y z, 3 \log_z y, 7 \log_x z$ are in A.P.

20 (b)

For $0 < x < \pi/2$, we have $0 < \sin^2 x < 1$

$\therefore y = \exp[(\sin^2 x + \sin^4 x + \sin^6 x + \cdots \infty) \log_e 2]$

$\Rightarrow y = \exp\left[\left(\frac{\sin^2 x}{1 - \sin^2 x}\right) \log_e 2\right] = \exp[\tan^2 x \log_2 2]$

$\Rightarrow y = e^{\log_e 2^{\tan^2 x}} = 2^{\tan^2 x}$

Since y satisfies the equation $x^2 - 9x + 8 = 0$. Therefore,

$y^2 - 9y + 8 = 0 \Rightarrow (y-1)(y-8) = 0 \Rightarrow y = 1$ or, $y = 8$

Now,

$y = 1 \Rightarrow 2^{\tan^2 x} = 1 \Rightarrow 2^{\tan^2 x} = 2^0 \Rightarrow \tan x = 0 \Rightarrow x = 0$
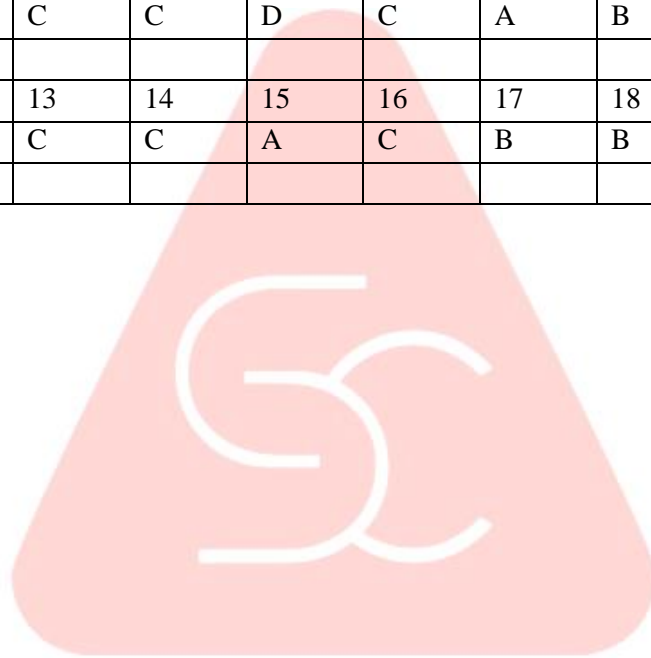
But, $0 < x < \pi/2$. Therefore, $y \neq 1$.

Consequently, we have

$y = 8 \Rightarrow 2^{\tan^2 x} = 2^3 \Rightarrow \tan^2 x = 3 \Rightarrow \tan x = \sqrt{3} \Rightarrow x = \pi/3$

$\therefore \dfrac{\sin x + \cos x}{\sin x - \cos x} = \dfrac{\tan x + 1}{\tan x - 1} = \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1} = 2 + \sqrt{3}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	C	C	D	C	A	B	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	C	C	A	C	B	B	A	B