

Class : XIth Subject : MATHS Date : **221 | 2010 | DPP** No. :2 **Solutions**

Topic :-STRAIGHT LINES

221 **(c)** Let the coordinate of M are (x_1, y_1) Since, the line *PM* is perpendicular to the given line $x + y = 3$ $\therefore \frac{y_1 - 3}{2}$ $\frac{51}{x_1 - 2}$ × (-1) = -1 \Rightarrow $y_1 - 3 = x_1 - 2$ \Rightarrow $x_1 - y_1 + 1 = 0$...(i) $\frac{x+y=3}{M(x_1, y_1)}$ and also the point lies on the given line. ∴ $x_1 + y_1 - 3 = 0$...(ii) On solving Eqs. (i) and (ii), we get $x_1 = 1, \quad y_1 = 2$ ∴ The coordinates of *M* are $(1, 2)$. 222 **(b)** The equation of line in new **position** is $y - 0 = \tan 15^{\circ} (x - 2)$ \Rightarrow $y = (2 - \sqrt{3})(x - 2)$ \Rightarrow $(2 - \sqrt{3})x - y - 4 + 2\sqrt{3} = 0$ 223 **(d)** Here $a = 1$, $h = 1$, $f = -4a$, $g = -4a$, $c = -9a$ Now, required distance $=\left|2\right|\frac{f^2-bc}{b(b+c)}$ $\frac{b(b+a)}{b(b+a)}$ \subset $=\left|2\right|\frac{16a^2+9a^2}{1(1+a)}$ $\frac{1(1+1)}{1(1+1)}$ $=$ $\left|2\right| \frac{25a^2}{2}$ 5 $\left| \frac{1}{2} \right|$ = ∙ 2 √2 $= 5\sqrt{2}a$ 224 **(c)** Let *ABC* be the equilateral triangle with centroid $O(0,0)$ and sides *BC* as $x + y - 2 = 0$. $0 + 0 - 2$ \therefore OD = $\left| \frac{1}{\sqrt{1^2+1^2}} \right| = \sqrt{2} \Rightarrow OA = 2\sqrt{2}$

Since AD is perpendicular to BC . Therefore, Slope of $AD = 1$

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 \Rightarrow AD makes 45° with *X*-axis

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B
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Clearly, A lies on 0A at a distance of $2\sqrt{2}$ units from 0. So, its coordinates are given by $x - 0$ $y - 0$ $\frac{1}{\cos \pi/4}$ $\frac{y}{\sin \pi/4} = \pm 2\sqrt{2} \Rightarrow x = \pm 2, y = \pm 2$ But, 0 and A lie on the same side of $x + y - 2 = 0$ Hence, the coordinates of A are $(-2, -2)$ 225 **(c)** The intersection point of lines $x - 2y = 1$ and $x + 3y = 2$ is 7 1 ($\frac{1}{5}$ $\frac{1}{5}$ Since, required is parallel to $3x + 4y = 0$ Therefore, the slope of required line = $-\frac{3}{4}$ 4 ∴ Equation of required line which pass<mark>es through (7</mark> $\frac{7}{5}, \frac{1}{5}$ $\frac{1}{5}$ and having slope $-\frac{3}{4}$, is 4 1 −3 7 $\mathcal{Y} \frac{1}{5}$ = $\frac{1}{4}$ $\left(x \frac{1}{5}$ $3x$ 21 1 ⇒ $\frac{1}{4} + y =$ $\frac{1}{20}$ + 5 $3x + 4y$ $21 + 4$ ⇒ $\frac{1}{4}$ = 20 \Rightarrow 3x + 4y = 5 \Rightarrow 3x + 4y - 5 = 0 226 **(b)** Required ratio is given by $3 \times 1 + 3 - 9$ $-\frac{1}{3 \times 2 + 7 - 9}$ TILE 3 = $\frac{1}{4}$ *ie*, 3: 4 internally 227 **(d)** The lines $4 x - 7 y + 10 = 0$ and $7 x + 4 y - 15 = 0$ are perpendicular and their point of intersection is $(1,2)$. Hence, the orthocentre is at (1,2) 228 **(b)** Since the distance between the parallel lines $lx + my + n = 0$ and $lx + my + n' = 0$ is same as the distance between parallel lines $mx + ly + n = 0$ and $mx + ly + n' = 0$. Therefore, the parallelogram is a rhombus. Also, the diagonals of a rhombus are at right angles. Therefore, the required angle is a right angle. 229 **(a)** Vertices are interception points of line $x + y = 2\sqrt{2}$...(i) with $y = x \tan(105^\circ)$ or $y = x \tan(165^\circ)$ (lines through centroid) $y = -x \tan 75^{\circ}$...(ii)

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For the interception point of Eqs. (i) and (ii) $x - x(2 + \sqrt{3}) = 2\sqrt{2}$ $\Rightarrow -x(1+\sqrt{3}) = 2\sqrt{2}$ $\Rightarrow x = -\frac{2\sqrt{2}(1-\sqrt{3})}{\sqrt{2}\sqrt{3}}$ $(1 + \sqrt{3})(1 - \sqrt{3})$ \Rightarrow $x = \sqrt{2} - \sqrt{6}$ ∴ $y = -(\sqrt{2} - \sqrt{6})(2 + \sqrt{3})$ $= -(2\sqrt{2} + \sqrt{6} - 2\sqrt{6} - 3\sqrt{2})$ $=\sqrt{2} + \sqrt{6}$ and its image about $y = x$ is $(\sqrt{2} + \sqrt{6}, \sqrt{2} - \sqrt{6})$ 230 **(a)** It is given that the lines $ax + 2y + 1 = 0$, $bx + 3y + 1 = 0$ and $cx + 4y + 1 = 0$ are concurrent \therefore |b 3 1| = 0 $|a \ 2 \ 1|$ $\begin{vmatrix} c & 4 & 1 \end{vmatrix}$ \Rightarrow $-a + 2b - c = 0 \Rightarrow 2b = a + c \Rightarrow a, b, c$ are in A.P. 231 **(b)** Let (h, k) be the centroid of the triangle whose vertices are $(a \cos t, a \sin t)$, $(b \sin t, -b \cos t)$ and $(1,0)$. Then, $3h = a \cos t + b \sin t + 1$ and $3k = a \sin t - b \cos t$ \Rightarrow $(3h-1)^2 + (3k)^2 = a^2 + b^2$ Hence, the locus of (h, k) is $(3x - 1)^2 + (3y)^2 = a^2 + b^2$ 234 **(c)** The equation representing the bisectors of the angles between the lines given by $ax^2 + 2hxy + by^2 = 0$ is $x^2 - y^2$ $\frac{b}{a-b} =$ xy ℎ $\Rightarrow hx^2 - (a - b)xy - hy^2 = 0$...(i) The combined equation of the bisectors of the angles between these lines is $x^2 - y^2$ $\frac{b}{h+h}$ = xy $-\frac{(a-b)}{2}$ 2 $\Rightarrow (a - b)(x^2 - y^2) + 4hxy = 0$ 235 **(a)** Given, $\sqrt{3} \sin \theta + 2 \cos \theta =$ 4 $\frac{1}{r}$... (i) Any line perpendicular to Eq.(i) is \Rightarrow √3 cos θ − 2 sin θ = \boldsymbol{k} \mathbf{r} It passes through (−1, π $\frac{1}{2}$), then √3 cos π $\frac{1}{2}$ – 2 sin π $\frac{1}{2}$ \boldsymbol{k} −1 $-2 =$ \boldsymbol{k} $\frac{1}{-1} \Rightarrow k = 2$ Thus, the equation is √3 cos θ − 2 sin θ = 2 r \Rightarrow $\sqrt{3}r \cos \theta - 2r \sin \theta = 2$ 236 **(b)** $P = |$ $a(4-3+4)+b(2+6-3)$ $\sqrt{(2a+b)^2 + (a-2b)^2}$ = $\sqrt{10}$

 \Rightarrow 25(a + b)² = 10(5a² + 5b²)

 \Rightarrow 25 $(a - b)^2 = 0 \Rightarrow a = b$ Only one line which is $3x - y + 1 = 0$ 237 **(b)** Let $\left(t, \frac{5-2t}{11}\right)$ be a point on the line $2x + 11y = 5$ Then, $p_1 = |$ $24t + 7\left(\frac{5-2t}{11}\right) - 20$ $\frac{(11)^{2}}{\sqrt{24^2+7^2}}$ = $|50t - 37|$ 55 and, $p_2 = |$ $4t - 3\left(\frac{5 - 2t}{11}\right) - 2$ $\frac{(11)^{2}}{\sqrt{4^{2}+(-3)^{2}}}$ = $|50t - 37|$ 55 Clearly, we have $p_1 = p_2$ ALITER Clearly, $2x + 11y = 5$ is the angle bisector of the two lines. Therefore, $p_1 = p_2$ 238 **(c)** The equation of lines are $\pm x \pm y = 0$. Now, we take the lines $x + y = 0$ and $x - y = 0$. ∴ The equation of bisector of the angles between these lines are $x + y$ $\sqrt{1}+1$ $=$ \pm $x - y$ $\sqrt{1}+1$ \Rightarrow $x + y = \pm (x - y)$ Taking positive sign, $x + y = x - y \Rightarrow y = 0$ Taking negative sign, $x + y = -(x - y) \Rightarrow x = 0$ 239 **(c)** Given pair of lines are $x^2 - 3xy + 2y^2 = 0$ and $x^2 - 3xy + 2y^2 + x - 2 = 0$ ∴ $(x - 2y)(x - y) = 0$ and $(x - 2y + 2)(x - y - 1) = 0$ \Rightarrow $x - 2y = 0, x - y = 0$ and $x - 2y + 2 = 0, x - y - 1 = 0$ Since, the lines $x - 2y = 0$, $x - 2y + 2 = 0$ and $x - y = 0$, $x - y - 1 = 0$ are parallel. Also, angle between $x - 2y = 0$ and $x - y = 0$ is not 90° ∴ It is a parallelogram. 240 **(b)** Let a and b the intercepts made by the straight line on the axes Given that, $a + b =$ ab 2 ⇒ $2a + 2b$ $\frac{1}{ab} = 1 \Rightarrow$ 2 $\frac{1}{a}$ 2 $\frac{1}{b} = 1$ On comparing with $\frac{d}{dx} + \frac{dy}{dx}$ $\frac{y}{b}$ = 1, we get $x = 2, y = 2$ ∴ Required point is (2, 2) So, the straight line passes through the point (2, 2)

