DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

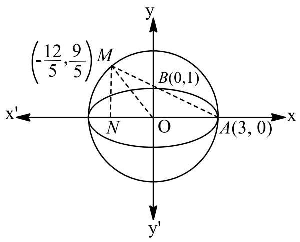
SUBJECT : MATHS
DPP NO. : 2

Topic :- CONIC SECTION

2 **(d)**

Equation of auxiliary circle is

$x^2+y^2=9$ ... (i)



Equation of AM is $\frac{x}{3}+\frac{y}{1}=1$ ... (ii)

On solving Eqs. (i) and (ii) , we get $M\left(-\frac{12}{5}, \frac{9}{5}\right)$

Now, area of $\Delta AOM=\frac{1}{2}.OA\times MN$

$=\dfrac{27}{10}$ sq unit

3 **(d)**

Equation of tangent to $y^2=4x$ is $y=mx+\frac{1}{m}$

Since, tangent passes through $(-1,-6)$

$\therefore -6=-m+\dfrac{1}{m} \Rightarrow m^2-6m-1=0$

Here, $m_1m_2=-1$

$\therefore$ Angle between them is $90°$

4 **(b)**

The equation of the ellipse is

$4(x^2+4x+4)+9(y^2-2y+1)=36 \Rightarrow \dfrac{(x+2)^2}{3^2}+\dfrac{(y-1)^2}{2^2}=1$

So, the coordinates of the centre are $(-2,1)$

5 **(c)**

The two circles are

$x^2+y^2-2\,ax+c^2=0$ and $x^2+y^2-2\,by+c^2=0$

Centres and radii of these two circles are :

Centres : $C_1(a,0)$ $\quad C_2(0,b)$

Radii : $r_1=\sqrt{a^2-c^2}$ $\quad r_2=\sqrt{b^2-c^2}$

Since the two circles touch each other externally.

$\therefore C_1C_2=r_1+r_2$

$\Rightarrow \sqrt{a^2+b^2}=\sqrt{a^2-c^2}+\sqrt{b^2-c^2}$

$\Rightarrow a^2+b^2=a^2-c^2+b^2-c^2+2\sqrt{a^2-c^2}\sqrt{b^2-c^2}$

$\Rightarrow c^4=a^2b^2-c^2(a^2+b^2)+c^4$

$\Rightarrow a^2b^2=c^2(a^2+b^2) \Rightarrow \dfrac{1}{a^2}+\dfrac{1}{b^2}=\dfrac{1}{c^2}$

6 **(a)**

It is given that $2ae = 8$ and $\frac{2a}{e} = 25$

$\Rightarrow 2ae \times \frac{2a}{e} = 8 \times 25 \Rightarrow 4a^2 = 200 \Rightarrow a = 5\sqrt{2} \Rightarrow 2a = 10\sqrt{2}$

7 **(c)**

Equation of chord joining points $P(a\cos\alpha, b\sin\alpha)$ and $Q(a\cos\beta, b\sin\beta)$ is

$\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right) + \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$

Now, $\beta = \alpha + 90°$

$\frac{x}{a}\cos\left(\frac{2\alpha+90°}{2}\right) + \frac{y}{b}\sin\left(\frac{2\alpha+90°}{2}\right) = \frac{1}{\sqrt{2}}$

now, compare it with $lx + my = -n$, we get

$\frac{\cos\left(\frac{2\alpha+90°}{2}\right)}{al} = \frac{\sin\left(\frac{2\alpha+90°}{2}\right)}{bm} = -\frac{1}{\sqrt{2}n}$

$\because \cos^2\theta + \sin^2\theta = 1$

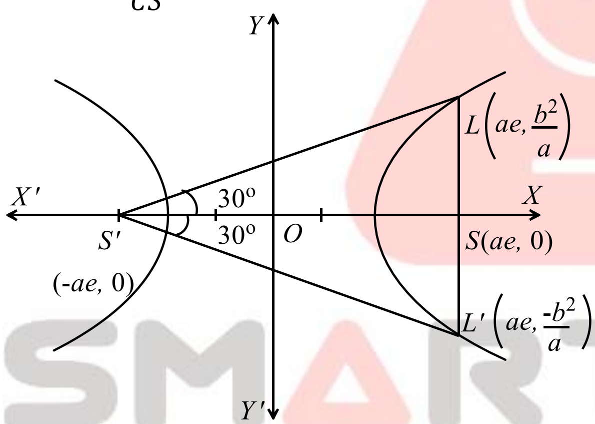
$\Rightarrow a^2l^2 + b^2m^2 = 2n^2$

8 **(c)**

Let LSL'' be a latusrectum and C be the centre of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. It is given that CLL'' is equilateral triangle. Therefore, $\angle LCS = 30°$

In ΔCSL, we have

$\tan 30° = \frac{SL}{CS}$



$\Rightarrow \frac{1}{\sqrt{3}} = \frac{b^2/a}{ae}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{b^2}{a^2e}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{e^2-1}{e} \Rightarrow \sqrt{3}\,e^2 - e - \sqrt{3} = 0 \Rightarrow e = \frac{1+\sqrt{13}}{2\sqrt{3}}$

9 **(c)**

Given equation can be rewritten as

$\Rightarrow 4(x^2 - 6x + 9) + 16(y^2 - 2y + 1) - 36 - 6 = 1$

$\Rightarrow \frac{(x-3)^2}{\frac{53}{4}} + \frac{(y-1)^2}{\frac{53}{16}} = 1$

Here, $a^2 = \frac{53}{4}$ and $b^2 = \frac{53}{16}$

$\therefore$ Eccentricity of ellipse is $e = \frac{\sqrt{a^2-b^2}}{a^2}$

$$\Rightarrow e=\frac{\sqrt{\frac{53}{4}-\frac{53}{16}}}{\frac{53}{4}}$$

$$\Rightarrow e=\frac{\sqrt{3}}{2}$$

10 **(d)**

The equation of hyperbola is

$4x^2-9y^2=36$

$\Rightarrow \frac{x^2}{9}-\frac{y^2}{4}=1$ ...(i)

The equation of the chords of contact of tangents from (x_1,y_1) and (x_2,y_2) to the given hyperbola are

$\frac{x\,x_1}{9}-\frac{y\,y_1}{4}=1$ ...(ii)

and $\frac{x\,x_2}{9}-\frac{y\,y_2}{4}=1$ ...(iii)

Lines (ii) and (iii) are at right angles.

$\therefore \frac{9}{4}.\frac{x_1}{y_1}\times\frac{4}{9}.\frac{x_2}{y_2}=-1$

$\Rightarrow \frac{x_1x_2}{y_1y_2}=-\left(\frac{9}{4}\right)^2=-\frac{81}{16}$

11 **(a)**

The circle having centre at the radical centre of three given circles and radius equal to the length of the tangent from it to any one of three circles cuts all the three circles orthogonally. The given circles are

$x^2+y^2-3\,x-6\,y+14=0$ ...(i)

$x^2+y^2-x-4\,y+8=0$ ...(ii)

$x^2+y^2+2\,c-6\,y+9=0$ ...(iii)

The radical axes of (i), (ii) and (ii), (iii) are respectively

$x+y-3=0$ ...(iv)

and, $3\,x-2y+1=0$ ...(v)

Solving (iv) and (v), we get $x=1, y=2$

Thus, the coordinates of the radical centre are (1,2)

The length of the tangent from (1,2) to circle (i) is given by

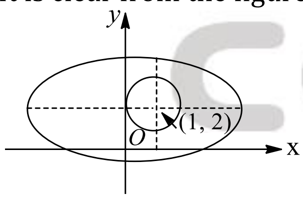
$r=\sqrt{1+4-3-12+14}=2$

Hence, the required circle is

$(x-1)^2+(y-2)^2=2^2\Rightarrow x^2+y^2-2x-4\,y+1=0$

12 **(d)**

It is clear from the figure that the two curves do not intersect each other



13 **(b)**

Directrix of $y^2=4(x+1)$ is $x=-2$. Any point on $x=-2$ is $(-2,k)$

Now mirror image (x,y) of $(-2,k)$ in the line $x+2y=3$ is given by

$$\frac{x+2}{1}=\frac{y-k}{2}=-2\left(\frac{-2+2k-3}{5}\right)$$

$$\Rightarrow x=\frac{10-4k}{5}-2$$

$\Rightarrow x=-\frac{4k}{5}$ ...(i)

And $y=\frac{20-8k}{5}+k$

$\Rightarrow y = \frac{20-3k}{5}$ ...(ii)

From Eqs. (i) and (ii), we get

$y = 4 + \frac{3}{5}\left(\frac{5x}{4}\right)$

$\Rightarrow y = 4 + \frac{3x}{4}$

$\Rightarrow 4y = 16 + 3x$ is the equation of the mirror image of the directrix

14 **(b)**

Putting $x = at^2$ in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

We get, $t^4 + \frac{y^2}{b^2} = 1$

$ie, y^2 = b^2(1 - t^4) = b^2(1 + t^2)(1 - t^2)$

y is real, if $1 - t^2 \geq 0$

$ie, |t| \leq 1$

16 **(a)**

The combined equation of the lines joining the origin to the points of intersection of $x\cos\alpha + y\sin\alpha = p$ and $x^2 + y^2 - a^2 = 0$ is a homogeneous equation of second degree given by

$x^2 + y^2 - a^2\left(\frac{x\cos\alpha + y\sin\alpha}{p}\right)^2 = 0$

$\Rightarrow x^2(p^2 - a^2\cos^2\alpha) + y^2(p^2 - a^2\sin^2\alpha) - (\alpha^2\sin 2\alpha)xy = 0$

The lines given by this equation are at right angle

Coeff. of x^2 + Coeff. of $y^2 = 0$

$\Rightarrow p^2 - a^2\cos^2\alpha + p^2 - a^2\sin^2\alpha = 0 \Rightarrow 2p^2 = a^2$

17 **(a)**

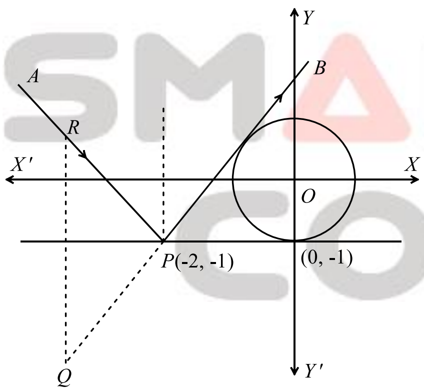
Using $S_1 - S_2 = 0$, we obtain $3x - 9 = 0$ or, $x = 3$ as the equation of the required common tangent

18 **(a)**

Since the difference of the radii of two circles is equal to the distance between their centres. Therefore, two circles touch each other internally and so only one common tangent can be drawn to given two circles

19 **(b)**

Clearly, the incidence ray passes through the point $P(-2,-1)$ and the image of any point Q on BP is $y = -1$



Let us find the equation of PB. Let its equation be

$y + 1 = m(x + 2)$

It touches the circle $x^2 + y^2 = 1$

$\therefore \left|\frac{2m - 1}{\sqrt{m^2 + 1}}\right| = 1 \Rightarrow m = 0, \frac{4}{3}$

So, the equation of PB is

$y + 1 = \frac{4}{3}(x + 2)$ or, $4x - 3y + 5 = 0$

Let $Q(-5,5)$ be a point on PB. The image of Q in $y = -1$ is $R(-5,3)$. So, the equation of RP is

$y-3=\frac{3+1}{-5+2}(x+5)$ or, $4x+3y+11=0$

20 **(c)**

The equation of the tangent to the given circle at the origin is $y=x$. Image of the point $A(2,5)$ in $y=x$ is $(5,2)$.

Thus, the coordinates of B are $(5,2)$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	C	D	D	B	C	A	C	C	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	B	B	A	A	A	A	B	C