DPP
DAILY PRACTICE PROBLEMS

Smart DPPs

SMARTLEARN COACHING

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :2

Topic :-LIMITS & DERIVATIVES

1 **(b)**

We have,

$$\lim_{x \to 0} \frac{1 + \sin x - \cos x + \log(1 - x)}{x^3}$$

$$= \lim_{x \to 0} \frac{1 + \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right) - \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots\right) + \left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}\right)}{x^3}$$

$$= \lim_{x \to 0} \left(-\frac{1}{3!} - \frac{1}{3}\right) + x^2 \left(\frac{1}{5!} - \frac{1}{5}\right) + \cdots = -\frac{1}{6} - \frac{1}{3} = -\frac{1}{2}$$

2 **(b)**

$$\lim_{x \to 0} \left\{\frac{1 + \tan x}{1 + \sin x}\right\}^{\cosec x}$$

$$= \lim_{x \to 0} \frac{\left[\left(1 + \frac{\sin x}{\cos x}\right)^{\frac{\cos x}{\sin x}}\right]^{1/\cos x}}{(1 + \sin x)^{1/\sin x}}$$

$$= \frac{e^{\lim_{x \to 0} \frac{1}{\cos x}}}{e}$$

$$= \frac{e}{e} = 1$$

3 **(b)**

$$\lim_{x \to \frac{\pi}{6}} \frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{4 \sin x \cos x + \cos x}{4 \sin x \cos x - 3 \cos x}$$

[by L'Hospital's rule]

$$= \lim_{x \to \frac{\pi}{6}} \frac{\cos x \,(4 \sin x + 1)}{\cos x \,(4 \sin x - 3)}$$

$$= \frac{4 \sin \frac{\pi}{6} + 1}{4 \sin \frac{\pi}{6} - 3} = -3$$

4 **(a)**

$$\lim_{x \to 0} \left(\frac{1 + 5x^2}{1 + 3x^2}\right)^{1/x^2} = \lim_{x \to 0} \left(1 + \frac{2x^2}{1 + 3x^2}\right)^{1/x^2}$$

$$= e^{\lim_{x \to 0} \frac{1}{x^2}\left(\frac{2x^2}{1 + 3x^2}\right)} = e^2$$

5 **(a)**

We have,

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} x = 0$ and, $\lim_{x \to 0^+} f(x) = \lim_{x \to 0} x^2 = 0$

Hence $\lim_{x \to 0} f(x) = 0$

6 **(a)**

If $x \in Q$, then $n! \pi x$ will be an integral multiple of π for large values of n. Therefore, $\cos(n! \pi x)$ will be either 1 or -1 and so $\cos^{2m}(n! \pi x) = 1$

$\therefore \lim_{m \to \infty} \lim_{n \to \infty} [1 + \cos^{2m}(n! \pi x)] = 1 + 1 = 2$

If $x \notin Q$, $n! \pi x$ will not be an integral multiple of π and so $\cos(n! \pi x)$ will lie between -1 and 1

Thus, $\lim_{m \to \infty} \cos^{2m}(n! \pi x) = 0$

$\Rightarrow \lim_{m \to \infty} \lim_{n \to \infty} [1 + \cos^{2m}(n! \pi x)] = 1 + 0 = 1$

7   (c)

We have,

$\lim_{x \to 1}(1 + \cos \pi x) \cot^2 \pi x$

$= \lim_{x \to 1} \frac{(1 + \cos \pi x)(\cos^2 \pi x)}{(1 - \cos^2 \pi x)} = \lim_{x \to 1} \frac{\cos^2 \pi x}{1 - \cos \pi x} = \frac{1}{2}$

8   (c)

$\lim_{x \to 0} \frac{(e^{kx} - 1) \sin kx}{x^2} = 4$

$\Rightarrow \lim_{x \to 0} \frac{e^{kx} - 1}{kx} \times k \times \frac{\sin kx}{kx} \times k = 4$

$\Rightarrow \quad k^2 = 4 \Rightarrow \quad k = \pm 2$

9   (b)

$\lim_{x \to 0} \frac{\log(1 + x^3)}{\sin^3 x} = \lim_{x \to 0} \frac{\left(\frac{x^3}{1} - \frac{(x^3)^2}{2} + \frac{(x^3)^3}{3} - \dots \infty\right)}{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \infty\right)^3}$

$= \lim_{x \to 0} \frac{\left(1 - \frac{x^3}{2} + \frac{(x^3)^2}{3} - \dots \infty\right)}{\left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \dots \infty\right)^3} = 1$

10   (c)

$l_1 = \lim_{x \to 2^+}(x + [x])$

$= \lim_{h \to 0} 2 + h + [2 + h] = 4$

$l_2 = \lim_{x \to 2^-}(2x - [x])$

$= \lim_{h \to 0}\{2(2 - h) - [2 - h]\}$

$= \lim_{h \to 0}\{2(2 - h) - 1\} = 3$

$l_3 = \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{x - \frac{\pi}{2}} = \lim_{x \to \frac{\pi}{2}} -\sin x = -1$

[by L'Hospital's rule]

Thus, $\quad l_3 < l_2 < l_1$

11   (b)

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin(1 + [x])}{[x]}$

$= \frac{\sin(1 - 1)}{-1} = 0$

12   (a)

$\lim_{x \to 0} \frac{(1 - \cos 2x) \sin 5x}{x^2 \sin 3x}$

$= \lim_{x \to 0} \frac{1 - \cos 2x}{x^2} \lim_{x \to 0} \frac{\sin 5x}{\sin 3x}$

$= \lim_{x \to 0} \frac{2 \sin^2 x}{x^2} \lim_{x \to 0} \frac{\sin 5x}{5x} \cdot \frac{3x}{\sin 3x} \cdot \frac{5x}{3x}$

$$= \lim_{x \to 0} 2\left(\frac{\sin x}{x}\right)^2 \cdot \frac{5}{3} = \frac{10}{3}$$

13 **(d)**

We have,

$$\Rightarrow \lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{\sin(e^{x-2} - 1)}{\log(x - 1)}$$

$$\Rightarrow \lim_{x \to 2} f(x) = \lim_{x \to 2} \left\{ \frac{\sin(e^{x-2} - 1)}{e^{x-2} - 1} \cdot \frac{e^{x-2} - 1}{x - 2} \cdot \frac{x - 2}{\log(1 + (x - 2))} \right\}$$

$$\Rightarrow \lim_{x \to 2} f(x) = 1 \times 1 \times 1 = 1$$

14 **(a)**

We have,

$$\lim_{n \to \infty} \frac{S_{n+1} - S_n}{\sqrt{\sum_{k=1}^{n} k}} = \lim_{n \to \infty} \frac{a_{n+1}}{\sqrt{\frac{n(n+1)}{2}}} = 0$$

15 **(c)**

$$\lim_{h \to 0} \frac{\sin\sqrt{x + h} - \sin\sqrt{x}}{h}$$

Applying L'Hospital's rule,

$$= \lim_{h \to 0} \frac{\frac{\cos\sqrt{x + h}}{2\sqrt{x + h}}}{1} = \frac{\cos\sqrt{x}}{2\sqrt{x}}$$

16 **(a)**

Let $y = \lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$

$$\Rightarrow \quad \log y = \lim_{x \to \frac{\pi}{2}} \tan x \log \sin x$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\log \sin x}{\cot x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\frac{1}{\sin x} \cdot \cos x}{-\csc^2 x} \qquad \text{[by L' Hospital's rule]}$$

$$= 0$$

$$\Rightarrow \quad y = e^0 = 1$$

17 **(c)**

We know that

$$\cos A \cos 2A \cos 4A \ldots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$$

$$\lim_{n \to \infty} \cos\left(\frac{x}{2}\right) \cos\left(\frac{x}{4}\right) \ldots \cos\left(\frac{x}{2^{n-1}}\right) \cos\left(\frac{x}{2^n}\right)$$

$$= \lim_{n \to \infty} \frac{\sin x}{2^n \sin(x/2^n)} \left[\text{put } A = \frac{x}{2^n}\right]$$

$$= \lim_{n \to \infty} \frac{\sin x}{x} \cdot \frac{(x/2^n)}{\sin(x/2^n)}$$

$$= \frac{\sin x}{x}$$

18 **(d)**

$$\lim_{x \to 1} \frac{\sin(e^{x-1} - 1)}{\log x}$$

$$= \lim_{h \to 0} \frac{\sin(e^h - 1)}{\log(1 + h)}$$

$$= \lim_{h \to 0} \frac{\sin(e^h - 1)}{(e^h - 1)} \times \frac{(e^h - 1)}{\log(1 + h)}$$

$$= 1 \times \lim_{h \to 0} \frac{\left(h + \frac{h^2}{2!} + \dots\right)}{\left(h - \frac{h^2}{2!} + \dots \infty\right)} = 1 \times 1 = 1$$

19 **(c)**

We have,

$$l = \lim_{x \to -2} \frac{\tan \pi x}{x + 2} + \lim_{x \to \infty} \left(1 + \frac{1}{x^2}\right)^x$$

$$\Rightarrow l = \lim_{x \to -2} \frac{\tan(2\pi + \pi x)}{x + 2} + \lim_{x \to \infty} \left(1 + \frac{1}{x^2}\right)^x$$

$$\Rightarrow l = \pi \lim_{x \to -2} \frac{\tan \pi (x + 2)}{\pi (x + 2)} + e^{\lim_{x \to \infty} \frac{x}{x^2}} = \pi + e^0 = \pi + 1$$

20 **(d)**

RHL$= \lim_{h \to 0} f(1 + h)$

$$= \lim_{h \to 0} \frac{\sqrt{1 - \cos 2h}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2} \sin h}{h} = \sqrt{2}$$

LHL$= \lim_{h \to 0} f(1 - h) \lim_{h \to 0} \frac{\sqrt{1 - \cos(-2h)}}{h}$

$$= \lim_{h \to 0} \sqrt{2} \frac{\sin h}{-h} = -\sqrt{2}$$

Here, LHL≠RHL

So, limit does not exist.

ANSWER-KEY									
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	B	A	A	A	C	C	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	D	A	C	A	C	D	C	D