DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutions

Subject : Maths
DPP No. : 3

Topic :-Binominal Theorem

1 **(a)**

Since, $x(1+x)^n = xC_0 + C_1x^2 + C_2x^3 + \ldots + C_nx^{n+1}$
On differentiating w.r.t. x, we get
$(1+x)^n + nx(1+x)^{n-1} = C_0 + 2C_1x + 3C_2x^2 + \ldots + (n+1)C_nx^n$
Put $x = 1$, we get
$C_0 + 2C_1 + 3C_2 + \ldots + (n+1)C_n = 2^n + n2^{n-1}$
$= 2^{n-1}(n+2)$

2 **(c)**

Let T_{r+1} denote the $(r+1)^{th}$ term in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$. Then,
$T_{r+1} = {}^nC_r\, x^{3n-5r}(-1)^r$
For this term to contain x^5, we must have
$3n - 5r = 5 \Rightarrow r = \dfrac{3n-5}{5}$
$\therefore$ Coefficient of $x^5 = {}^nC_{\frac{3n-5}{5}}(-1)^{\frac{3n-5}{5}}$
Similarly,
Coefficient of $x^{10} = {}^nC_{\frac{3n-10}{5}}(-1)^{\frac{3n-10}{5}}$
Now,
Coefficient of x^5 + Coefficient of $x^{10} = 0$
$\Rightarrow {}^nC_{\frac{3n-5}{5}}(-1)^{\frac{3n-5}{5}} + {}^nC_{\frac{3n-10}{5}}(-1)^{\frac{3n-10}{5}} = 0$
$\Rightarrow {}^nC_{\frac{3n-5}{5}} = {}^nC_{\frac{3n-10}{5}}$
$\Rightarrow \dfrac{3n-5}{5} + \dfrac{3n-10}{5} = n$
$\Rightarrow 6n - 15 = 5n$
$\Rightarrow n = 15$

3 **(b)**

$(1+x+x^2+x^3)^6 = (1+x)^6(1+x^2)^6$
$= ({}^6C_0 + {}^6C_1x + {}^6C_2x^2 + {}^6C_3x^3 + {}^6C_4x^4 + {}^6C_5x^5 + {}^6C_6x^6) \times ({}^6C_0 + {}^6C_1x^2 + {}^6C_2x^4 + {}^6C_3x^6 + {}^6C_4x^8 + {}^6C_5x^{10} + {}^6C_6x^{12})$
$\therefore$ Coefficient of x^{14} in $(1+x+x^2+x^3)^6$
$= {}^6C_2 \cdot {}^6C_6 + {}^6C_4 \cdot {}^6C_5 + {}^6C_6 \cdot {}^6C_4$
$= 15 + 90 + 15 = 120$

4 **(c)**

The 14th term from the end in the expansion of $\left(\sqrt{x}-\sqrt{y}\right)^{17}$ is the $(18-14+1)^{\text{th}}$ i.e. 5th term from the beginning and is given by

$^{17}C_4\left(\sqrt{x}\right)^{13}\left(-\sqrt{y}\right)^4 = {}^{17}C_4\, x^{13/2}y^2$

5 **(d)**

Put $x=1$, we get

$(1+2+3+\cdots+n)^2=\sum n^3$

6 **(d)**

We have,

$(1+x+x^2)^n=a_0+a_1x+a_2x^3+\ldots.+a_{2n}x^{2n}$

On differentiating both sides, we get

$n(1+x+x^2)^{n-1}(1+2x)=a_1+2a_2x+3a_3x^2$
$+\ldots+2na_{2n}x^{2n-1}$

Now, on putting $x=1$, we get

$n(3)^{n-1}\cdot(3)=a_1+2a_2+3a_3+\ldots+2na_{2n}$

$\Longrightarrow a_1+2a_2+3a_3+\ldots.+2na_{2n}=n\cdot 3^n$

7 **(c)**

There are total $(n+1)$ factors, let $P(x)=0$

Let $(x+{}^nC_0)(x+3\,{}^nC_1)(x+5\,{}^nC_2)\ldots[x+(2n+1)\,{}^nC_n]$

$=a_nx^n+a_{n-1}x^{n-1}+\ldots+a_1x+a_0$

Clearly, $a_n=1$

and roots of the equation $P(x)=0$ are $-{}^nC_0, -3\,{}^nC_1, \ldots$

Sum of roots $=-a_{n-1}/a_n$

$=-{}^nC_0-3\,{}^nC_1-5\,{}^nC_2\ldots$

$\Rightarrow\ a_{n-1}=(n+1)2^n$

8 **(b)**

$^{n-2}C_r+2\cdot{}^{n-2}C_{r-1}+{}^{n-2}C_{r-2}$

$=({}^{n-2}C_r+{}^{n-2}C_{r-1})+({}^{n-2}C_{r-1}+{}^{n-2}C_{r-2})$

$={}^{n-1}C_r+{}^{n-1}C_{r-1}\ \ (\because {}^nC_{r-1}+{}^nC_r={}^{n+1}C_r)$

$={}^nC_r$

9 **(d)**

$\because \dfrac{1}{(x-1)^2(x-2)}=\dfrac{1}{-2(1-x)^2\left(1-\frac{x}{2}\right)}$

$=-\dfrac{1}{2}\left[(1-x)^{-2}\left(1-\dfrac{x}{2}\right)^{-1}\right]$

$=-\dfrac{1}{2}\left[(1+2x+\ldots)\left(1+\dfrac{x}{2}+\ldots\right)\right]$

$\therefore$ Coefficient of constant term is $-\frac{1}{2}$

10 **(b)**

In the expansion of $\left(x^2+\dfrac{a}{x}\right)^5$, the general term is

$T_{r+1}={}^5C_r(x^2)^{5-r}\left(\dfrac{a}{x}\right)^r={}^5C_r\cdot a^r\cdot x^{10-3r}$

For the coefficient of x, put

$10-3r=1\ \Longrightarrow r=3$

$\therefore$ Coefficient of $x = {}^5C_3 a^3 = 10a^3$

12 **(b)**

Coefficient of x^r in the expansion of $(1+x)^{10}$ is ${}^{10}C_r$ and it is maximum for $r = \frac{10}{2} = 5$

Hence, Greatest coefficient= ${}^{10}C_5 = \frac{10\,!}{(5\,!)^2}$

13 **(c)**

Given expansion is $\left(\frac{a}{x} + bx\right)^{12}$

$\therefore$ General term, $T_{r+1} = {}^{12}C_r \left(\frac{a}{x}\right)^{12-r} (bx)^r$

$= {}^{12}C_r (a)^{12-r} b^r x^{-12+2r}$

For coefficient of x^{-10}, put

$-12 + 2r = -10$

$\Rightarrow\ r = 1$

Now, the coefficient of x^{-10} is

${}^{12}C_1 (a)^{11} (b)^1 = 12a^{11}b$

15 **(a)**

We have,

$$T_{r+1} = {}^{21}C_r \left(\sqrt[3]{\frac{a}{\sqrt{b}}}\right)^{21-r} \left(\sqrt{\frac{b}{\sqrt[3]{a}}}\right)^r$$

$\Rightarrow T_{r+1} = {}^{21}C_r\, a^{7-\frac{r}{2}}\, b^{\frac{2}{3}r - \frac{7}{2}}$

Since the powers of a and b are the same

$\therefore 7 - \frac{r}{2} = \frac{2}{3}r - \frac{7}{2} \Rightarrow r = 9$

16 **(b)**

$$(1-x)^{-4} = 1.x^0 + 4x^1 + \frac{4.5}{2}x^2 + \ldots$$

$$= \left[\frac{1.2.3}{6}x^0 + \frac{2.3.4}{6}x + \frac{3.4.5}{6}x^2 + \frac{4.5.6}{6}x^3 + \ldots + \frac{(r+1)(r+2)(r+3)}{6}x^r + \ldots\right]$$

Therefore, $T_{r+1} = \frac{(r+1)(r+2)(r+3)}{6}x^r$

17 **(a)**

We have,

$$y = \frac{1}{3} + \frac{1\cdot 3}{3\cdot 6} + \frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \cdots$$

$$\Rightarrow y + 1 = 1 + \frac{1}{3} + \frac{1\cdot 3}{3\cdot 6} + \frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \cdots$$

Comparing the series on RHS with

$1 + n\,x + \frac{n(n-1)}{2\,!}x^2 + \cdots$, we get

$n\,x = \frac{1}{3}$ ... (i)

and, $\frac{n(n-1)}{2}x^2 = \frac{1}{6}$ ... (ii)

Dividing (ii) by square of (i), we get

$\frac{n-1}{2n}=\frac{9}{6}\Rightarrow n=-\frac{1}{2}$

$\Rightarrow x=-\frac{2}{3}$ [putting $n=-\frac{1}{2}$ in (i)]

$\therefore y+1=(1+x)^n$

$\Rightarrow y+1=\left(1-\frac{2}{3}\right)^{-1/2}$

$\Rightarrow y+1=\left(\frac{1}{3}\right)^{-1/2}$

$\Rightarrow (y+1)^2=\left(\frac{1}{3}\right)^{-1}\Rightarrow y^2+2y+1=3\Rightarrow y^2+2y=2$

18 **(b)**

$S(k)=1+3+5....+(2k-1)=3+k^2$

Put $k=1$ in both sides, we get

LHS $=1$ and RHS$=3+1=4$

$\Longrightarrow$ LHS $\neq$ RHS

Put $(k+1)$ in both sides in the place of k, we get

LHS $=1+3+5....+(2k-1)+(2k+1)$

RHS $=3+(k+1)^2=3+k^2+2k+1$

Let LHS $=$ RHS

Then, $1+3+5....+(2k-1)+(2k+1)$

$=3+k^2+2k+1$

$\Longrightarrow 1+3+5+...+(2k-1)=3+k^2$

If $S(k)$ is true, then $S(k+1)$ is also true.

Hence, $S(k)\Longrightarrow S(k+1)$

19 **(b)**

The general term in the expansion of $(5^{1/6}+2^{1/8})^{100}$ is given by

$T_{r+1}={}^{100}C_r\left(5^{1/6}\right)^{100-r}(2^{1/8})^r$

As 5 and 2 are relatively prime, T_{r+1} will be rational, if

$\frac{100-r}{6}$ and $\frac{r}{8}$ are both integers *ie*, if $100-r$ is a multiple of 6 and r is a multiple of 8. As $0\leq r\leq 100$, multiples of 8 upto 100 and corresponding value of $100-r$ are

$r=0,8,16,24,\dots,88,96$

ie, $100-r=100,92,84,76,\dots,12,4$

Out of $100-r$, multiples of 6 are 84, 60, 36, 12

$\therefore$ There are four rational terms

Hence, number of irrational terms is $101-4=97$

20 **(b)**

We have,

$T_r={}^{10}C_{r-1}\left(\frac{x}{3}\right)^{10-r+1}\left(-\frac{2}{x^2}\right)^{r-1}$

$\Rightarrow T_r={}^{10}C_{r-1}\left(\frac{1}{3}\right)^{11-r}(-2)^{r-1}x^{13-3r}$

For this term to contain x^4, we must have

$13-2r=4\Rightarrow r=3$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	C	D	D	C	B	D	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	C	A	A	B	A	B	B	B