

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :3

Topic :-SEQUENCES AND SERIES

1 (c)

$$\left(a + \frac{1}{p} - \frac{1}{q}\right)^2 + \left(a + \frac{1}{q} - \frac{1}{r}\right)^2 + \left(a + \frac{1}{r} - \frac{1}{s}\right)^2 \leq 0$$

$$\Rightarrow \frac{1}{p} - \frac{1}{q} = \frac{1}{q} - \frac{1}{r} = \frac{1}{r} - \frac{1}{s}$$

$\Rightarrow p, q, r, s$ are in HP.

2 (d)

$$\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} - \dots = \left(1 - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) - \dots$$

$$= 2 \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots\right) - 1 = \log_e \frac{4}{e}$$

3 (b)

We have,

$$x^{\log_x(x^2 - 4x + 5)} = x - 1$$

$$\Rightarrow x^2 - 4x + 5 = x - 1 \Rightarrow x^2 - 5x + 6 = 0 \Rightarrow x = 2, 3$$

4 (b)

It is given that

$y - x, 2(y - a), (y - z)$ are in H.P.

$\Rightarrow \frac{1}{y-x}, \frac{1}{2(y-a)}, \frac{1}{y-z}$ are in A.P.

$$\Rightarrow \frac{1}{2(y-a)} - \frac{1}{y-x} = \frac{1}{y-z} - \frac{1}{2(y-a)}$$

$$\Rightarrow \frac{2a - y - x}{y-x} = \frac{y+z-2a}{y-z}$$

$$\Rightarrow \frac{(x-a) + (y-a)}{(x-a) - (y-a)} = \frac{(y-a) + (z-a)}{(y-a) - (z-a)}$$

$$\Rightarrow \frac{x-a}{y-a} = \frac{z-a}{y-a}$$

$x - a, y - a, z - a$ are in G.P.

5 (b)

The sum of n terms of given series = $\frac{n(n+1)^2}{2}$ if n is even. Let n is odd i.e., $n = 2m + 1$

Then, $S_{2m+1} = S_{2m} + (2m + 1)$ th term

$$= \frac{(n-1)n^2}{2} + n$$



$$= \frac{(n-1)n^2}{2} + n^2 \quad [\because n \text{ is odd} = 2m+1]$$

$$= n^2 \left[\frac{n-1+2}{2} \right] = \frac{(n+1)n^2}{2}$$

6 (c)

$$\text{LHS} = \frac{1(1-\lambda^{n+1})}{1-\lambda} = \frac{1-\lambda^{n+1}}{1-\lambda}$$

$$\text{And RHS} = (1+\lambda)(1+\lambda^2)(1+\lambda^4)$$

$$(1+\lambda^8)(1+\lambda^{16})$$

$$(1-\lambda)(1+\lambda)(1+\lambda^2)$$

$$= \frac{(1+\lambda^4)(1+\lambda^8)(1+\lambda^{16})}{(1-\lambda)}$$

$$(1-\lambda^2)(1+\lambda^2)(1+\lambda^4)$$

$$= \frac{(1+\lambda^8)(1+\lambda^{16})}{(1-\lambda)}$$

$$= \frac{1-\lambda}{(1-\lambda^{32})}$$

$$= \frac{1-\lambda}{1-\lambda^{n+1}} = \frac{1-\lambda^{32}}{1-\lambda}$$

$$\therefore \frac{1-\lambda}{1-\lambda^{n+1}} = \frac{1-\lambda}{1-\lambda^{32}}$$

$$\Rightarrow 1-\lambda^{n+1} = 1-\lambda^{32}$$

$$\therefore n+1 = 32 \Rightarrow n = 31$$

7 (a)

$$\therefore (x+1) + (x+4) + (x+7) + \dots + (x+28) = 155$$

Let n be the number of terms in the AP on LHS.

$$\therefore x+28 = (x+1) + (n-1)3$$

$$\Rightarrow n = 10$$

$$\therefore \frac{10}{2} [(x+1) + (x+28)] = 155$$

$$\Rightarrow x = 1$$

8 (a)

Let ' r ' be the common ratio,

$$\therefore \frac{\sum_{n=1}^{100} a_{2n}}{\sum_{n=1}^{100} a_{2n-1}} = \frac{a_2 + a_4 + a_6 + \dots + a_{200}}{a_1 + a_3 + a_5 + \dots + a_{199}}$$

$$= \frac{a_1(r+r^3+r^5+\dots+r^{199})}{a_1(1+r^2+r^4+\dots+r^{198})} = r$$

$$\Rightarrow \frac{\alpha}{\beta} = r$$

9 (a)

Let $S = 2 + 7 + 14 + 23 + 34 + \dots + T_n \dots$ (i)

and $S = 2 + 7 + 14 + 23 + 34 + \dots + T_{n-1} + T_n \dots$ (ii)

On subtracting Eqs. (i) from (ii), we get

$$\therefore S - S = 2 + [5 + 7 + 9 + 11 + \dots + T_n - T_{n-1}] - T_n$$

$$\Rightarrow T_n = 2 + \left[\frac{n-1}{2} \{2 \times 5 + (n-2)2\} \right]$$

$$\Rightarrow T_n = 2 + (n-1)(n+3)$$

$$\therefore T_{99} = 2 + 98 \times 102 = 9998$$

10 (b)

Here, $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$

$$\Rightarrow (a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) \leq 0$$

$$\Rightarrow (ap - b)^2 + (bp - c)^2 + (cp - d)^2 \leq 0$$

(Since sum of squares is never less than zero).



⇒ Each of the square is zero.

$$\therefore (ap - b)^2 = (bp - c)^2 = (cp - d)^2 = 0$$

$$\Rightarrow p = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

∴ a, b, c, d are in GP.

11 (c)

$$\text{Since, } \frac{2p}{3} + \frac{2p}{3} + \frac{2p}{3} + \underbrace{\frac{3q}{5} + \dots + \frac{3q}{5}}_{5\text{times}} + \underbrace{\frac{4r}{7} + \dots + \frac{4r}{7}}_{7\text{times}}$$

$$\geq 15 \sqrt{\left(\frac{2p}{3}\right)^3 \left(\frac{3q}{5}\right)^5 \left(\frac{4r}{7}\right)^7} \quad (\because \text{AM} \geq \text{GM})$$

$$\Rightarrow p^3 q^5 r^7 \frac{2^3 3^5 4^7}{3^3 5^5 7^7} \leq 1$$

$$\Rightarrow p^3 q^5 r^7 \leq \frac{5^5 7^7}{2^3 3^2 4^7}$$

12 (c)

$$\text{Let } S = 1 + 10 + 10^2 + \dots + 10^{90}$$

$$= \frac{1 \cdot (10^{91} - 1)}{10 - 1} = \frac{(10^{13})^7 - 1}{10^{13} - 1} \times \frac{10^{13} - 1}{10 - 1}$$

$$= [(10^{13})^6 + (10^{13})^5 + (10^{13})^4 + \dots + 1] \times (10^{12} + 10^{11} + \dots + 1)$$

∴ It is the product of two integers and hence not prime.

13 (d)

$$\text{Let } S = 1 + 2x + 3x^2 + 4x^3 + \dots \infty \quad \dots(i)$$

$$xS = x + 2x^2 + 3x^3 + \dots \infty \quad \dots(ii)$$

Subtracting Eq. (ii) from Eq. (i), we get

$$(1 - x)S = 1 + x + x^2 + x^3 + \dots \infty$$

$$\Rightarrow S = \frac{1}{(1 - x)} \left(\frac{1}{1 - x} \right) = \frac{1}{(1 - x)^2}$$

14 (c)

We have,

$$5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$$

$$= 5^x - 7^{\frac{1}{x}}, \text{ where } x = \sqrt{\log_5 7}$$

$$= 5^x - (5^{x^2})^{\frac{1}{x}} \quad [\because x = \sqrt{\log_5 7} \Rightarrow x^2 = \log_5 7 \Rightarrow 7 = 5^{x^2}]$$

$$= 5^x - 5^x = 0$$

15 (b)

We have, $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \dots, \frac{1}{x_n}$ are in AP.

$$\therefore \frac{1}{x_2} - \frac{1}{x_1} = \frac{1}{x_3} - \frac{1}{x_2} = \dots = \frac{1}{x_n} - \frac{1}{x_{n-1}} = d \quad (\text{say})$$

$$\therefore \frac{x_2 - x_1}{x_1 x_2} = \frac{x_3 - x_2}{x_2 x_3} = \dots = \frac{x_{n-1} - x_n}{x_{n-1} x_n} = d$$

Now, $x_1 x_2 + x_2 x_3 + \dots + x_{n-1} x_n$

$$= \frac{1}{d} [x_1 - x_2 + x_2 - x_3 + \dots + x_{n-1} - x_n]$$

$$= \frac{x_1 - x_n}{d}$$

$$\text{But } \frac{1}{x_n} - \frac{1}{x_1} = (n - 1)d$$

$$\therefore \frac{x_1 - x_n}{d} = (n - 1)d$$

$$\text{or } \frac{x_1 x_n}{d} = (n - 1) x_1 x_n$$



$$\therefore x_1x_2 + x_2x_3 + \dots + x_{n-1}x_n = (n-1)x_1x_n$$

16 (d)

Given a, b, c are in GP and $4a, 5b, 4c$ are in AP.

$$\therefore b^2 = ac \text{ and } 5b = \frac{4a+4c}{2}$$

$$\Rightarrow b^2 = ac \text{ and } 5b = 2a + 2c$$

$$\text{Now, } a + b + c = 70 \quad (\text{given})$$

$$\Rightarrow 2a + 2c + 2b = 140$$

$$\Rightarrow 5b + 2b = 140$$

$$\Rightarrow b = 20$$

17 (d)

Since, p, q and r in HP.

$$\Rightarrow q = \frac{2pr}{p+r} \Rightarrow \frac{q}{2} = \frac{pr}{p+r} = K \quad (\text{say})$$

$$\Rightarrow q = 2K, pr = (p+r)K$$

Also, p^2, q^2, r^2 are in AP.

$$\therefore 2q^2 = p^2 + r^2 = (p+r)^2 - 2pr$$

$$\Rightarrow 8K^2 = (p+r)^2 - 2(p+r)K$$

$$\Rightarrow (p+r)^2 - 2(p+r)K - 8K^2 = 0$$

$$\Rightarrow p+r = 4K, -2K$$

When $p+r = 4K$, then $pr = 4K^2$

$$\therefore (p-r)^2 = (p+r)^2 - 4pr = 16K^2 - 16K^2 = 0$$

$$\Rightarrow p = r$$

But this is not possible ($\because p \neq r$)

$$\therefore p+r = -2K \Rightarrow pr = -2K \cdot K = -2K^2$$

Now, $(p-r)^2 = (p+r)^2 - 4pr$

$$= 4K^2 - 4(-2K^2) = 12K^2$$

$$\Rightarrow p-r = \pm 2\sqrt{3}K$$

$$\Rightarrow p = (-1 \pm \sqrt{3})K$$

$$\text{And } 2r = -2K \mp \sqrt{3}K$$

$$\Rightarrow r = (-1 \mp \sqrt{3})K$$

$$\therefore p : q : r = (-1 \mp \sqrt{3})K : 2K : (-1 \mp \sqrt{3})K$$

$$= -1 \mp \sqrt{3} : 2 : -1 \mp \sqrt{3}$$

$$= (-1 \mp \sqrt{3}) : (-2) : (-1 \mp \sqrt{3})$$

18 (a)

$$\text{Given, } x = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \dots$$

$$\Rightarrow x = e^2$$

$$\Rightarrow x^{-1} = e^{-2}$$

19 (a)

$$\text{Since, } \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \infty = \frac{\pi^4}{90}$$

$$\Rightarrow \frac{\pi^4}{90} = \left(\frac{1}{1^4} + \frac{1}{3^4} + \dots \infty \right) + \left(\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \dots \infty \right)$$

$$\Rightarrow \frac{\pi^4}{90} = \left(\frac{1}{1^4} + \frac{1}{3^4} + \dots \infty \right) + \frac{1}{2^4} \left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots \infty \right)$$

$$\Rightarrow \frac{\pi^4}{90} = \left(\frac{1}{1^4} + \frac{1}{3^4} + \dots \infty \right) + \frac{1}{16} \left(\frac{\pi^4}{90} \right)$$

$$\Rightarrow \frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \dots \infty$$

20 (a)



We have,

$$\begin{aligned} & (x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \dots \infty \\ &= \frac{x^2 - y^2}{x - y} + \frac{x^3 - y^3}{x - y} + \frac{x^4 - y^4}{x - y} + \dots \text{ to } \infty \\ &= \frac{1}{x - y} \{(x^2 + x^3 + x^4 + \dots) - (y^2 + y^3 + y^4 + \dots)\} \\ &= \frac{1}{x - y} \left\{ \frac{x^2}{1 - x} - \frac{y^2}{1 - y} \right\} = \frac{x + y - xy}{1 - x - y + xy} \end{aligned}$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	C	D	B	B	B	C	A	A	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	D	C	B	D	D	A	A	A