DPP
DAILY PRACTICE PROBLEMS

CLASS : XIth
DATE :

Solutions

SUBJECT : MATHS
DPP NO. :3

Topic :-LIMITS & DERIVATIVES

1 **(a)**

We have,

$\lim_{x\to 2}\frac{2x^2-4f'(x)}{x-2}=\lim_{x\to 0}\frac{4x-4f''(x)}{1}$ [Using L' Hospital's Rule]

$$\Rightarrow \lim_{x\to 2}\frac{2x^2-4f'(x)}{x-2}=8-4f''(2)=8-4=4$$

2 **(a)**

$$\lim_{x\to\pi/4}\frac{\int_2^{\sec^2 x} f(t)dt}{x^2-\pi^2/16}$$

$=\lim_{x\to\pi/4}\frac{2\sec^2 x\tan x\, f(\sec^2 x)}{2x}$ [Using Leibniz and L' Hospital's rules]

$$=\frac{\sec^2\frac{\pi}{2}\, f\left(\sec^2\frac{\pi}{4}\right)\tan\frac{\pi}{4}}{\pi/4}=\frac{8}{\pi}f(2)$$

3 **(d)**

$$\lim_{x\to a}\frac{f(a)g(x)-f(x)g(a)}{x-a}\left[\frac{0}{0}\text{from}\right]$$

$=\lim_{x\to a}\frac{f(a)g\prime(x)-f\prime(x)g(a)}{1-0}$ [by L' Hospital's rule]

$=f(a)g'(a)-f'(a)g(a)$

$=2(-1)-1(3)=-2-3=-5$

4 **(a)**

We have,

$$\lim_{x\to\infty}f(x)=\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$$

$$\Rightarrow \lim_{x\to\infty}f(x)=\lim_{x\to\infty}\left(1+\frac{4x+1}{x^2+x+2}\right)^x=e^{\lim_{x\to\infty}\frac{x(4x+1)}{x^2+x+2}}=e^4$$

5 **(c)**

$$\lim_{x\to 0}\frac{\sin^{-1}x-x}{x^3\cos x}=\lim_{x\to 0}\frac{\frac{1}{\sqrt{1-x^2}}-1}{x^3(-\sin x)+3x^2\cos x}$$

[using L'Hospital's rule]

$$=\lim_{x\to 0}\frac{1-\sqrt{1-x^2}}{\sqrt{1-x^2}.x^2(-x\sin x+3\cos x)}\times\frac{1+\sqrt{1-x^2}}{1+\sqrt{1-x^2}}$$

$$=\lim_{x\to 0}\frac{1}{\left[\begin{array}{c}\sqrt{1-x^2}\left(1+\sqrt{1-x^2}\right)\\(-x\sin x+3\cos x)\end{array}\right]}$$

$$=\frac{1}{1(1+1)(3)}=\frac{1}{6}$$

6 **(c)**

Here, $\lim_{x\to 0}(\sin x)^{1/x} + \lim_{x\to 0}\left(\frac{1}{x}\right)^{\sin x} = 0 + \lim_{x\to 0} e^{\log\left(\frac{1}{x}\right)^{\sin x}}$

$\left[\begin{array}{l}\lim_{x\to 0}(\sin x)^{\frac{1}{x}} \to 0 \\ \text{as,} \quad 0 < \sin x < 1\end{array}\right]$

$= e^{\lim_{x\to 0}\frac{\log(1/x)}{\operatorname{cosec} x}} = e^{\lim_{x\to 0}\frac{x\left(-\frac{1}{x^2}\right)}{-\operatorname{cosec} x \cot x}}$

[by L'Hospital's rule]

$= e^{\lim_{x\to 0}\frac{\sin x}{x}\tan x} = e^0 = 1$

7 **(b)**

$\lim_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}}$

$= \lim_{x\to\infty}\left[1 + \frac{-6}{3x+2}\right]^{\frac{x+1}{3}}$

$= \left[\lim_{x\to\infty}\left\{1 + \frac{-6}{3x+2}\right\}^{\frac{3x+2}{-6}}\right]^{\frac{-6}{3x+2}\times\frac{x+1}{3}}$

$= [e]^{\lim_{x\to\infty}\frac{-6}{3x+2}\times\frac{x+1}{3}} \left[\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e\right]$

$= e^{\lim_{x\to\infty}\frac{-2x-2}{3x+2}} = e^{-2/3}$

8 **(c)**

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

As $x \to 0 \Rightarrow \theta \to 0$

$\therefore \lim_{\theta\to 0}\frac{1}{\tan\theta}\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \lim_{\theta\to 0}\frac{1}{\tan\theta}\sin^{-1}(\sin 2\theta)$

$= \lim_{\theta\to 0}\frac{2\theta}{\tan\theta} = 2$

9 **(c)**

$$\lim_{x\to 0}\frac{2\sin^2 3x}{x^2} = \lim_{x\to 0} 2\left(\frac{\sin 3x}{3x}\right)^2 \times \frac{9}{1} = 18$$

10 **(a)**

$\lim_{x\to 0}\frac{a^x + a^{-x} - 2}{x^2} = \lim_{x\to 0}\frac{a^x \log a - a^{-x}\log a}{2x}$

[by L' Hospital's rule]

$= \lim_{x\to 0}\frac{a^x(\log a)^2 + a^{-x}(\log a)^2}{2}$

$= (\log a)^2$ [by L' Hospital's rule]

11 **(b)**

LHL= $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = 1$

[$\because$ $(0-h)$ is rational]

RHL= $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = 1$

[$\because$ $(0+h)$ is rational]

Hence, LHL=RHL=1

12 **(c)**

$$\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x = \lim_{x\to\infty}\left[\frac{1-\frac{3}{x}}{1+\frac{2}{x}}\right]^x$$

$$=\frac{e^{-3}}{e^2}=e^{-5}$$

13 **(b)**

We have,

$$\lim_{x\to\infty}\frac{x}{2}\sin\left(\frac{\pi}{2x}\right)$$

$$=\lim_{x\to\infty}\frac{\sin\left(\frac{\pi}{2x}\right)}{\frac{\pi}{2x}}\cdot\frac{\pi}{4}=\frac{\pi}{4}\lim_{y\to 0}\frac{\sin y}{y}=\frac{\pi}{4}, \text{where } y=\frac{\pi}{2x}$$

14 **(b)**

$$\lim_{x\to 0}\frac{f(x)}{x}=\lim_{x\to 0}\frac{f'(x)}{1}$$

$$=\lim_{x\to 0}\frac{\tan^4 x}{1}=0$$

15 **(c)**

RHL$=\lim_{x\to 1^+}\frac{1}{2}\{g(x)+(x)\}\sin x$

$$=\lim_{x\to 1^+}\frac{1}{2}\{1+x\}\sin x$$

$$=\frac{1}{2}.(1+1)\sin 1=\sin 1$$

and LHL$=\lim_{x\to 1^-}\frac{\sin x}{x}=\sin 1$

Since, RHL=LHL=sin 1

$\therefore \quad \lim_{x\to 1} f(x)=\sin 1$

16 **(c)**

Given, $\lim_{x\to\infty}\left[\frac{x^3+1}{x^2+1}-(ax+b)\right]=2$

$$\Rightarrow \quad \lim_{x\to\infty}\left[\frac{x(1-a)-b-\frac{a}{x}+\frac{(1-b)}{x^2}}{1+\frac{1}{x^2}}\right]=2$$

This limit will exist, if

$1-a=0$

and $b=-2$

$\Rightarrow \quad a=1$

and $\quad b=-2$

17 **(b)**

$$\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=\lim_{x\to 2}\frac{\frac{x-2}{x^2-3x+2}-1}{x-2}$$

$$=\lim_{x\to 2}\frac{x-2-(x^2-3x+2)}{(x-2)(x^2-3x+2)}$$

$$=\lim_{x\to 2}\frac{-(x-2)^2}{(x-2)(x-2)(x-1)}$$

$$=-\lim_{x\to 2}\frac{1}{x-1}$$

$$=-1$$

18 **(c)**

$$\lim_{x\to 3}\frac{\int_3^{f(x)} 2t^3 dt}{x-3} = \lim_{x\to 3}\frac{2[f(x)]^3 . f'(x)}{1}$$
$$= 2[f(3)]^3 . f'(3) = 2\times 3^3 \times \frac{1}{2}$$
$$= 27$$

19 **(a)**

Given, $\lim_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)} = 4$

Applying L' Hospital's rule,

$$\Rightarrow \quad \lim_{x\to a}\frac{f(a)g'(x)-g(a)f'(x)}{g'(x)-f'(x)} = 4$$
$$\Rightarrow \quad \lim_{x\to a}\frac{kg'^{(x)}-kf'(x)}{g'(x)-f'(x)} = 4$$
$$\Rightarrow \quad k = 4$$

20 **(d)**

We have,

$$\lim_{x\to 0}\frac{\sin x}{\sqrt{x^2}} = \lim_{x\to 0}\frac{\sin x}{|x|}$$

Now, $\lim_{x\to 0^-}\frac{\sin x}{|x|} = \lim_{x\to 0}\frac{\sin x}{x|} = -1$

and, $\lim_{x\to 0^+}\frac{\sin x}{|x|} = \lim_{x\to 0}\frac{\sin x}{x} = 1$

Hence, $\lim_{x\to 0}\frac{\sin x}{|x|}$ does not exist

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	D	A	C	C	B	C	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	B	B	C	C	B	C	A	D