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MATHEMATICS

Sets

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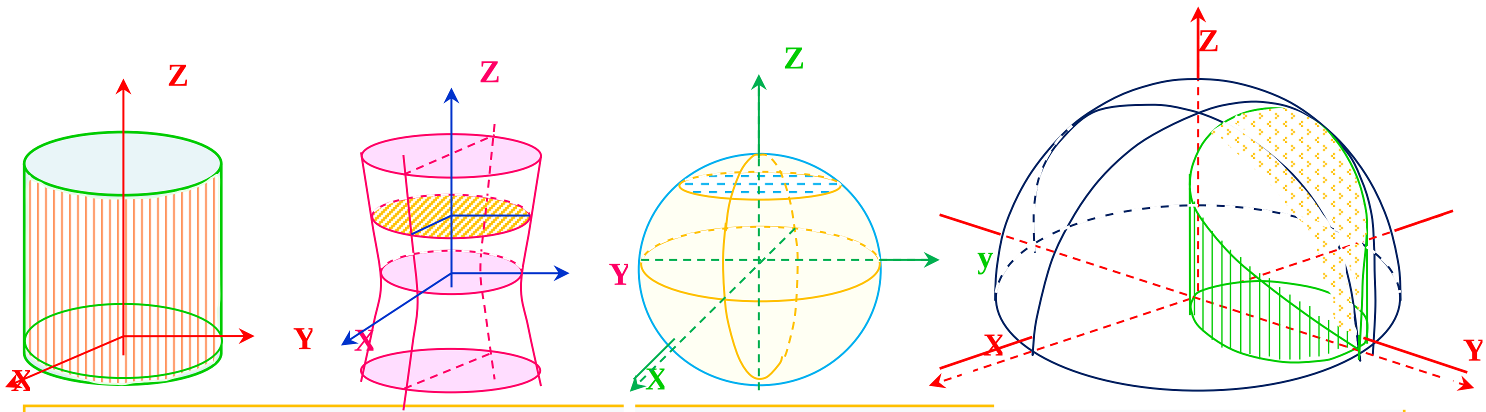


MATHS BOOKLET FOR JEE (MAINS & ADVANCE) & BOARDS

India's First Colour Smart Notes

Sets

1 SETS

A set is a collection of well defined distinct objects i.e. the objects follow a given rule or rules.

If we say that we have a collection of short students in a class, then this collection is not a set as short student is not well defined. If however, we say that we have a collection of students whose heights is less then 5 feet, then it represent a set.

Examples:

1. $A = \{1, 4, 5, 4, 8\}$, the elements of this collection are distinguishable but not distinct, hence *A* is not a set.
2. Let $A =$ collection of all vowels in English alphabates, then $A = \{a, e, i, o, u\}$. Hence elements of *A* are distinguishable as well as distinct, then *A* is a set.
3. The collection of all positive integers is a set.
4. The collection of all students of IIT (Delhi) is a set.

Some standard notation for some special sets:

1. The set of all natural number i.e., the set of all positive integers, is denoted by *N*.
2. The set of all integer number is denoted by *I* or *Z*.
3. The set of rational number is denoted by *Q*.
4. The set of all irrational number is denoted by Q'.
5. The set of all real number is denoted by *R*.

6. The set of all positive number is denoted by R^+. (zero is not included)
7. The set of all negative real number is denoted by R^-. (zero is not included)
8. The set of complex number is denoted by C.

1.1 REPRESENTATION OF SETS

- **Tabular form or Roster form**

In this method of describing a set, the elements of the set are listed separated by coma within braces.

Example: The set of prime number less then 10 can be described as {2, 3, 5, 7}

- **Set Builder form or Rule method**

In this method of describing a set, a variable x which stands for each element of the set is written under braces and then after giving a semicolor or oblique line the property or properties $P(x)$ possessed by each element of set is written the braces itself.

Example1: The set A of all even natural number can be written as $A = \{2x : x \in N\}$

Example2: The set $A = \{1, 3, 5\}$ can be written as $A = \{x : x$ is an odd natural number $\leq 5\}$

1.2 FINITE AND INFINITE SETS

A set having finite number of elements is called a **finite set**.

Example: A = {1, 2, 3, 4}. A is a finite set as it contains 4 elements.

A set which is not a finite set is called an **infinite set**. Thus a set A is said to be an infinite set if the number of elements of set A is not finite.

Example: Let A = set of all points on a particular straight line.

1.3 CARDINAL NUMBER OF A FINITE SET

The number of elements in a finite set A is called the cardinal number of set A and is denoted by $n(A)$

Example: Let A = {1, 2, 3, 4, 5}, then $n(A) = 5$

1.4 EQUIVALENT SETS

Two finite sets A and B are said to be equivalent if they have the same cardinal number. Thus set A and B are equivalent iff $n(A) = n(B)$.

If sets A and B are equivalent, we write $A \approx B$

Example: Let A = {1, 2, 3, 4, 5}, B = {a, e, i, o, u}

Here $n(A) = n(B) = 5$

Therefore, sets A and B are equivalent.

1.5 EQUAL SETS

Two set A and B are said to be equal set if each element of set A is an element of set B and each element of B is an element of set A. Thus two sets A and B are equal if they have exactly the same elements. The order in which the elements in the two sets have been written is immaterial.

If set A and B are equal we can write A = B

Example1: Let A = {1, 2, 3, 4, 5}, $B = \{x : x \in N$ and $1 \leq x \leq 5\}$

Here A and B are equal.

2 DIFFERENT TYPES OF SETS

2.1 NULL SET (OR EMPTY SET OR VOID A SET)

A set having no element is called null set or empty set or void set. It is denoted by ϕ or { }.

Example: The set of odd numbers divisible by 2.

2.2 SINGLETON SET

A set having single element is called a singleton set. It is represented by writing down the element within the braces.

Example: {2}, {0}, {ϕ}.

2.3 UNIVERSAL SET

A set consisting of all possible elements which occur in the discussion is called a universal set and is denoted by *U*.

2.4 PAIR SET

A set having two elements is called a pair set.

Example: {1, 2}, {2, 0}.

2.5 SET OF SETS

A set *S* having all its elements as set is called a set of sets or a family of sets or a class of sets.

Example1: $S = \{\{1, 2, 3\}, 3, \{4\}\}$ is not a set of sets as 3 is not a set.

Example2: {ϕ} is a singleton set of set having null set ϕ as its elements.

3 SUBSETS, SUPERSETS, PROPER SUBSETS

3.1 SUBSETS OF A SET

A set A is said to be a subset of a set *B* if each element of *A* is also an element of *B*. If *A* is a subset of set *B*, we write $A \subseteq B$

Thus, $A \subseteq B \Leftrightarrow [x \in A \Rightarrow x \in B]$

Example: Let *A* = {1, 2, 3}, *B* = {2, 3, 4, 1, 5}, then $A \subseteq B$.

The statement $A \subseteq B$ can also be expressed equivalently by writing $B \supseteq A$ (read '*B* is a superset of *A*')

If *A* is not a subset of *B* i.e., if there is an element in *A* which is not an element of *B*, then we write $A \not\subseteq B$ or $B \not\supseteq A$.

- **Some important properties of subset**
  - Every set is its own subset.
    Let *A* be any set ; $x \in A \Rightarrow x \in A$
    Hence $A \subseteq A$
  - Empty set is a subset of each set.
  - Let *A* and *B* be any two sets:
    then $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$
  - Let *A, B, C* be three sets.
    If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

3.2 PROPER SUBSET OF A SET

A set *A* is said to be a proper subset of a set *B,* if *A* is a subset of *B* and $A \neq B$ i.e. if

Every element of *A* is an elements of *B* and *B* has at least one element which is not an element of *A*. This fact is expressed by writing $A \subset B$ or $B \supset A$.

If A is not a proper subset of *B*, then we write $A \not\subset B$.

Example: Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4, 1, 5\}$, then $A \subset B$ and $B \supset A$.

3.3 SUPERSET OF SETS

A set *A* is said to be a super set of set *B*, if *B* is a subset of *A* i.e., each elements of *B* is an elements of *A*. If *A* is a super set of *B*, then $A \supseteq B$.

Example: Let *A* = {1, 2, 3, 4, 5) and *B* = {2, 5, 4}.

Here *B* is a subset of *A*, therefore *A* is a superset of *B*.

3.4 POWER SET

The set or family of all the subsets of a given set A is said to be the power set of A and is denoted by $P(A)$

Example: If $A = \{1, 2\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

If A has n elements then $P(A)$ has 2^n elements.

Illustration 1

Question: **List all the subsets and all the proper subsets of the set {−1, 0, 1}.**

Solution: Let $A = \{-1, 0, 1\}$.

Subset of A having no element is : ϕ

Subsets of A having one element are : {−1}, {0}, {1}.

Subsets of A having two elements are : {−1, 0}, {0, 1}, {−1, 1}.

Subsets of A having three elements are : {−1, 0, 1}.

Thus, all the subsets of A are ϕ, {−1}, {0}, {1}, {−1, 0}, {0, 1}, {−1, 1}, {−1, 0, 1}.

Proper subsets of A are ϕ, {−1}, {0}, {1}, {−1, 0}, {0, 1}, {−1, 1}.

Illustration 2

Question: **Make correct statements by filling the blanks by suitable symbols $\subseteq, \not\subseteq$.**

(i) $\{x : x$ is an even natural number$\}$ ------ $\{x : x$ is an integer$\}$

(ii) $\{x : x$ is a triangle in the plane$\}$ ------ $\{x : x$ is a rectangle in the plane$\}$

(iii) $\{x : x$ is isosceles triangle in the plane$\}$ ------ $\{x : x$ is an equilateral triangle in the plane$\}$

(iv) a ------ $\{a, \{b\}, c\}$

(v) $\{\{a\}\}$ ____ $\{a, \{b\}, c\}$

Solution: (i) Since every even natural number is an integer, therefore,
$\{x : x$ is an even natural number$\} \subseteq \{x : x$ is an integer$\}$.

(ii) Since a triangle is not a rectangle, therefore
$\{x : x$ is a triangle in the plane $\} \not\subseteq \{x : x$ is a rectangle in the plane$\}$.

(iii) Since an isosceles triangle is not necessarily an equilateral triangle, therefore
$\{x : x$ is an isosceles triangle$\} \not\subseteq \{x : x$ is an equilateral triangle$\}$.

(iv) Since a is not a set, therefore, $a \not\subseteq \{a, \{b\}, c\}$.

(v) Since $\{\{a\}\}$ is a set containing exactly one element $\{a\}$ and $\{a\}$ is not an element of the set $\{a, \{b\}, c\}$, therefore, $\{\{a\}\} \not\subseteq \{a, \{b\}, c\}$.

Illustration 3

Question: **How many elements are in the set**

$A = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$

$B = \{x : x$ is even integer and $x < 19\}$

$C = \{x : 0 \le x \le 1$ and x is a rational number$\}$

Solution: The elements of A are ϕ, $\{\phi\}$, $\{\phi, \{\phi\}\}$. So A has three elements.

$B = \{x : x = 0, \pm 2, \pm 4, \pm 6, \ldots \text{ and } x < 19\} = \{\ldots, -4, -2, 0, 2, 4, 6, \ldots, 18\}$

$\therefore$ B is an infinite set.

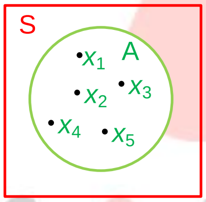
C is also infinite set because $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ are all elements of C.

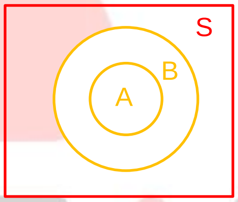
Important formulae/points

- *The order in which the elements of a set are written is immaterial thus the set {1, 2,3} and {2, 1, 3} are same.*
- *Two sets A and B are equal if* $x \in A \Rightarrow x \in B$ *and* $x \in B \Rightarrow x \in A$.
- *The set {0} is not an empty set as it contains one element 0.*
- *The set {ϕ} is not an empty set as it contain one element ϕ.*
- $A \subseteq B \Rightarrow P(A) \subseteq P(B)$
- *If A has n elements then P(A) has* 2^n *elements.*

4 VENN DIAGRAMS

Statements involving sets can be easily understood with pictorial representation of the sets. A set is represented by circle or a closed geometrical figure A, inside the universal set *S,* which is represented by a rectangular region. Elements of a set *A* are represented by points within the circle which represents *A*.





$A \subseteq B$

4.1 OPERATION ON SETS

In algebra of numbers, the operation of addition (+) when applied on two numbers gives a third number $a + b$. Likewise we discuss the operation union ($\cup$), intersection ($\cap$) and difference (−) applicable on any two sets.

- **Union of two sets**

The union of two sets *A* and *B* is the set of all those elements which are either in *A* or in *B* or in both. This set is denoted by $A \cup B$ (read as '*A* union *B*').

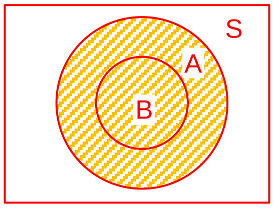
$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

$$= \{x : x \in A \vee x \in B\} \qquad \{\vee \text{ denotes 'or'}\}$$

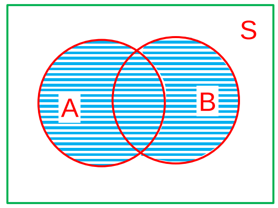
Also, $x \in A \cup B \Leftrightarrow x \in A \text{ or } x \in B$

Example: Let $A = \{1, 2, 3\}$ and $B = \{2, 1, 5, 6\}$, then $A \cup B = \{1, 2, 3, 5, 6\}$.

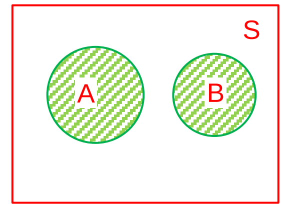
The union of two sets can be represented by Venn diagram as shown in the figure below:



$A \cup B$ when $A \subseteq B$



$A \cup B$ when neither $A \subseteq B$ nor $B \subseteq A$



$A \cup B$
when A and B disjoint sets

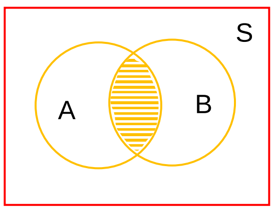
Here shaded portions are $A \cup B$.

- **Intersection of two sets**

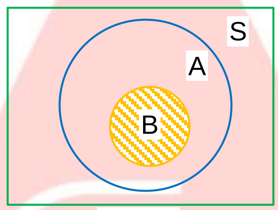
The intersection of two sets A and B is the set of all the elements which are common in A and B. This set is denoted as $A \cap B$ and read as A intersection B.

$$A \cap B = \{x : x \in A \text{ and } x \in B\} = \{x : x \in A \wedge x \in B\} \quad (\wedge \text{ denotes and})$$

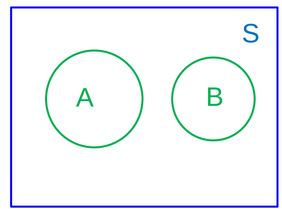
$$x \in A \cap B \iff x \in A \text{ and } x \in B$$



$A \cap B$ when neither $A \subseteq B$ nor $B \subseteq A$



$A \cap B$
when $B \subseteq A$, $A \cap B = B$



$A \cap B = \phi$
no shaded region
when A and B are disjoint sets

Example: Let $A = \{1, 2, 3\}$ and $B = \{2, 1, 5, 6\}$, then $A \cap B = \{1, 2\}$

Now, $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

5 DIFFERENCE AND COMPLEMENTS

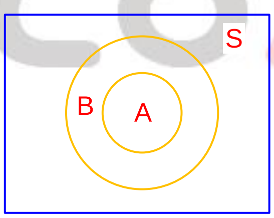
5.1 DIFFERENCE OF TWO SETS

The difference of two sets A and B is the set of all those elements of A which are not elements of B. It is denoted by $A - B$.

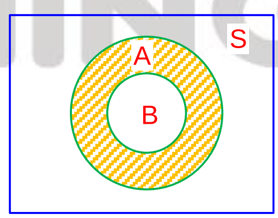
$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

thus $x \in A - B \iff x \in A$ and $x \notin B$

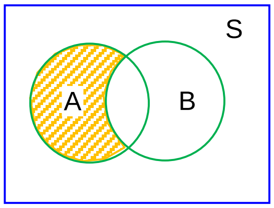
$A - B$ can be represented by Venn diagram (shaded region) as below:



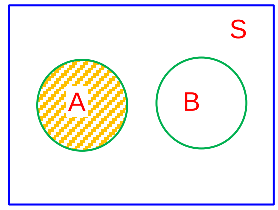
$A - B = \phi$
when $A \subseteq B$



$A - B$ when $B \subseteq A$



$A - B$ when neither $A \subseteq B$ nor $B \subseteq A$



$A - B = A$
when A and B are disjoint sets

Example: Let $A = \{1, 3, 5, 6, 7\}$; $B = \{2, 3, 4, 5\}$, then $A - B = \{1, 6, 7\}$; $B - A = \{2, 4\}$.

5.2 COMPLEMENT OF A SET

The complement of a set *A* is a set of all those elements of universal set *S* which are not elements of *A*. It is denoted by A^C or A'.

$$A' = S - A$$

6 LAWS OF ALGEBRA OF SETS

(i) Associative Law

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) Commutative Law

- $A \cap B = B \cap A$
- $A \cup B = B \cup A$

(iii) Idempotent Law

- $A \cap A = A$
- $A \cup A = A$

(iv) Law of *U*

- $A \cap U = A$
- $A \cup U = U$

(v) Identity Law

- $A \cup \phi = A$
- $A \cap \phi = \phi$

(vi) Distributive Law

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(vii) De Morgan's Law

- $(A \cap B)' = A' \cup B'$
- $(A \cup B)' = A' \cap B'$

(viii) Complement Law

- $(A \cap A') = \phi$
- $(A \cup A') = U$
- $\phi' = U$

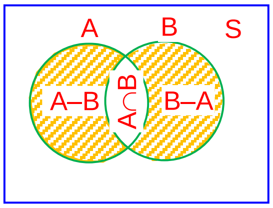
- $U' = \phi$

(ix) Involution Law (Law of double complementation)

- $(A')' = A$
-

7 SOME PRACTICAL APPLICATIONS OF SET THEORY

Here we shall study the use of set theory in practical problems.
The number of distinct elements of a finite set A is denoted by $n(A)$.



Use the following results whichever is required

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

(iii) $n(A \cup B) = n(A) + n(B - A)$

(iv) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A \cap B = \phi$

(v) $n(A) = n(A - B) + n(A \cap B)$

(vi) $n(B) = n(B - A) + n(A \cap B)$

(vii) Number of elements belonging to exactly one of A and B
$= n(A - B) + n(B - A)$
$= n(A \cup B) - n(A \cap B) = n(A) + n(B) - 2n(A \cap B)$

(viii) Number of elements belonging to exactly two of A, B and C
$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(ix) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(x) Number of elements belonging to exactly one of A, B and C
$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

(xi) $n(A' \cap B') = n(S) - n(A \cup B) = n(A \cup B)'$

(xii) $n(A' \cup B') = n(S) - n(A \cap B)$

Illustration 4

Question: **If $A = \{1, 3, 5, 6, 7\}$, $B = \{2, 3, 6, 8\}$ and $C = \{1, 2, 3, 4\}$, then find (i) $A \cap B$ (ii) $A \cup B$ (iii) $A - B$ (iv) $B - A$**

Solution:
(i) $A \cap B = \{x : x \in A \text{ and } x \in B\} = \{3, 6\}$
(ii) $A \cup B = \{x : x \in A \text{ or } x \in B\} = \{1, 2, 3, 5, 6, 7, 8\}$
(iii) $A - B = \{x : x \in A \text{ and } x \notin B\} = \{1, 5, 7\}$
(iv) $B - A = \{x : x \in B \text{ and } x \notin A\} = \{2, 8\}$

Illustration 5

Question: **If universal set $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5, 6\}$, $C = \{2, 3, 7\}$, then find (i) A' (ii) $(A - B)'$ (iii) $B' - A'$ (iv) $A' \cap B$ (v) $A \cup B'$**

(vi) $(A \cap C)'$.

Solution: (i) $A' = \{x : x \in S \text{ and } x \notin A\} = \{0, 5, 6, 7, 8, 9\}$

(ii) $A - B = \{x : x \in A \text{ and } x \notin B\} = \{1, 4\}$

$\therefore \quad (A - B)' = S - (A - B) = S - \{1, 4\} = \{0, 2, 3, 5, 6, 7, 8, 9\}$

(iii) $B' = S - B = S - \{2, 3, 5, 6\} = \{0, 1, 4, 7, 8, 9\}$

$A' = S - A = S - \{1, 2, 3, 4\} = \{0, 5, 6, 7, 8, 9\}$

$B' - A' = \{0, 1, 4, 7, 8, 9\} - \{0, 5, 6, 7, 8, 9\} = \{1, 4\}$.

(iv) $A' \cap B = \{0, 5, 6, 7, 8, 9\} \cap \{2, 3, 5, 6\} = \{5, 6\}$

(v) $A \cup B' = \{1, 2, 3, 4\} \cup \{0, 1, 4, 7, 8, 9\} = \{0, 1, 2, 3, 4, 7, 8, 9\}$

(vi) $A \cap C = \{1, 2, 3, 4\} \cap \{2, 3, 7\} = \{2, 3\}$

$\therefore \quad (A \cap C)' = S - (A \cap C) = \{0, 1, 4, 5, 6, 7, 8, 9\}$

Illustration 6

Question: **If $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7, 8\}$, $C = \{2, 3, 4, 5, 6, 7\}$. Then verify that**

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (ii) $(A \cup B)' = A' \cap B'$

Solution:

(i) $B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A \cap B = \{1, 8\}$, $A \cap C = \{2, 4, 6\}$

Now, $A \cap (B \cup C) = \{x : x \in A \text{ and } x \in B \cup C\} = \{1, 2, 4, 6, 8\}$

$(A \cap B) \cup (A \cap C) = \{1, 2, 4, 6, 8\}$

$\therefore \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(ii) $A \cup B = \{x : x \in A \text{ or } x \in B\} = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$(A \cup B)' = \{x : x \in S \text{ and } x \notin A \cup B\} = \{9\}$

$A' = \{x : x \in S \text{ and } x \notin A\} = \{3, 5, 7, 9\}$

$B' = \{x : x \in S \text{ and } x \notin B\} = \{2, 4, 6, 9\}$

$A' \cap B' = \{9\}$

$\therefore \quad (A \cup B)' = A' \cap B'$.

Illustration 7

Question: **For any three sets A, B, C, prove the following (by using different laws on operations of sets):**

(i) $A - B = B' - A'$ (ii) $(A \cup B) \cap (A \cup B') = A$

(iii) $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

Solution:

(i) $A - B = (A \cap B') = B' \cap A = B' \cap (A')' = B' - A'$.

(ii) $(A \cup B) \cap (A \cup B') = A \cup (B \cap B')$ [By distributive law]

$= A \cup \phi = A$

(iii) $(A - B) \cup (B - A) \cup (A \cap B) = [(A \cup B) - (A \cap B)] \cup (A \cap B)$

$= [(A \cup B) \cap (A \cap B)'] \cup (A \cap B)$

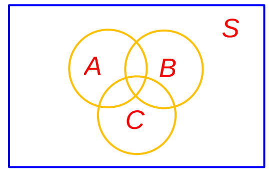
$= [(A \cup B) \cup (A \cap B)] \cap [(A \cap B)' \cup (A \cap B)]$ [By distributive law]

$= (A \cup B) \cap S$, where S is the universal set

$= A \cup B$.

Illustration 8

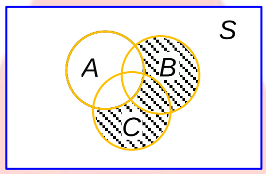
Question: **In the given figure shade the following sets:**



(i) $A' \cap (B \cup C)$

(ii) $A' \cap (C - B)$

Solution: (i) Shaded region represents $A' \cap (B \cup C)$.



(ii) Shaded region represents $A' \cap (C - B)$

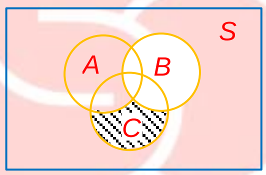


Illustration 9

Question: **In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?**

Solution: Let A = set of people who like coffee

B = set of people who like tea

Given, $n(A \cup B) = 70$, $n(A) = 37$, $n(B) = 52$

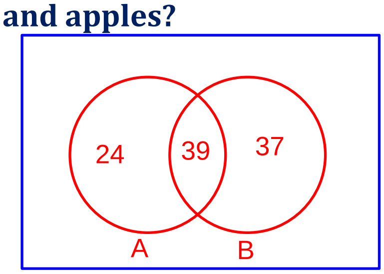
To find $n(A \cap B)$

$$n(A \cap B) = n(A) + n(B) - n(A \cup B) = 37 + 52 - 70 = 19$$

Illustration 10

Question: **If 53% of persons like oranges where 66% like apples, what can be said about the percentage of persons who like both oranges and apples?**

Solution: Let the total number of persons = 100 $\Rightarrow$ $n(A \cup B) = 100$



Let $A = \{x : x$ like oranges$\}$

$B = \{x : x$ likes apples)

Then $n(A) = 53$, $n(B) = 66$

$\therefore \quad A \cap B = \{x : x \text{ likes oranges and apples both}\}$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore \quad n(A \cap B) = n(A) + n(B) - n(A \cup B) = 53 + 66 - 100 = 19$

Illustration 11

Question: **Let A has 3 elements and B has 6 elements. What can minimum number of elements in $A \cup B$?**

Solution: Clearly $A \cup B$ will contain minimum number of elements if $A \subseteq B$ or $B \subseteq A$

But $n(A) = 3 < 6 = n(B)$

$\therefore \quad B \not\subseteq A \quad \therefore \; A \subset B$

Thus $A \cup B = B \quad \therefore \; n(A \cup B) = n(B) = 6$

Thus $A \cup B$ contains at least 6 elements

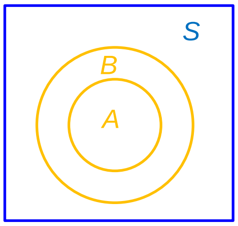


Illustration 12

Question: **In a group of 2000 people, there are 1500, who can speak Hindi and 800, who can speak Bengali. How many can speak Hindi only? How many can speak Bengali only? How many can speak both Hindi and Bengali?**

Solution: Let $A = \{x : x \text{ speaks Hindi}\}$, $B = \{x : x \text{ speaks Bengali}\}$

Then $A - B = \{x : x$ speaks Hindi and can not speak Bengali}

$B - A = \{x : x$ speaks Bengali and can not speak Hindi}

$A \cap B = \{x : x$ speaks Hindi and Bengali both)

Given, $n(A) = 1500$, $n(B) = 800$, $n(A \cup B) = 2000$

Now, $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 1500 + 800 - 2000 = 300$

$\therefore$ Number of people speaking Hindi and Bengali both is 300

$n(A) = n(A - B) + n(A \cap B)$

$\Rightarrow \quad n(A - B) = n(A) - n(A \cap B) = 1500 - 300 = 1200$

Also, $n(B - A) = n(B) - n(A \cap B) = 800 - 300 = 500$

Thus number of people speaking Hindi only = 1200

And number of people speaking Bengali only = 500

Illustration 13

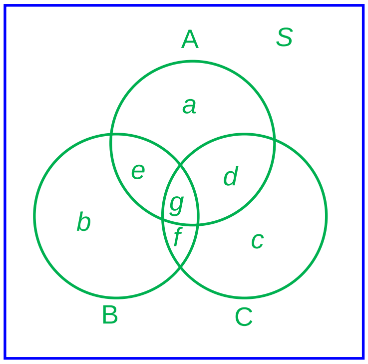
Question: **A class has 175 students. Following is the description showing the number of students studying one or more of the following subjects in this class.**

Mathematics 100, Physics 70, Chemistry 46; Physics and Chemistry 23; Mathematics and Physics 30; Mathematics and Chemistry 28; Mathematics, Physics and Chemistry 18.

How many students are enrolled in Mathematics alone, Physics alone and Chemistry alone? Are there students who have not offered any of these three subjects.

Solution: Let *A, B* and *C* denote the sets of students studying Mathematics, Physics and Chemistry respectively.

Let us denote the number of elements (students) contained in the bounded region as shown in the diagram by *a, b, c, d, e, f* and *g* respectively.



Then $a+d+g+e=100$

$b+f+g+e=70$

$c+f+g+d=46$

$g+e=30$

$g+d=28$

$g+f=23$

$g=18$

Solving these, we get

$g=18,\ f=5,\ d=10,\ e=12, c=13, b=35,\ a=60$

$\therefore \quad a+b+c+d+e+f+g=153$

So, the number of students who have not offered any these three subjects

175 – 153 = 22

Students studying Mathematics only = *a* = 60

Students studying Physics only = *b* = 35

Students studying Chemistry only = *c* = 13

Illustration 14

Question: **For any sets *A, B, C.* Using logical method, prove that**

(i) $A\cap(B-A)=\phi$ **(ii)** $A-B=B-A \Leftrightarrow A=B$

(iii) $(A\cup B)-(A\cap B)=(A-B)\cup(B-A)$ **(iv)** $A\cup B=(A-B)\cup(B-A)\cup(A\cap B)$

Solution: (i) Let $x\in A\cap(B-A) \Rightarrow \quad x\in A$ and $x\in(B-A)$

$\Rightarrow \quad x\in A$ and $(x\in B \text{ and } x\notin A)$

$\Rightarrow \quad (x\in A \text{ and } x\in B)$ and $(x\in A \text{ and } x\notin A)$

$\Rightarrow \quad x\in(A\cap B)$ and $x\in\phi$

$\Rightarrow \quad x\in\phi \qquad$ [$\because$ ϕ has no element]

Hence $A\cap(B-A)\subseteq\phi \qquad$ ...(i)

But ϕ is a subset of each set.

$\therefore\ \phi\subseteq A\cap(B-A) \qquad$ ...(ii)

From (i) and (ii), we have, $A\cap(B-A)=\phi$

(ii) $A-B=B-A\Leftrightarrow A=B$

Only if part: Let $A-B=B-A \qquad$ ...(i)

To prove $A=B$

Let $x\in A \Leftrightarrow \ (x\in A \text{ and } x\notin B)$ or $(x\in A \text{ and } x\in B)$

$\Leftrightarrow \quad x\in(A-B)$ or $x\in(A\cap B)$

$\Leftrightarrow \quad x\in(B-A)$ or $x\in A\cap B \qquad$ [from (i)]

$\Leftrightarrow \quad (x \in B \text{ and } x \notin A)$ or

$\Leftrightarrow \quad x \in B$

Hence $A = B$

If part: Let ...(ii)

To prove

Now, $A - B = A - A = \phi$ $\quad [\because B = A]$

and $B - A = A - A = \phi$ $\quad [\because B = A]$

$\therefore \quad A - B = B - A$

Thus $A = B \Rightarrow A - B = B - A$

(iii) $x \in (A \cup B) - (A \cap B)$

$\Leftrightarrow \quad x \in (A \cup B) \wedge x \notin (A \cap B)$ $\quad$ [$\wedge$ stands for 'and']

$\Leftrightarrow \quad (x \in A \vee x \in B) \wedge (x \notin A \vee x \notin B)$ $\quad$ [$\vee$ stands for 'or']

$\Leftrightarrow \quad [(x \in A \vee x \in B) \wedge (x \notin A)] \vee [(x \in A \vee x \in B) \wedge x \notin B]$

$\Leftrightarrow \quad [x \in B - A] \vee [x \in A - B]$

$\Leftrightarrow \quad x \in (B - A) \cup (A - B)$

$\Leftrightarrow \quad x \in (A - B) \cup (B - A)$ $\quad [\because A \cup B = B \cup A]$

Thus $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

(iv) $x \in A \cup B \Leftrightarrow x \in A$ or $x \in B$

$\Leftrightarrow$ ($x \in A$ and $x \notin B$) or ($x \notin A$ and $x \in B$) or ($x \in A$ or $x \in B$)

$\Leftrightarrow$ $x \in A - B$ or $x \in B - A$ or $x \in A \cap B$

$\Leftrightarrow$ $x \in (A - B) \cup (B - A) \cup (A \cap B)$

$\therefore \quad A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

Important formulae/points

- $x \notin A \cup B \Leftrightarrow x \notin A$ and $x \notin B$
- If $A \subseteq B$, then $A \cup B = B$
- $x \in A^C \Leftrightarrow x \in S$ and $x \notin A$
- $n(A) = n(A - B) + n(A \cap B)$
- $n(B) = n(B - A) + n(A \cap B)$
- $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$
- $n(A' \cap B') = n(S) - n(A \cup B) = n(A \cup B)'$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- $n(B) = n(B - A) + n(A \cap B)$
- $n(A \cup B) = n(A) + n(B - A)$

8 CARTESIAN PRODUCT OF SETS

Let a be an arbitrary element of a given set A i.e. $a \in A$ and b be an arbitrary element of B i.e. $b \in B$. Then the pair (a, b) is an ordered pair. Obviously $(a, b) \neq (b, a)$. The Cartesian product of two sets A and B is defined as the set of ordered pairs (a, b). The Cartesian product is denoted $A \times B$.

$\Rightarrow \quad A \times B = \{(a, b);\ a \in A,\ b \in B\}$.

In general $A \times B \neq B \times A$ and if A or B is a null set, then $A \times B = \phi$.

Moreover, $n(A \times B) = n(A) \cdot n(B)$.

Note :
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
(iii) $A \times (B - C) = (A \times B) - (A \times C)$
(iv) $(A - B) \times C = (A \times C) - (B \times C)$
(v) $(A \cap B) \times C = (A \times C) \cap (B \times C)$
(vi) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Illustration 15

Question: **If A = {2, 5}, B = {3, 4, 7} and C = {3, 4, 8} then evaluate $A \times B$, $B \times A$, $A \times A$ and verify that**

(i) $A \times (B - C) = (A \times B) - (A \times C)$

(ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Solution: Here $A \times B = \{2, 5\} \times \{3, 4, 7\} = \{(2, 3), (2, 4), (2, 7), (5, 3), (5, 4), (5, 7)\}$

$B \times A = \{3, 4, 7\} \times \{2, 5\} = \{(3, 2), (3, 5), (4, 2), (4, 5), (7, 2), (7, 5)\}$ and

$A \times A = \{2, 5\} \times \{2, 5\} = \{(2, 2), (2, 5), (5, 2), (5, 5)\}$.

Also $A \times C = \{2, 5\} \times \{3, 4, 8\} = \{(2, 3), (2, 4), (2, 8), (5, 3), (5, 4), (5, 8)\}$

$B - C = \{3, 4, 7\} - \{3, 4, 8\} = \{7\}$

$\Rightarrow \quad A \times (B - C) = \{2, 5\} \times \{7\} = \{(2, 7)\ (5, 7)\}$

$(A \times B) - (A \times C) = \{(2, 3), (2, 4), (2, 7), (5, 3), (5, 4), (5,7)\}$

$- \{(2, 3), (2, 4), (2, 8), (5, 3), (5, 4), (5, 8)\} = \{(2, 7), 5, 7\} = A \times (B - C)$.

To verify (ii), we write $B \cup C = \{3, 4, 7, 8\}$

$\Rightarrow \quad A \times (B \cup C) = \{2, 5\} \times \{3, 4, 7, 8\}$

$= \{(2, 3), (2, 4), (2, 7), (2, 8), (5, 3), (5, 4), (5, 7), (5, 8)\}$

and $(A \times B) \cup (A \times C) = \{(2, 3), (2, 4), (2, 7), (2, 8), (5, 3), (5, 4), (5, 7), (5, 8)\}$

$= A \times (B \cup C)$

Important formulae/points

- *The Cartesian product is denoted A × B* $\Rightarrow A \times B = \{(a, b);\ a \in A,\ b \in B\}$.
- *The elements of A × B are also called 2-tuples.*
- *If A = ϕ or B = ϕ i.e. if at least one of A and B is an empty set, then A × B = ϕ.*
- $A \times B \neq \phi \Leftrightarrow A \neq \phi$ and $B \neq \phi$
- *A × B may or may not be equal to B × A.*
- *A × B = B × A* if and only if *A = B.*

EXERCISE

1. Which of the following collections are sets? Justify your answer.
(i) The collection of all most talented writers of India.
(ii) The collection of all months of a year beginning with the letter J.
(iii) The collection of handsome boys of the world.

2. Are the following sets equal?
(i) $A = \{x : x^3 - 8 = 0 \text{ and } x \text{ is a real number}\}$.
$B = \{x : x^2 + 7x - 18 = 0 \text{ and } x > 0\}$.

(ii) A = the set of letters in the word 'ALLOY'.
B = the set of letters in the word 'LOYAL'.

3. Which of the following sets are infinite sets?
(i) The set of all circles passing through the origin.
(ii) The set of prime numbers less than 99.

4. If $A = \{2, 3, 4, 5, 6\}$, $B = \{3, 4, 5, 6, 7\}$, $C = \{4, 5, 6, 7, 8\}$. Then find
(i) $(A \cup B) \cap (A \cup C)$ (ii) $A - (B \cup C)$
(iii) $A - (B - C)$ (iv) $(A \cap B) \cup (A \cap C)$

5. For sets A and B, prove the following using the properties of sets:
(i) $(A \cup B) - A = B - A$ (ii) $A \cup (B - A) = A \cup B$
(iii) $A - (A - B) = B \Leftrightarrow B \subseteq A$ (iv) $A \cup (A \cap B) = A$
(v) $(A \cap B) \cup (A - B) = A$ (vi) $(A \cup B) \cap (A \cup B') = A$

6. For sets A, B and C, prove the following using the properties of sets:
(i) $A - (B - C) = (A - B) \cup (A \cap C)$ (ii) $(A - B) - C = A - (B \cup C)$
(iii) $A \cap (A \cup B) = A$

7. For any three sets A, B, C, prove the following (by using different laws on operations of sets):
(i) $(A - B) \cup A = A$ (ii) $(A - B) \cap (B - A) = \phi$
(iii) $A - (A - B) = A \cap B$ (iv) $A \cap (B - C) = (A \cap B) - (A \cap C)$

8. If A and B are two non-empty sets having n elements in common, then show that $A \times B$ and $B \times A$ have n^2 elements in common.

9. Find x and y, if $(2x, x + y) = (6, 2)$.

10. If A and B are two sets such that $n(A)=17,\ n(B)=23$ and $n(A\cup B)=38$, find $n(A\cap B)$.

11. Use logical method to prove the following:

(i) For any set A, prove that $(A')'=A$.

(ii) For any two sets A and B, prove that $A\subseteq B \Leftrightarrow B'\subseteq A'$.

(iii) For any two set A and B, prove that $A-B=\phi$ iff $A\subseteq B$.

12. In a group of people, 50 speak both English and Hindi and 30 people speak English but not Hindi. All the people speak at least one of the two languages. How many people speak English?

13. In a group of 1000 people, 750 can speak Hindi and 400 can speak Bengali. All the people speak at least one of the two languages. How many can speak Hindi only? How many can speak Bengali only? How many can speak both Hindi and Bengali?

14. A college awarded 38 medals in Football, 15 in Basketball and 20 in Cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports?

15. In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all three subjects. Find the number of students that had taken (i) only chemistry (ii) only mathematics (iii) only physics (iv) physics and chemistry but not mathematics (v) mathematics and physics but not chemistry (vi) only one of the subjects (vii) at least one of three subjects (viii) none of three subjects.

16. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey, 80 played cricket and basketball and 40 played cricket and hockey, 24 played all the three games. Find the number of boys who did not play any game.

17. If $A=\{x : x^2-5x+6=0\}, B=\{2, 4\}, C=\{4,\ 5\},$ then find $A\times(B\cap C)$.

18. If $n(A)=4,\ n(B)=3,\ n(A\times B\times C)=24,$ then find $n(C)$.

19. If $A=\{2, 3, 5\}, B=\{2, 5, 6\}$, then find $(A-B)\times(A\cap B)$.

20. Shade the following sets:

(i) $A' \cap B'$ (ii) $A' \cup B'$
(iii) $(A \cup B) \cap (A \cup C)$ (iv) $(A \cap B) \cup (A \cap C)$

ANSWERS TO EXERCISE

1. (ii)

2. (i) $A = B$
(ii) $A = B$

3. (i) is infinite set

4. (i) {2, 3, 4, 5, 6, 7}
(ii) {2}
(iii) {2, 4, 5, 6}
(iv) {3, 4, 5, 6}

9. $x = 3, y = -1$

10. 2

12. 80

13. The number of people speaking Hindi only = 600, Bengali only = 250, both Hindi and Bengali = 150

14. Number of people who got medal in exactly two of the three sports = 9

15. (i) 5
(ii) 4
(iii) 2
(iv) 1
(v) 6
(vi) 11
(viii) 23
(viii) 2

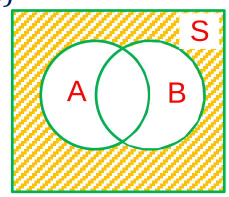
16. 160

17. $\{(2, 4), (3, 4)\}$

18. 2

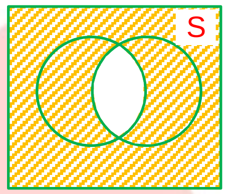
19. $\{(3, 2), (3, 5)\}$

20. (i)



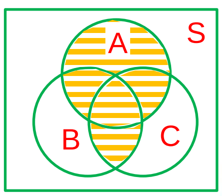
(i) $A' \cap B'$

(ii)



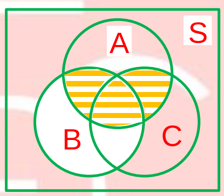
(ii) $A' \cup B'$

(iii)



(iii) $(A \cup B) \cap (A \cup C)$

(iv)



(iv) $(A \cap B) \cup (A \cap C)$

IMPORTANT PRACTICE QUESTION SERIES FOR IIT-JEE EXAM – 1

1. Let R_1 be a relation defined by
$R_1 = \{(a, b)|a \geq b, a, b \in R\}$. Then, R_1 is
a) An equivalence relation on R
b) Reflexive, transitive but not symmetric
c) Symmetric, transitive but not reflexive
d) Neither transitive not reflexive but symmetric
2. On the set of human beings a relation R is defined as follows:
"aRb iff a and b have the same brother". Then R is
a) Only reflexive b) Only symmetric c) Only transitive d) Equivalence
3. In a class of 35 students, 17 have taken Mathematics, 10 have taken Mathematics but not Economics. If each student has taken either Mathematics or Economics or both, then the number of students who have taken Economics but not Mathematics is
a) 7 b) 25 c) 18 d) 32
4. $\{n(n+1)(2n+1): n \in Z\} \subset$
a) $\{6k : k \in Z\}$ b) $\{12k : k \in Z\}$ c) $\{18k : k \in Z\}$ d) $\{24k : k \in Z\}$
5. If $A = \{1, 2, 3, 4, 5\}, B = \{2, 4, 6\}, C = \{3, 4, 6\}$, then $(A \cup B) \cap C$ is
a) $\{3, 4, 6\}$ b) $\{1, 2, 3\}$ c) $\{1, 4, 3\}$ d) None of these
6. Let A be the set of all students in a school. A relation R is defined on A as follows:
"aRb iff a and b have the same teacher"
a) Reflexive b) Symmetric c) Transitive d) Equivalence
7. If P is the set of all parallelograms, and T is the set of all trapeziums, then $P \cap T$ is
a) P b) T c) ϕ d) None of these
8. A and B are any two non-empty sets and A is proper subset of B. If $n(A) = 5$, then find the minimum possible value of $n(A \Delta B)$
a) Is 1
b) Is 5
c) Cannot be determined
d) None of these
9. If $n(A) = 4, n(B) = 3, n(A \times B \times C) = 240$, then $n(C)$ is equal to
a) 288 b) 1 c) 12 d) 2
10. In a class, 70 students wrote two tests viz;test-I and test-II. 50% of the students failed in test-I and 40% of the students in test-II. How many students passed in both tests?
a) 21 b) 7 c) 28 d) 14
11. Let Z denote the set of all integers and $A = \{(a, b): a^2 + 3b^2 = 28, \quad a, b \in Z\}$ and $B = \{(a, b): a > b, a, b \in Z\}$. Then, the number of elements in $A \cap B$ is
a) 2 b) 3 c) 4 d) 6
12. Let L be the set of all straight lines in the Euclidean plane. Two lines l_1 and l_2 are said to be related by the relation R iff l_1 is parallel to l_2. Then, the relation R is not
a) Reflexive b) Symmetric c) Transitive d) None of these
13. Let R be a relation on the set N be defined by $\{(x, y)|x, y \in N, 2x + y = 41\}$. Then, R is
a) Reflexive b) Symmetric c) Transitive d) None of these
14. In an office, every employee likes at least one of tea, coffee and milk. The number of employees who like only tea, only coffee, only milk and all the three are all equal. The number of employees who like only tea and coffee, only coffee and milk and only tea and milk are equal and each is equal to the number of employees who like all the three. Then a possible

value of the number of employees in the office is

a) 65 b) 90 c) 77 d) 85

15. Which of the following cannot be the number of elements in the power set of any finite set?

a) 26 b) 32 c) 8 d) 16

16. The relation 'is subset of' on the power set $P(A)$ of a set A is

a) Symmetric b) Anti-symmetric c) Equivalence relation d) None of these

17. Let A and B be two non-empty subsets of a set X such that A is not a subset of B. Then,

a) A is a subset of complement of B

b) B is a subset of A

c) $A$and B are disjoint

d) A and the complement of B are non-disjoint

18. If A, B and C are three sets such that $A \supset B \supset C$, then $(A \cup B \cup C) - (A \cap B \cap C) =$

a) $A - B$ b) $B - C$ c) $A - C$ d) None of these

19. A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and apples, then

a) $x = 39$ b) $x = 63$ c) $39 \leq x \leq 63$ d) None of these

20. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1): n \in N\}$, then $X \cup Y$ is equal to

a) X b) Y c) N d) None of these

21. Let $A = \{x: x \text{is a multiple of } 3\}$ and $B = \{x: x \text{is a multiple of } 5\}$. Then, $A \cap B$is given by

a) {3, 6, 9, ...} b) {5, 10, 15, 20, ...} c) {15, 30, 45, ...} d) None of these

22. If $n(A \times B) = 45$, then $n(A)$ cannot be

a) 15 b) 17 c) 5 d) 9

23. In order that a relation R defined on a non-empty set A is an equivalence relation, it is sufficient, if R

a) Is reflective

b) Is symmetric

c) Is transitive

d) Possesses all the above three properties

24. For real numbers x and y, we write $xRy \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then, the relation R is

a) Reflexive b) Symmetric c) Transitive d) None of these

25. In a class of 45 students, 22 can speak Hindi and 12 can speak English only. The number of students, who can speak both Hindi and English, is

a) 9 b) 11 c) 23 d) 17

26. A, B and C are three non-empty sets. If $A \subset B$ and $B \subset C$, then which of the following is true?

a) $B - A = C - B$ b) $A \cap B \cap C = B$ c) $A \cup B = B \cap C$ d) $A \cup B \cup C = A$

27. $\left\{x \in R: \frac{2x-1}{x^3+4x^2+3x} \in R\right\}$equals

a) $R - \{0\}$ b) $R - \{0, 1, 3\}$ c) $R - \{0, -1, -3\}$ d) $R - \left\{0, -1, -3, +\frac{1}{2}\right\}$

28. If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is

a) 2^{mn} b) $2^{mn} - 1$ c) $2mn$ d) m^n

29. If $A = \{(x, y): y^2 = x; x, y \in R\}$ and
$B = \{(x, y): y = |x|; x, y \in R\}$, then

a) $A \cap B = \phi$

b) $A \cap B$ is a singleton set

c) $A \cap B$ contains two elements only

d) $A \cap B$ contains three elements only

30. Which of the following is an equivalence relation?
a) Is father of b) Is less than c) Is congruent to d) Is an uncle of
31. From 50 students taking examinations in Mathematics, Physics and Chemistry, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 passed Mathematics and Chemistry and at most 20 passed Physics and Chemistry. The largest possible number that could have passed all three examinations is
a) 11 b) 12 c) 13 d) 14
32. Let A be the non-void set of the children in a family. The relation $'x$ is a brother of y' on A is
a) Reflexive b) Symmetric c) Transitive d) None of these
33. In a class of 30 pupils 12 take needls work, 16 take physics and 18 take history. If all the 30 students take at least one subject and no one takes all three, then the number of pupils taking 2 subjects is
a) 16 b) 6 c) 8 d) 20
34. If R is a relation on a finite set having n elements, then the number of relations on A is
a) 2^n b) 2^{n^2} c) n^2 d) n^n
35. The void relation on a set A is
a) Reflexive
b) Symmetric and transitive
c) Reflexive and symmetric
d) Reflexive and transitive
36. Suppose $A_1, A_2, \dots, A_{30}$ are thirty sets, each having 5 elements and $B_1, B_2, \dots, B_n$ are n sets each with 3 elements, let
$\cup_{i=1}^{30} A_i = \cup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly 10 of the A_i's and exactly 9 of the B_j's. Then, n is equal to
a) 115 b) 83 c) 45 d) None of these
37. If A is a finite set having n elements, then $P(A)$ has
a) $2n$ elements b) 2^n elements c) n elements d) None of these
38. Let A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$?
a) 3 b) 6 c) 9 d) 18
39. Let R be a reflexive relation on a set A and I be the identity relation on A. Then,
a) $R \subset I$ b) $I \subset R$ c) $R = I$ d) None of these
40. If $A_1, A_2, \dots, A_{100}$ are sets such that $n(A_i) = i + 2, A_1 \subset A_2 \subset A_3 \dots \subset A_{100}$ and $\cap_{i=3}^{100} A_i = A$, then $n(A) =$
a) 3 b) 4 c) 5 d) 6
41. If A and B are two given sets, then $A \cap (A \cap B)^c$ is equal to
a) A b) B c) Φ d) $A \cap B^c$
42. If a set has 13 elements and R is a reflexive relation on A with n elements, then
a) $13 \le n \le 26$ b) $0 \le n \le 26$ c) $13 \le n \le 169$ d) $0 \le n \le 169$
43. Let X be the set of all engineering colleges in a state of Indian Republic and R be a relation on X defined as two colleges are related iff they are affiliated to the same university, then R is
a) Only reflexive b) Only symmetric c) Only transitive d) Equivalence
44. In the above question, the number of families which buy none of A, B and C is
a) 4000 b) 3300 c) 4200 d) 5000
45. If A and B are two sets, then $A \cap (A \cup B)$ equals
a) A b) B c) ϕ d) None of these
46. If $A = \{1,3,5,7,9,11,13,15,17\}, B = \{2,4 \dots ,18\}$ and N is the universal set, then $A' \cup ((A \cup B) \cap B')$ is

a) A b) N c) B d) none of these

47. If $A = \{\phi, \{\phi\}\}$, then the power set of A is
a) A b) $\{\phi, \{\phi\}, A\}$ c) $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$ d) None of these

48. Let $A = \{(x, y): y = e^x, x \in R\}$,
$B = \{(x, y): y = e^{-x}, x \in R\}$. Then,
a) $A \cap B = \phi$ b) $A \cap B \neq \phi$ c) $A \cup B = R^2$ d) None of these

49. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha R \beta \Leftrightarrow \alpha \perp \beta, \alpha, \beta \in L$. Then R is
a) Reflexive b) Symmetric c) Transitive d) None of these

50. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then
a) $A = C$ b) $B = C$ c) $A \cap B = \phi$ d) $A = B$

51. Let $S = \{1, 2, 3, 4\}$. The total number of unordered pairs of disjoint subsets of S is equal to
a) 25 b) 34 c) 42 d) 41

52. If $A = \{(x, y): x^2 + y^2 = 4;\ x, y \in R\}$ and
$B = \{(x, y): x^2 + y^2 = 9; x, y \in R\}$, then
a) $A - B = \phi$ b) $B - A = B$ c) $A \cap B \neq \phi$ d) $A \cap B = A$

53. Let $n(\mathcal{U}) = 700, n(A) = 200, n(B) = 300$ and $n(A \cap B) = 100$. Then, $n(A^c \cap B^c) =$
a) 400 b) 600 c) 300 d) 200

54. If $A = \left\{\theta : \cos\theta > -\frac{1}{2}, 0 \leq \theta \leq \pi\right\}$ and
$B = \left\{\theta : \sin\theta > \frac{1}{2}, \frac{\pi}{3} \leq \theta \leq \pi\right\}$, then
a) $A \cap B = \{\theta : \pi/3 \leq \theta \leq 2\pi/3\}$
b) $A \cap B = \{\theta : -\pi/3 \leq \theta \leq 2\pi/3\}$
c) $A \cup B = \{\theta: -5\pi/6 \leq \theta \leq 5\pi/6\}$
d) $A \cup B = \{\theta : 0 \leq \theta \leq \pi/6\}$

55. In a set of ants in a locality, two ants are said to be related iff they walk on a same straight line, then the relation is
a) Reflexive and symmetric
b) Symmetric and transitive
c) Reflexive and transitive
d) Equivalence

56. If $A = \{1, 2, 3\}, B = \{a, b\}$, then $A \times B$ mapped A to B is
a) $\{(1, a), (2, b), (3, b)\}$ b) $\{(1, b), (2, a)\}$
c) $\{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$ d) $\{(1, a), (2, a), (2, b), (3, b)\}$

57. If A_n is the set of first n prime numbers, then $\bigcup_{n=2}^{10} A_n =$
a) $\{2,3,5,7,11,13,17,19\}$ b) $\{2,3,5,7,11,13,17,19,2$ c) $\{3,5\}$ d) $\{2,3\}$

58. If $A = \{4, 6, 10, 12\}$ and R is a relation defined on A as "two elements are related iff they have exactly one common factor other than 1". Then the relation R is
a) Antisymmetric b) Only transitive c) Only symmetric d) Equivalence

59. If R is a relation from a set A to a set B and S is a relation from B to a set C, then the relation SoR
a) Is from A to C b) Is from C to A c) Does not exist d) None of these

60. Let n be a fixed positive integer. Define a relation R on the set Z of integers by, $aRb \Leftrightarrow n \mid a - b$. Then, R is not
a) Reflexive b) Symmetric c) Transitive d) None of these

61. If $n(A_i) = i + 1$ and $A_1 \subset A_2 \subset A_3 \subset \cdots \subset A_{99}$, then $n(\bigcup_{i=1}^{99} A_i) =$
a) 99 b) 98 c) 100 d) 101

62. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more

than the total number of subsets of the second set. The values of m and n are
a) $m = 7, n = 6$ b) $m = 6, n = 3$ c) $m = 5, n = 1$ d) $m = 8, n = 7$

63. Let A be the set of all animals. A relation R is defined as "aRb iff a and b are in different zoological parks". Then R is
a) Only reflexive b) Only symmetric c) Only transitive d) Equivalence

64. Let X and Y be the sets of all positive divisors of 400 and 1000 respectively (including 1 and the number). Then, $n(X \cap Y)$ is equal to
a) 4 b) 6 c) 8 d) 12

65. Let R be a relation from a set A to a set B, then
a) $R = A \cup B$ b) $R = A \cap B$ c) $R \subseteq A \times B$ d) $R \subseteq B \times A$

66. If X and Y are two sets, then $X \cap (Y \cup X)'$ equals
a) X b) Y c) ϕ d) None of these

67. If $A = \{1, 2, 3, 4, 5, 6\}$, then how many subsets of A contain the elements 2, 3 and 5?
a) 4 b) 8 c) 16 d) 32

68. For any three sets A_1, A_2, A_3, let $B_1 = A_1, B_2 = A_2 - A_1$ and $B_3 = A_3 - (A_1 \cup A_2)$, then which one of the following statement is always true
a) $A_1 \cup A_2 \cup A_3 \supset B_1 \cup B_2 \cup B_3$
b) $A_1 \cup A_2 \cup A_3 = B_1 \cup B_2 \cup B_3$
c) $A_1 \cup A_2 \cup A_3 \subset B_1 \cup B_2 \cup B_3$
d) None of these

69. If A is a non-empty set, then which of the following is false?
p : There is at least one reflexive relation on A
q : There is at least one symmetric relation on A
a) p alone b) q alone c) Both p and q d) Neither p nor q

70. In an election, two contestants A and B contested $x\%$ of the total voters voted for A and $(x + 20)\%$ for B. If 20% of the voters did not vote, then $x =$
a) 30 b) 25 c) 40 d) 35

71. Let $A = \{1,2,3,4\}$, and let $R = \{(2,2), (3,3), (4,4), (1,2)\}$ be a relation on A. Then, R is
a) Reflexive b) Symmetric c) Transitive d) None of these

72. In a rehabilitation programme, a group of 50 families were assured new houses and compensation by the government. Number of families who got both is equal to the number of families who got neither of the two. The number of families who got new houses is 6 greater than the number of families who got compensation. How many families got houses?
a) 22 b) 28 c) 23 d) 25

73. Let $\mathcal{U}$ be the universal set for sets A and B such that $n(A) = 200, n(B) = 300$ and $n(A \cap B) = 100$. Then, $n(A' \cap B')$ is equal to 300, provided that $n(\mathcal{U})$ is equal to
a) 600 b) 700 c) 800 d) 900

74. An integer m is said to be related to another integer n if m is a multiple of n. Then, the relation is
a) Reflexive and symmetric
b) Reflexive and transitive
c) Symmetric and transitive
d) Equivalence relation

75. Three sets A, B, C are such that $A = B \cap C$ and $B = C \cap A$, then
a) $A \subset B$ b) $A \supset B$ c) $A \equiv B$ d) $A \subset B'$

76. Let R be a relation on the set N of natural numbers defined by $nRm \Leftrightarrow n$ is a factor of m(i.e. $n|m$). Then, R is
a) Reflexive and symmetric

b) Transitive and symmetric
c) Equivalence
d) Reflexive, transitive but not symmetric

77. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N$ are relatively prime, then
a) $d = bc$ b) $c = bd$ c) $b = cd$ d) None of these

78. In rule method the null set is represented by
a) { } b) Φ c) $\{x : x \neq x\}$ d) $\{x : x = x\}$

79. Let A be a set represented by the squares of natural number and x, y are any two elements of A. Then,
a) $x - y \in A$ b) $xy \in A$ c) $x + y \in A$ d) $\frac{x}{y} \in A$

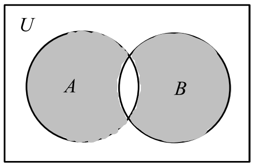
80. Let $A_1, A_2, A_3 \dots, A_{100}$ be 100 sets such that $n(A_i) = i + 1$ and $A_1 \subset A_2 \subset A_3 \subset \cdots \subset A_{100}$, then $\cup_{i=1}^{100} A_i$ contains… elements
a) 99 b) 100 c) 101 d) 102

81. In a certain town 25% families own a cell phone, 15% families own a scooter and 65% families own neither a cell phone nor a scooter. If 1500 families own both a cell phone and a scooter, then the total number of families in the town is
a) 10000 b) 20000 c) 30000 d) 40000

82. If A, B and C are three non-empty sets such that any two of them are disjoint, then $(A \cup B \cup C) \cap (A \cap B \cap C) =$
a) A b) B c) C d) ϕ

83. If $R = \{(a, b): a + b = 4\}$ is a relation on N, then R is
a) Reflexive b) Symmetric c) Antisymmetric d) Transitive

84. The shaded region in the figure represents



a) $A \cap B$ b) $A \cup B$ c) $B - A$ d) $(A - B) \cup (B - A)$

85. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 3, 5, 7, 9\}$. Which of the following is/are not relations from X to Y?
a) $R_1 = \{(x, y) | y = 2 + x, x \in X, y \in Y\}$
b) $R_2 = \{(1,1), (2,1), (3,3), (4,3), (5,5)\}$
c) $R_3 = \{(1,1), (1,3), (3,5), (3,7), (5,7)\}$
d) $R_4 = \{(1,3), (2,5), (2,4), (7,9)\}$

86. Given the relation $R = \{(1,2), (2,3)\}$ on the set $A = \{1,2,3\}$, the minimum number of ordered pairs which when added to R make it an equivalence relation is
a) 5 b) 6 c) 7 d) 8

87. If sets A and B are defined as
$A = \left\{(x, y): y = \frac{1}{x}, 0 \neq x \in R\right\}$,
$B = \{(x, y): y = -x, x \in R\}$, then
a) $A \cap B = A$ b) $A \cap B = B$ c) $A \cap B = \phi$ d) None of these

88. Let R be an equivalence relation on a finite set A having n elements. Then, the number of ordered pairs in R is
a) Less than n
b) Greater than or equal to n
c) Less than or equal to n

d) None of these

89. If $A_1 \subset A_2 \subset A_3 \subset \cdots \subset A_{50}$ and $n(A_i) = i - 1$, then $n\left(\bigcap_{i=11}^{50} A_i\right) =$
a) 49 b) 50 c) 11 d) 10

90. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N$ then
a) $d = bc$ b) $c = bd$ c) $b = cd$ d) None of these

91. X is the set of all residents in a colony and R is a relation defined on X as follows:
"Two persons are related iff they speak the same language"
The relation R is
a) Only symmetric
b) Only reflexive
c) Both symmetric and reflexive but not transitive
d) Equivalence

92. If S is a set with 10 elements and $A = \{(x, y): x, y \in S, x \neq y\}$, then the number of elements in A is
a) 100 b) 90 c) 50 d) 45

93. Let $A = \{\text{ONGC, BHEL, SAIL, GAIL, IOCL}\}$ and R be a relation defined as "two elements of A are related if they share exactly one letter". The relation R is
a) Anti-symmetric b) Only transitive c) Only symmetric d) Equivalence

94. The finite sets A and B have m and n elements respectively. if the total number of subsets of A is 112 more than the total number of subsets of B, then the volume of m is
a) 7 b) 9 c) 10 d) 12

95. Let $R = \{(a, a)\}$ be a relation on a set A. Then, R is
a) Symmetric
b) Antisymmetric
c) Symmetric and antisymmetric
d) Neither symmetric nor antisymmetric

96. If $A = \{p: p = \frac{(n+2)(2n^5+3n^4+4n^3+5n^2+6)}{n^2+2n}, n, p \in Z^+\}$ then the number of elements in the set A, is
a) 2 b) 3 c) 4 d) 6

97. If $A = \{x : x \text{is a multiple of } 3\}$ and,
$B = \{x : x \text{is a multiple of } 5\}$, then $A - B$ is
a) $\bar{A} \cap B$ b) $A \cap \bar{B}$ c) $\bar{A} \cap \bar{B}$ d) $\overline{A \cap B}$

98. An investigator interviewed 100 students to determine the performance of three drinks milk, coffee and tea. The investigator reported that 10 students take all three drinks milk, coffee and tea; 20 students take milk and coffee, 30 students take coffee and tea, 25 students take mile and tea, 12 students take milk only, 5 students take coffee only and 8 students take tea only. Then, the number of students who did not take any of the three drinks, is
a) 10 b) 20 c) 25 d) 30

99. Consider the following statements:
(i) Every reflexive relation is antisymmetric
(ii) Every symmetric relation is antisymmetric
Which one among (i) and (ii) is true?
a) (i) alone is true
b) (ii) alone is true
c) Both (i) and (ii) true
d) Neither (i) and (ii) is true

100. Given $n(U) = 20, n(A) = 12, n(B) = 9, n(A \cap B) = 4$, where U is the universal set, A and B are subsets of U, then $n[(A \cup B)^c]$ equals to
a) 10 b) 9 c) 11 d) 3

101. Let Z denote the set of integers, then $\{x \in Z: |x-3| < 4\} n \{x \in Z: |x-4| < 5\} =$

a) $\{-1,0,1,2,3,4\}$ b) $\{-1,0,1,2,3,4,5\}$ c) $\{0,1,2,3,4,5,6\}$ d) $\{-1,0,1,2,3,5,6,7,8,9\}$

102. Let R be a reflexive relation on a finite set A having n elements, and let there be m ordered pairs in R. Then,

a) $m \geq n$ b) $m \leq n$ c) $m = n$ d) None of these

103. Let $A = \{1, 2, 3\}, B = \{3, 4\}, C = \{4, 5, 6\}$. Then, $A \cup (B \cap C)$ is

a) $\{3\}$ b) $\{1, 2, 3, 4\}$ c) $\{1, 2, 5, 6\}$ d) $\{1, 2, 3, 4, 5, 6\}$

104. If $A = \left\{(x, y) : y = \frac{4}{x}, x \neq 0\right\}$ and $B = \{(x, y): x^2 + y^2 = 8, x, y \in R\}$, then

a) $A \cap B = \phi$

b) $A \cap B$ contains one point only

c) $A \cap B$ contains two points only

d) $A \cap B$ contains 4 points only

105. If $R = \{(a, b): |a + b| = a + b\}$ is a relation defined on a set $\{-1, 0, 1\}$, then R is

a) Reflexive b) Symmetric c) Anti symmetric d) Transitive

106. If $n(A \cap B) = 5, n(A \cap C) = 7$ and $n(A \cap B \cap C) = 3$, then the minimum possible value of $n(B \cap C)$ is

a) 0 b) 1 c) 3 d) 2

107. The relation $R = \{(1,3), (3,5)\}$ is defined on the set with minimum number of elements of natural numbers. The minimum number of elements to be included in R so that R is an equivalence relation, is

a) 5 b) 6 c) 7 d) 8

108. If $A = \{1, 2, 3\}$, then the relation $R = \{(1,1), (2,2), (3,1), (1,3)\}$ is

a) Reflexive b) Symmetric c) Transitive d) Equivalence

109. Let R be a relation on a set A such that $R = R^{-1}$, then R is

a) Reflexive b) Symmetric c) Transitive d) None of these

110. In Q.No. 6, $\bigcap_{n=3}^{10} A_n =$

a) $\{3,5,7,11,13,17,19\}$ b) $\{2,3,5\}$ c) $\{2,3,5,7,11,13,17\}$ d) $\{3,5,7\}$

111. The number of elements in the set $\{(a, b): 2a^2 + 3b^2 = 35, a, b \in Z\}$, where Z is the set of all integers, is

a) 2 b) 4 c) 8 d) 12

112. If $A = \{a, b, c\}, B = \{b, c, d\}$ and $C = \{a, d, c\}$, then $(A - B) \times (B \cap C)$ is equal to

a) $\{(a, c), (a, d)\}$ b) $\{(a, b), (c, d)\}$ c) $\{(c, a), (d, a)\}$ d) $\{(a, c), (a, d), (b, d)\}$

113. A class has 175 students. The following data shows the number of students opting one or more subjects. Mathematics 100; Physics 70; Chemistry 40; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 23; Mathematics, Physics and Chemistry 18. Hoe many students have offered Mathematics alone?

a) 35 b) 48 c) 60 d) 22

114. If $A = \{1, 2, 3\}, B\{3, 4\}, C\{4, 5, 6\}$. Then, $A \cup (B \cap C)$ is

a) $\{1, 2\}$ b) $\{\phi\}$ c) $\{4, 5\}$ d) $\{1, 2, 3, 4\}$

115. If $A \subseteq B$, then $B \cup A$ is equal to

a) $B \cap A$ b) A c) B d) None of these

116. If $n(u) = 100, n(A) = 50, n(B) = 20$ and $n(A \cap B) = 10$, then $n\{(A \cup B)^c\}$

a) 60 b) 30 c) 40 d) 20

117. If A is a non-empty set, then which of the following is false?
p : Every reflexive relation is a symmetric relation
q : Every antisymmetric relation is reflexive
Which of the following is/are true?

a) p alone b) q alone c) Both p and q d) Neither p nor q

118. Two points P and Q in a plane are related if $OP = OQ$, where O is a fixed point. This relation is

a) Partial order relation
b) Equivalence relation
c) Reflexive but not symmetric
d) Reflexive but not transitive

119. In a city 20% of the population travels by car, 50% travels by bus and 10% travels by both car and bus. Then, persons travelling by car or bus is

a) 80% b) 40% c) 60% d) 70%

120. If $n(A \cap B = 10, n(B \cap C) = 20)$ and $n(A \cap C) = 30$, then the greatest possible value of $n(A \cap B \cap C)$ is

a) 15 b) 20 c) 10 d) 4

121. If S is the set of squares and R is the set of rectangles, then $(S \cup R) - (S \cap S)$ is

a) S
b) R
c) Set of squares but not rectangles
d) Set of rectangles but not squares

122. Let X be a family of sets and R be a relation on X defined by $'A$ is disjoint from B'. Then, R is

a) Reflexive b) Symmetric c) Antisymmetric d) Transitive

123. If $A = \{x, y\}$, then the power set of A is

a) $\{x^y, y^x\}$ b) $\{\phi, x, y\}$ c) $\{\phi, \{x\}, \{2y\}\}$ d) $\{\phi, \{x\}, \{y\}, \{x, y\}\}$

124. In a town of 10,000 familiesit was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then the number of families which buy A only is

a) 3100 b) 3300 c) 2900 d) 1400

125. Let R and S be two equivalence relations on a set A. Then,

a) $R \cup S$ is an equivalence relation on A
b) $R \cap S$ is an equivalence relation on A
c) $R - S$ is an equivalence relation on A
d) None of these

126. Which of the following is true?

a) $A \cap \phi = A$ b) $A \cap \phi = \phi$ c) $A \cap \phi = U$ d) $A \cap \phi = A'$

127. Let $A = \{p, q, r\}$. Which of the following is not an equivalence relation on A?

a) $R_1 = \{(p,q),(q,r),(p,r),(p,p)\}$
b) $R_2 = \{(r,q),(r,p),(r,r),(q,q)\}$
c) $R_3 = \{(p,p),(q,q),(r,r).(p,q)\}$
d) None of these

128. Let $A = \{1, 2, 3, 4\}, B = \{2, 4, 6\}$. Then, the number of sets C such that $A \cap B \subseteq C \subseteq A \cup B$ is

a) 6 b) 9 c) 8 d) 10

129. If $A = \left\{p \in N: p \text{is } a \text{ prime and } p = \frac{7n^2+3n+3}{n} \text{ for some} n \in N\right\}$, then the number of elements in the set A, is

a) 1 b) 2 c) 3 d) 4

130. Let $Y = \{1, 2, 3, 4, 5\}, A\{1, 2\}, B = \{3, 4, 5\}$ and ϕ denotes null set. If $(A \times B)$ denotes cartesian product of the sets A and B; then $(Y \times A) \cap (Y \times B)$ is

a) Y b) A c) B d) ϕ

131. If $n(A)$ denotes the number of elements in the set A and if $n(A) = 4, n(B) = 5$ and $n(A \cap B) = 3$, then $n[(A \times B) \cap (B \times A)]$ is equal to

a) 8 b) 9 c) 10 d) 11

132. Universal set, $U = \{x: x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$
And $A = \{x: x^2 - 5x + 6 = 0\}$
$B = \{x: x^2 - 3x + 2 = 0\}$
Then, $(A \cap B)'$ is equal to

a) $\{1, 3\}$ b) $\{1, 2, 3\}$ c) $\{0, 1, 3\}$ d) $\{0, 1, 2, 3\}$

133. If R be a relation $<$ from $A = \{1,2,3,4\}$ to $B = \{1,3,5\}$ $i.e.$ $(a, b) \in R \Leftrightarrow a < b$, then RoR^{-1} is

a) $\{(1,3),(1,5),(2,3),(2,5),(3,5),(4,5)\}$
b) $\{(3,1),(5,1),(3,2),(5,2),(5,3),(5,4)\}$
c) $\{(3,3),(3,5),(5,3),(5,5)\}$
d) $\{(3,3),(3,4),(4,5)\}$

134. A relation between two persons is defined as follows:
$aRb \Leftrightarrow a$ and b born in different months. Then, R is

a) Reflexive b) Symmetric c) Transitive d) Equivalence

135. If A and B are two sets such that $n(A \cap \bar{B}) = 9, n(\bar{A} \cap B) = 10$ and $n(A \cup B) = 24$, then $n(A \times B) =$

a) 105 b) 210 c) 70 d) None of these

136. If A and B are two sets, then $A - (A - B)$ is equal to

a) B b) $A \cup B$ c) $A \cap B$ d) $B - A$

137. If $A = \{1, 2, 3, 4\}$, then the number of subsets of A that contain the element 2 but not 3, is

a) 16 b) 4 c) 8 d) 24

138. Let A be a set of compartments in a train. Then the relation R defined on A as aRb iff "a and b have the link between them", then which of the following is true for R?

a) Reflexive b) Symmetric c) Transitive d) Equivalence

139. Let R and S be two relations on a set A. Then, which one of the following is not true?

a) R and S are transitive, then $R \cup S$ is also transitive

b) R and S are transitive, then $R \cap S$ is also transitive
c) R and S are reflexive, then $R \cap S$ is also reflexive
d) R and S are symmetric, then $R \cup S$ is also symmetric

140. The relation "is a factor of" on the set N of all natural numbers is not

a) Reflexive b) Symmetric c) Antisymetric d) Transitive

141. If $R \subset A \times B$ and $S \subset B \times C$ be relations, then $(SoR)^{-1} =$

a) $S^{-1}oR^{-1}$ b) $R^{-1}oS^{-1}$ c) SoR d) RoS

142. If relation R is defined as :
aRb if "a is the father of b". Then, $R$is

a) Reflexive b) Symmetric c) Transitive d) None of these

143. In a set of teachers of a school, two teachers are said to be related if they "teach the same subject", then the relation is

a) Reflexive and symmetric
b) Symmetric and transitive
c) Reflexive and transitive
d) Equivalence

144. In a battle 70% of the combatants lost one eye, 80% an ear, 75% an arm, 85% a leg, x% lost all the four limbs. The minimum value of x is

a) 10 b) 12 c) 15 d) None of these

145. If $A = \{1, 2, 3, 4\}$, then the number of subsets of set A containing element 3, is

a) 24 b) 28 c) 8 d) 16

146. The relation $R = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\}$ on set $A = \{1,2,3\}$ is

a) Reflexive but not symmetric
b) Reflexive but not transitive
c) Symmetric and transitive
d) Neither symmetric nor transitive

147. The value of $(A \cup B \cup C) \cap (A \cap B^C \cap C^C)^C \cap C^C$is

a) $B \cap C^C$ b) $B^C \cap C^C$ c) $B \cap C$ d) $A \cap B \cap C$

148. If a set A contains n elements, then which of the following cannot be the number of reflexive relations on the set A?

a) 2^n b) 2^{n-1} c) 2^{n^2-1} d) 2^{n+1}

149. If A and B are two sets such that $n(A) = 7, n(B) = 6$ and $(A \cap B) \neq \phi$. The least possible value of $n(A\Delta B)$, is

a) 1 b) 7 c) 6 d) 13

150. Set builder form of the relation
$R = \{(-2, -7), (-1, -4), (0, -1), (1, 2), (2, 5)\}$ is

a) $\{(a, b): b = 2a - 3; a, b, \in Z\}$
b) $((x, y): y = 3x - 1; x, y \in Z\}$
c) $\{(a, b): b = 3a - 1; a, b \in N\}$
d) $\{(u, v): v = 3u - 1; -2 \leq u < 3$ and $u \in Z\}$

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1)	b	2)	d	3)	c	4)	a
5)	a	6)	d	7)	a	8)	a
9)	d	10)	b	11)	d	12)	d
13)	d	14)	c	15)	a	16)	b
17)	d	18)	c	19)	c	20)	b
21)	c	22)	b	23)	d	24)	a
25)	b	26)	c	27)	c	28)	a
29)	d	30)	c	31)	d	32)	c
33)	a	34)	b	35)	b	36)	c
37)	b	38)	b	39)	b	40)	c
41)	d	42)	c	43)	d	44)	a
45)	a	46)	b	47)	c	48)	b
49)	b	50)	b	51)	d	52)	b
53)	c	54)	a	55)	d	56)	c
57)	b	58)	c	59)	a	60)	d
61)	c	62)	b	63)	b	64)	d
65)	c	66)	c	67)	b	68)	a
69)	d	70)	a	71)	c	72)	b
73)	b	74)	b	75)	c	76)	d
77)	a	78)	c	79)	b	80)	c
81)	c	82)	d	83)	b	84)	d
85)	d	86)	c	87)	c	88)	b
89)	d	90)	d	91)	d	92)	b
93)	c	94)	a	95)	c	96)	c
97)	b	98)	b	99)	d	100)	d
101)	c	102)	a	103)	b	104)	c
105)	b	106)	c	107)	a	108)	b
109)	b	110)	b	111)	c	112)	a
113)	c	114)	d	115)	c	116)	c
117)	d	118)	b	119)	c	120)	c
121)	d	122)	b	123)	d	124)	b
125)	b	126)	b	127)	d	128)	c
129)	a	130)	d	131)	b	132)	c
133)	c	134)	b	135)	b	136)	c
137)	b	138)	b	139)	a	140)	b
141)	b	142)	d	143)	d	144)	a
145)	c	146)	a	147)	b	148)	d
149)	a	150)	d				

1 **(b)**
For any $a \in R$, we have $a \geq a$
Therefore, the relation R is reflexive.
R is not symmetric as $(2,1) \in R$ but $(1,2) \notin R$. The relation R is transitive also, because $(a,b) \in R, (b,c) \in R$ imply that $a \geq b$ and $b \geq c$ which in turn imply that $a \geq c$

2 **(d)**
Clearly, R is an equivalence relation

3 **(c)**
Let M and E denote the sets of students who have taken Mathematics and Economics respectively. Then, we have
$n(M \cup E) = 35, n(M) = 17$ and $n(M \cap E') = 10$
Now,
$n(M \cap E') = n(M) - n(M \cap E)$
$\Rightarrow 10 = 17 - n(M \cap E) \Rightarrow n(M \cap E) = 7$
Now,
$n(M \cup E) = n(M) + n(E) - n(M \cap E)$
$\Rightarrow 35 = 17 + n(E) - 7 \Rightarrow n(E) = 25$
$\therefore n(E \cap M') = n(E) - n(E \cap M) = 25 - 7 = 18$

4 **(a)**
Let $A = \{n(n+1)(2n+1): n \in Z\}$
Putting $n = \pm1, \pm2, \ldots.$, we get $A = \{\ldots - 30, -6, 0, 6, 30, \ldots\}$
$\Rightarrow \quad \{n(n+1)(2n+1): n \in Z\} \subset \{6k: k \in Z\}$

5 **(a)**
$\because \quad A \cup B = \{1, 2, 3, 4, 5, 6\}$
$\therefore \quad (A \cup B) \cap C = \{1, 2, 3, 4, 5, 6\} \cap \{3, 4, 6\}$
$= \{3, 4, 6\}$

6 **(d)**
We have,
$n(A \cap \bar{B}) = 9, n(\bar{A} \cap B) = 10$ and $n(A \cup B) = 24$
$\Rightarrow n(A) - n(A \cap B) = 9, n(B) - n(A \cap B) = 10$ and, $n(A) + n(B) - n(A \cap B) = 24$
$\Rightarrow n(A) + n(B) - 2n(A \cap B) = 19$ and $n(A) + n(B) - n(A \cap B) = 24$
$\Rightarrow n(A \cap B) = 5$
$\therefore n(A) = 14$ and $n(B) = 15$
Hence, $n(A \times B) = 14 \times 15 = 210$

7 **(a)**
Clearly, $P \subset T$
$\therefore P \cap T = P$

8 **(a)**
It is given that A is a proper subset of B
$\therefore A - B = \phi \Rightarrow n(A - B) = 0$
We have, $n(A) = 5$. So, minimum number of elements in B is 6
Hence, the minimum possible value of $n(A\Delta B)$ is $n(B) - n(A) = 6 - 5 = 1$

9 **(d)**
$\because \quad n(A \times B \times C) = n(A) \times n(B) \times n(C)$
$\therefore \quad n(C) = \frac{24}{4 \times 3} = 2$

10 **(b)**

Use $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

11 **(d)**

$\because A = \{(a, b): a^2 + 3b^2 = 28, a, b \in Z\}$

=\{(5, 1), (-5, -1), (5, -1), (-5, 1), (1, 3), (-1, -3), (-1, 3), (1, -3), (4, 2), (-4, -2), (4, -2), (-4, 2)\}

And $B = \{(a, b): a > b, a, b \in Z\}$

$\therefore \quad A \cap B = \{(-1, -5), (1, -5), (-1, -3), (1, -3), (4, 2), (4, -2)\}$

$\therefore$ Number of elements in $A \cap B$ is 6.

13 **(d)**

We have

$R = \{(1,39), (2,37), (3,35), (4,33), (5,31), (6,29), (7,27), (8,25), (9,23), (10,21), (11,19), (12,17), (13,15), (14,13), (15,11), (16,9), (17,7), (18,5), (19,3), (20,1)\}$

Since $(1,39) \in R$, but $(39,1) \notin R$

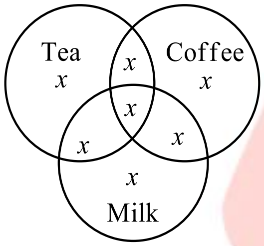
Therefore, R is not symmetric

Clearly, R is not reflexive. Now, $(15,11) \in R$ and $(11,19) \in R$ but $(15,19) \notin R$

So, R is not transitive

14 **(c)**

Total number of employees $= 7x$ i.e. a multiple of 7. Hence, option (c) is correct



15 **(a)**

The power set of a set containing n elements has 2^n elements.

Clearly, 2^n cannot be equal to 26

16 **(b)**

The relation is not symmetric, because $A \subset B$ does not imply that $B \subset A$. But, it is anti-symmetric because

$A \subset B$ and $B \subset A \Rightarrow A = B$

18 **(c)**

We have, $A \supset B \supset C$

$\therefore A \cup B \cup C = A$ and $A \cap B \cap C = C$

$\Rightarrow (A \cup B \cup C) - (A \cap B \cap C) = A - C$

19 **(c)**

Given, $n(C) = 63, n(A) = 76$ and $n(C \cap A) = x$

We know that,

$n(C \cup A) = n(C) + n(A) - n(C \cap A)$

$\Rightarrow \quad 100 = 63 + 76 - x \Rightarrow x = 139 - 100 = 39$

And $n(C \cap A) \leq n(C)$

$\Rightarrow \quad x \leq 63 \qquad \therefore 39 \leq x \leq 63$

20 **(b)**

We have,

$X =$ Set of some multiple of 9

and, Y = Set of all multiple of 9
$\therefore X \subset Y \Rightarrow X \cup Y = Y$

21 **(c)**
$A \cap B = \{x: x\text{a multiple of }3\}\text{and }\{x: x\text{is a multiple of }5\}$
$= \{x: x\text{is a multiple of }15\}$
$= \{15, 30, 45, \dots \dots \dots .\}$

22 **(b)**
We have,
$n(A \times B) = 45$
$\Rightarrow n(A) \times n(B) = 45$
$\Rightarrow n(A)$and $n(B)$ are factors of 45 such that their product is 45
Hence, $n(A)$ cannot be 17

24 **(a)**
For any $x \in R$, we have
$x - x + \sqrt{2} = \sqrt{2}$ an irrational number
$\Rightarrow xRx$ for all x
So, R is reflexive
R is not symmetric, because $\sqrt{2}R\ 1$ but $1 \not R \sqrt{2}$
R is not transitive also because $\sqrt{2}R\ 1$ and $1\ R\ 2\sqrt{2}$ but $\sqrt{2} \not R\ 2\sqrt{2}$

25 **(b)**
We have,
$n(H) - n(H \cap E) = 22, n(E) - n(H \cap E) = 12, n(H \cup E) = 45$
$\therefore n(H \cup E) = n(H) + n(E) - n(H \cap E)$
$\Rightarrow 45 = 22 + 12 + n(H \cup E)$
$\Rightarrow n(H \cap E) = 11$

26 **(c)**
We have, $A \subset B$ and $B \subset C$
$\therefore A \cup B = B\text{and } B \cap C = B$
$\Rightarrow A \cup B = B \cap C$

27 **(c)**
Let $A = \left\{x \in R: \frac{2x-1}{x^3+4x^2+3x}\right\}$
Now, $x^3 + 4x^2 + 3x = x(x^2 + 4x + 3)$
$= x(x+3)(x+1)$
$\therefore \quad A = R - \{0, -1, -3\}$

29 **(d)**
Clearly, $y^2 = x$ and $y = |x|$ intersect at $(0,0), (1,1)$ and $(-1, -1)$. Hence, option (d) is correct

31 **(d)**
Let M, P and C be the sets of students taking examinations in Mathematics, Physics and Chemistry respectively.
We have,
$n(M \cup P \cup C) = 50, n(M) = 37, n(P) = 24, n(C) = 43$
$n(M \cap P) < 19, n(M \cap C) \leq 29, n(P \cap C) \leq 20$
Now,
$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P)$
$-n(M \cap C) - n(P \cap C) + n\ (M \cap P \cap C)$
$\Rightarrow 50 = 37 + 24 + 43 - \{n(M \cap P) + n(M \cap C) + n(P \cap C)\}$

$+n(M \cap P \cap C)$
$\Rightarrow n(M \cap P \cap C) = n(M \cap P) + n(M \cap C) + n(P \cap C) - 54$
$\Rightarrow n(M \cap P) + n(M \cap C) + n(P \cap C)$
$= n(M \cap P \cap C) + 54$ ...(i)
Now,
$n(M \cap P) \leq 19, n(M \cap C) \leq 29, n(P \cap C) \leq 20$
$\Rightarrow n(M \cap P) + n(M \cap C) + n(P \cap C) \leq 19 + 29 + 20$ [Using (i)]
$\Rightarrow n(M \cap P \cap C) + 54 \leq 68$
$\Rightarrow n(M \cap P \cap C) \leq 14$

33 **(a)**
Given, $n(N) = 12, n(P) = 16, n(H) = 18,$
$n(N \cup P \cup H) = 30$
And $n(N \cap P \cap H) = 0$
Now, $n(N \cup P \cup H) = n(N) + n(P) + n(H)$
$-n(N \cap P) - n(P \cap H) - n(H \cap N)$
$+n(N \cap P \cap H)$
$\Rightarrow n(N \cap P) + n(P \cap H) + n(H \cap N) = (12 + 16 + 18) - 30$
$= 46 - 30 = 16$

35 **(b)**
The void relation R on A is not reflexive as $(a, a) \notin R$ for any $a \in A$. The void relation is symmetric and transitive

36 **(c)**
Given, A's are 30 sets with five elements each, so
$\sum_{i=1}^{30} n(A_i) = 5 \times 30 = 150$ ...(i)
If the m distinct elements in S and each elements of S belongs to exactly 10 of the A_i's, then
$\sum_{i=1}^{30} n(A_i) = 10m$ ...(ii)
From Eqs. (i) and (ii), $m = 15$
Similarly, $\sum_{j=1}^{n} n(B_j) = 3n$ and $\sum_{j=1}^{n} n(B_j) = 9m$
$\therefore \quad 3n = 9m$
$\Rightarrow \quad n = \frac{9m}{3} = 3 \times 15 = 45$

38 **(b)**
$A \cup B$ will contain minimum number of elements if $A \subset B$ and in that case, we have
$n(A \cup B) = n(B) = 6$

40 **(c)**
It is given that $A_1 \subset A_2 \subset A_3 \subset \cdots \subset A_{100}$
$\therefore \bigcup_{i=3}^{100} A_i = A \Rightarrow A_3 = A \Rightarrow n(A) = n(A_3) = 3 + 2 = 5$

41 **(d)**
We have,
$A \cap (A \cap B)^c = A \cap (A^c \cup B^c)$
$\Rightarrow A \cap (A \cap B)^c = (A \cap A^c) \cup (A \cap B^c)$
$\Rightarrow A \cap (A \cap B)^c = \phi \cup (A \cap B^c) = A \cap B^c$

42 **(c)**
Since R is a reflexive relation on A.
$\therefore (a, a) \in R$ for all $a \in A$

$\Rightarrow n(A) \le n(R) \le n(A \times A) \Rightarrow 13 \le n(R) \le 169$

43 **(d)**

Clearly, R is reflexive symmetric and transitive. So, it is an equivalence relation

44 **(a)**

We have,

Required number of families

$= n(A' \cap B' \cap C')$

$= n(A \cup B \cup C)'$

$= N - n(A \cup B \cup C)$

$= 10000 - \{n(A) + n(B) + n(C) - n(A \cap B)\}$

$-n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)\}$

$= 10000 - 4000 - 2000 - 1000 + 500 + 300 + 400 - 200$

$= 4000$

45 **(a)**

We have,

$A \subset A \cup B$

$\Rightarrow A \cap (A \cup B) = A$

46 **(b)**

We have,

$(A \cup B) \cap B' = A$

$\therefore \big((A \cup B) \cap B'\big) \cup A' = A \cup A' = N$

48 **(b)**

The set A consists of all points on $y = e^x$ and the set Bconsists of points on $y = e^{-x}$, these two curves intersect at (0, 1). Hence, $A \cap B$ consists of a single point

50 **(b)**

Given, $A \cap B = A \cap C$ and $A \cup B = A \cup C$

$\Rightarrow \quad B = C$

51 **(d)**

Required number

$$= \frac{3^4 + 1}{2} = 41$$

52 **(b)**

Clearly, A is the set of all points on a circle with centre at the origin and radius 2 and B is the set of all points on a circle with centre at the origin and radius 3. The two circles do not intersect. Therefore,

$A \cap B = \phi \Rightarrow B - A = B$

53 **(c)**

We have,

$n(A^c \cap B^c)$

$= n\{(A \cup B)^c\}$

$= n(\mathcal{U}) - n(A \cup B)$

$= n(\mathcal{U}) - \{n(A) + n(B) - n(A \cap B)\}$

$= 700 - (200 + 300 - 100) = 300$

54 **(a)**

We have,

$\cos\theta > -\frac{1}{2}$ and $0 \le \theta \le \pi$

$\Rightarrow 0 \le \theta \le 2\pi/3$ and $0 \le \theta \le \pi$

$\Rightarrow 0 \le \theta \le \frac{2\pi}{3} \Rightarrow A = \{\theta : 0 \le \theta \le 2\pi/3\}$

Also,

$\sin\theta > \frac{1}{2}$ and $\pi/3 \le \theta \le \pi$

$\Rightarrow \frac{\pi}{3} \le \theta \le \frac{5\pi}{6} \Rightarrow B = \left\{\theta : \frac{\pi}{3} \le \theta \le \frac{5\pi}{6}\right\}$

$\therefore A \cap B = \left\{\theta : \frac{\pi}{3} \le \theta \le \frac{2\pi}{3}\right\}$ and $A \cup B = \left\{\theta : 0 \le \theta \le \frac{5\pi}{6}\right\}$

55 **(d)**

Clearly, R is an equivalence relation

56 **(c)**

Given, $A = \{1, 2, 3\}, B = \{a, b\}$

$\therefore \quad A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

57 **(b)**

Clearly,

$A_2 \subset A_3 \subset A_4 \subset \cdots \subset A_{10}$

$\therefore \bigcup_{n=2}^{10} A_n = A_{10} = \{2,3,5,7,11,13,17,19,23,29\}$

58 **(c)**

Clearly,

$R = \{(4,6), (4,10), (6,4), (10,4)(6,10), (10,6), (10,12), (12,10)\}$

Clearly, R is symmetric

$(6,10) \in R$ and $(10,12) \in R$ but $(6,12) \notin R$

So, R is not transitive

Also, R is not reflexive

61 **(c)**

It is given that

$A_1 \subset A_2 \subset A_3 \ldots \subset A_{99}$

$\bigcup_{i=1}^{999} A_i = A_{99}$

$\Rightarrow n\left(\bigcup_{i=1}^{99} A_i\right) = n(A_{99}) = 99 + 1 = 100$

62 **(b)**

It is given that $2^m - 2^n = 56$

Obviously, $m = 6, n = 3$ satisfy the equation

63 **(b)**

Clearly, $(a, a) \in R$ for any $a \in A$

Also,

$(a, b) \in R$

$\Rightarrow a$ and b are in different zoological parks

$\Rightarrow b$ and a are in different zoological parks

$\Rightarrow (b, a) \in R$

Now, $(a, b) \in R$ and $(b, a) \in R$ but $(a, a) \notin R$

So, R is not transitive

64 **(d)**

$X \cap Y = \{1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200\}$

$\therefore \quad n(X \cap Y) = 12$

66 **(c)**

We have,

$X \cap (Y \cup X)' = X \cap (Y' \cap X') = (X \cap X') \cap Y' = \phi \cap Y' = \phi$

67 **(b)**

The number of subsets of A containing 2, 3 and 5 is same as the number of subsets of set $\{1, 4, 6\}$ which is equal to $2^3 = 8$

68 **(a)**

We have,

$B_1 = A_1 \Rightarrow B_1 \subset A_1$

$B_2 = A_2 - A_1 \Rightarrow B_2 \subset A_2$

$B_3 = A_3 - (A_1 \cup A_2) \Rightarrow B_3 \subset A_3$

$\therefore B_1 \cup B_2 \cup B_3 \subset A_1 \cup A_2 \cup A_3$

69 **(d)**

The identity relation on a set A is reflexive and symmetric both. So, there is always a reflexive and symmetric relation on a set

70 **(a)**

Let the total number of voters be n. Then,

Number of voters voted for $A = \frac{nx}{100}$

Number of voters voted for $B = \frac{n(x+20)}{100}$

$\therefore$ Number of voters who voted for both

$= \frac{nx}{100} + \frac{n(x+20)}{100}$

$= \frac{n(2x+20)}{100}$

Hence, $n - \frac{n(2x+20)}{100} = \frac{20n}{100} \Rightarrow x = 30$

71 **(c)**

Since $(1,1) \notin R$. So, R is not reflexive

Now, $(1,2) \in R$ but, $(2,1) \notin R$. Therefore, R is not symmetric.

Clearly, R is transitive

72 **(b)**

Let A and B denote respectively the sets of families who got new houses and compensation

It is given that

$n(A \cap B) = n(\overline{A \cup B})$

$\Rightarrow n(A \cap B) = 50 - n(A \cup B)$

$\Rightarrow n(A) + n(B) = 50$

$\Rightarrow n(B) + 6 + n(B) = 50 \quad [\because n(A) = n(B) + 6 \text{ (given)}]$

$\Rightarrow n(B) = 22 \Rightarrow n(A) = 28$

73 **(b)**

We have,

$n(A' \cap B') = n((A \cup B)')$

$\Rightarrow n(A' \cap B') = n(\mathcal{U}) - n(A \cup B)$

$\Rightarrow n(A' \cap B') = n(\mathcal{U}) - \{n(A) + n(B) - n(A \cap B)\}$

$\Rightarrow 300 = n(\mathcal{U}) - \{200 + 300 - 100\}$

$\Rightarrow n(\mathcal{U}) = 700$

74 **(b)**

For any integer n, we have

$n|n \Rightarrow nRn$

So, nRn for all $n \in Z$

$\Rightarrow R$ is reflexive

Now, 2|6 but 6 does not divide 2

$\Rightarrow (2, 6) \in R$ but $(6,2) \notin R$

So, R is not symmetric

Let $(m, n) \in R$ and $(n, p) \in R$. Then,

$$\left.\begin{matrix}(m, n) \in R \Rightarrow m|n \\ (n, p) \in R \Rightarrow n|p\end{matrix}\right\} \Rightarrow m|p \Rightarrow (m, p) \in R$$

So, R is transitive

Hence, R is reflexive and transitive but it is not symmetric

75 **(c)**

Since, $A = B \cap C$ and $B = C \cap A$,

Then $A \equiv B$

76 **(d)**

Since $n|n$ for all $n \in N$. Therefore, R is reflexive. Since $2|6$ but $6 \nmid 2$, therefore R is not symmetric

Let nRm and mRp

$\Rightarrow nRm$ and mRp

$\Rightarrow n|m$ and $m|p \Rightarrow n|p \Rightarrow nRp$

So, R is transitive

77 **(a)**

We have,

$bN = \{bx | x \in \mathrm{N}\}$ = Set of positive integral multiples of b

$cN = \{cx | x \in N\}$ = Set positive integral multiples of c

$\therefore bN \cap cN$ = Set of positive integral multiples of bc

$\Rightarrow bN \cap cN = bcN$ [$\because$ b and c are prime]

Hence, $d = bc$

79 **(b)**

Let $x, y \in A$. Then,

$x = m^2, y = n^2$ for some $m, n \in N$

$\Rightarrow xy = (mn)^2 \in A$

80 **(c)**

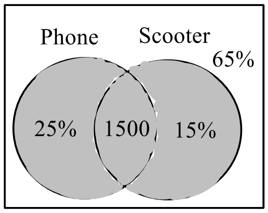
We have,

$A_1 \subset A_2 \subset A_3 \subset \cdots \subset A_{100}$

$$\therefore \bigcup_{i=1}^{100} A_i = A_{100} \Rightarrow n\left(\bigcup_{i=1}^{100} A_i\right) = n(A_{100}) = 101$$

81 **(c)**

Let the total population of town be x.



$$\therefore \quad \frac{25x}{100}+\frac{15x}{100}-1500+\frac{65x}{100}=x$$

$$\Rightarrow \quad \frac{105x}{100}-x=1500$$

$$\Rightarrow \quad \frac{5x}{100}=1500$$

$$\Rightarrow \quad x=30000$$

82 **(d)**

As A, B, C are pair wise disjoints. Therefore,

$A \cap B = \phi, B \cap C = \phi$ and $A \cap C = \phi$

$\Rightarrow A \cap B \cap C = \phi \Rightarrow (A \cup B \cup C) \cap (A \cap B \cap C) = \phi$

83 **(b)**

Clearly, $R = \{(1,3), (3,1), (2,2)\}$

We observe that R is symmetric only

84 **(d)**

Given figure clearly represents

$(A - B) \cup (B - A)$

85 **(d)**

R_4 is not a relation from A to B, because $(7,9) \in R_4$ but $(7,9) \notin A \times B$

86 **(c)**

R is reflexive if it contains $(1,1), (2,2), (3,3)$

$\because (1,2) \in R, (2,3) \in R$

$\because R$ is symmetric, if $(2,1), (3,2) \in R$

Now, $R = \{(1,1), (2,2), (3,3), (2,1), (3,2), (2,3), (1,2)\}$

R will be transitive, if $(3,1), (1,3) \in R$

Thus, R becomes an equivalence relation by adding

$(1,1)(2,2)(3,3), (2,1)(3,2), (1,3), (3,1)$. Hence, the total number of ordered pairs is 7

87 **(c)**

The set A is the set of all points on the hyperbola $xy = 1$ having its two branches in the first and third quadrants, while the set B is the set of all points on $y = -x$ which lies in second and four quadrants. These two curves do not intersect.

Hence, $A \cap B = \phi$.

88 **(b)**

Since R is an equivalence relation on set A. Therefore $(a, a) \in R$ for all $a \in A$.

Hence, R has at least n ordered pairs

89 **(d)**

It is given $A_1 \subset A_2 \subset A_3 \subset A_4 \ldots \subset A_{50}$

$$\therefore \bigcup_{i=11}^{50} A_i = A_{11}$$

$$\Rightarrow n\left(\bigcup_{i=11}^{50} A_i\right) = n(A_{11}) = 11 - 1 = 10$$

90 **(d)**
We have,
$bN = \{bx|x \in \mathrm{N}\}$ = Set of positive integral multiples of b
$cN = \{cx|x \in N\}$ = Set of positive integral multiples of c
$\therefore cN = \{cx \mid x \in N\}$ = Set of positive integral multiples of b and c both
$\Rightarrow d = 1.\text{c. m. of}b\text{and}c$

91 **(d)**
Clearly, R is an equivalence relation

92 **(b)**
Number of element is $S = 10$
And $A = \{(x, y); x, y \in S, x \neq y\}$
$\therefore$ Number of element in $A = 10 \times 9 = 90$

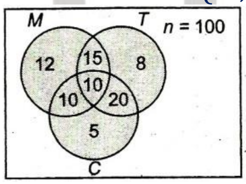
93 **(c)**
Clearly,
$R = \{(\text{BHEL}, \text{SAIL}), (\text{SAIL}, \text{BHEL}), (\text{BHEL}, \text{GAIL}),$
$(\text{GAIL}, \text{BHEL}), (\text{BHEL}, \text{IOCL}), (\text{IOCL}, \text{BHEL})\}$
We observe that R is symmetric only

94 **(a)**
According to the given condition,
$2^m = 112 + 2^n$
$\Rightarrow \quad 2^m - 2^n = 112$
$\Rightarrow \quad m = 7, n = 4$

96 **(c)**
We have,
$$p = \frac{(n+2)(2n^5 + 3n^4 + 4n^3 + 5n^2 + 6)}{n^2 + 2n}$$
$$\Rightarrow p = 2n^4 + 3n^3 + 4n^2 + 5n + \frac{6}{n}$$
Clearly, $p \in Z^+$ iff $\text{n} = 1, 2, 3, 6$. So, A has 4 elements

97 **(b)**
Clearly,
$x \in A - B \Rightarrow x \in A$ but $x \notin B$
$\Rightarrow x$ is a multiple of 3 but it is not a multiple of 5
$\Rightarrow x \in A \cap \bar{B}$

98 **(b)**
Total drinks=3(ie, milk, coffee, tea).



Total number of students who take any of the drink is 80.
$\therefore$The number of students who did not take any of three drinks= $100 - 80 = 20$

100 **(d)**
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 12 + 9 - 4 = 17$
Hence, $n[(AUB)^c] = n(U) - n(A \cup B)$
$= 20 - 17 = 3$

101 **(c)**

We have,
$\{x \in Z: |x-3| < 4\} = \{x \in Z: -1 < x < 7\} = \{0,1,2,3,4,5,6\}$
and,
$\{x \in Z: |x-4| < 5\} = \{x \in Z: -1 < x < 9\}$
$= \{0,1,2,3,4,5,6,7,8\}$
$\therefore \{x \in Z: |x-3| < 4\} \cap \{x \in Z: |x-4| < 5\}$
$= \{0,1,2,3,4,5,6\}$

102 **(a)**
Since R is reflexive relation on A
$\therefore (a,a) \in R$ for all $a \in A$
$\Rightarrow$ The minimum number of ordered pairs in R is n
Hence, $m \geq n$

104 **(c)**
We have, $y = \frac{4}{x}$ and $x^2 + y^2 = 8$
Solving these two equations, we have
$x^2 + \frac{16}{x^2} = 8 \Rightarrow (x^2 - 4) = 0 \Rightarrow x = \pm 2$
Substituting $x = \pm 2$ in $y = \frac{4}{x}$, we get $y = \pm 2$
Thus, the two curves intersect at two points only (2, 2) and (−2, 2). Hence, $A \cap B$ contains just two points

105 **(b)**
Let $(a,b) \in R$. Then,
$|a+b| = a+b \Rightarrow |b+a| = b+a \Rightarrow (b,a) \in R$
$\Rightarrow R$ is symmetric

106 **(c)**
Minimum possible value of $n(B \cap C)$ is $n(A \cap B \cap C) = 3$

107 **(a)**
To make R a reflexive relation, we must have (1,1), (3,3) and (5,5) in it. In order to make R a symmetric relation, we must inside (3,1) and (5,3) in it.
Now, $(1,3) \in R$ and $(3,5) \in R$. So, to make R a transitive relation, we must have, $(1,5) \in R$.
But, R must be symmetric also. So, it should also contain (5,1). Thus, we have
$R = \{(1,1), (3,3), (5,5), (1,3), (3,5), (3,1), (5,3), (1,5), (5,1)\}$
Clearly, it is an equivalence relation on $A\{1,3,5\}$

108 **(b)**
Clearly, $(3,3) \notin R$. So, R is not reflexive. Also, (3,1) and (1,3) are in R but $(3,3) \notin R$. So, R is not transitive
But, R is symmetric as $R = R^{-1}$

109 **(b)**
Let $(a,b) \in R$. Then,
$(a,b) \in R \Rightarrow (b,a) \in R^{-1}$ [By def. of R^{-1}]
$\Rightarrow (b,a) \in R [\because R = R^{-1}]$
So, $R$is symmetric

110 **(b)**
We have,
$A_2 \subset A_3 \subset A_4 \subset \cdots \subset A_{10}$
$\therefore \bigcap_{n=3}^{10} A_n = A_3 = \{2,3,5\}$

111 **(c)**
The possible sets are $\{\pm 2, \pm 3\}$ and $\{\pm 4, \pm 1\}$; therefore, number of elements in required set is 8.
112 **(a)**
Given, $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, d, c\}$
Now, $A - B = \{a, b, c\} - \{b, c, d\} = \{a\}$
And $B \cap C = \{b, c, d\} \cap \{a, d, c\} = \{c, d\}$
$\therefore (A - B) \times (B \cap C) = \{a\} \times \{c, d\}$
$= \{(a, c), (a, d)\}$
113 **(c)**
Given, $n(M) = 100, n(P) = 70, \quad n(C) = 40$
$n(M \cap P) = 30, \quad n(M \cap C) = 28,$
$n(P \cap C) = 23$ and $n(M \cap P \cap C) = 18$
$\therefore n(M \cap P' \cap C') = n[M \cap (P \cap C')]$
$= n(M) - n[M \cap (P \cap C)]$
$= n(M) - [n(M \cap P) + n(M \cap C) - n(M \cap P \cap C)]$
$= 100 - [30 + 28 - 18 = 60]$
114 **(d)**
$B \cap C = \{4\}$.
$\therefore A \cup (B \cap C) = \{1, 2, 3, 4\}$
115 **(c)**
$\because \quad A \subseteq B$
$\therefore \quad B \cup A = B$
116 **(c)**
$n((A \cup B)^c\} = n(\mathcal{U}) - n(A \cup B)$
$= n(\mathcal{U}) - \{n(A) + n(B) - n(A \cap B)\}$
$= 100 - (50 + 20 - 10) = 40$
117 **(d)**
If $A = \{1,2,3\}$, then $R = \{(1,1), (2,2), (3,3), (1,2)\}$ is reflexive on A but it is not symmetric
So, a reflexive relation need not be symmetric
The relation 'is less than' on the set Z of integers is antisymmetric but it is not reflexive
119 **(c)**
Clearly,
Required percent $= 20 + 50 - 10 = 60\%$
$[\because n(A \cup B) = n(A) + n(B) - n(A \cap B)]$
120 **(c)**
The greatest possible value of $n(A \cap B \cap C)$ is the least amongst the values $n(A \cap B), n(B \cap C)$ and $n(A \cap C)$ i.e. 10
121 **(d)**
Clearly, $S \subset R$
$\therefore S \cup R = R$and $S \cap R = S$
$\Rightarrow (S \cap R) - (S \cap R) =$Set of rectangles which are not squares
122 **(b)**
Clearly, the relation is symmetric but it is neither reflexive nor transitive
123 **(d)**
Since, power set is a set of all possible subsets of a set.
$\therefore \quad P(A) = \{\phi, \{x\}, \{y\}, \{x, y\}\}$
124 **(b)**

We have,
$N = 10{,}000, n(A) = 40\%$ of $10{,}000 = 4000$,
$n(B) = 2000, n(C) = 1000, n(A \cap B) = 500$,
$n(B \cap C) = 300, n(C \cap A) = 400, n(A \cap B \cap C) = 200$
Now,
Required number of families =
$n(A \cap \bar{B} \cap \bar{C}) = n(A \cap (B \cup C)')$
$= n(A) - n(A \cap (B \cup C))$
$= n(A) - n\big((A \cap B) \cup (A \cap C)\big)$
$= n(A) - \{n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)\}$
$= 4000 - (500 + 400 - 200) = 3300$

126 **(b)**
$A \cap \phi = \phi$ is true.

128 **(c)**
$A \cap B = \{2, 4\}$
$\{A \cap B\} \subseteq \{1, 2, 4\}, \{3, 2, 4\}, \{6, 2, 4\}, \{1, 3, 2, 4\}$,
$\{1, 6, 2, 4\}, \{6, 3, 2, 4\}, \{2, 4\}, \{1, 3, 6, 2, 4\} \subseteq A \cup B$
$\Rightarrow \quad n(C) = 8$

129 **(a)**
We have,
$p = \dfrac{7n^2 + 3n + 3}{n} \Rightarrow p = 7n + 3 + \dfrac{3}{n}$
It is given that $n \in N$ and p is prime. Therefore, $n = 1$
$\therefore n(A) = 1$

130 **(d)**
$(Y \times A) = \{(1, 1), (1, 2), (2, 1), (2, 2),$
$(3, 1), (3, 2), (4, 1), (4, 2), (5, 1), (5, 2)\}$
And$(Y \times B) = \{(1, 3), (1, 4), (1, 5), (2, 3),$
$(2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 3),$
$(4, 4), (4, 5), (5, 3), (5, 4), (5, 5)\}$
$\therefore (Y \times A) \cap (Y \times B) = \phi$

131 **(b)**
Given, $n(A) = 4, \quad n(B) = 5$ and $\quad n(A \cap B) = 3$
$\therefore n[(A \times B) \cap (B \times A)] = 3^2 = 9$

132 **(c)**
$U = \{x: x^5 + 6x^4 + 11x^3 - 6x^2 = 0\} = \{0, 1, 2, 3\}$
$A = \{x: x^2 - 5x + 6 = 0\} = \{2, 3\}$
And $\quad B = \{x: x^2 - 3x + 2 = 0\} = \{2, 1\}$
$\therefore \quad (A \cap B)' = U - (A \cap B)$
$= \{0, 1, 2, 3\} - \{2\} = \{0, 1, 3\}$

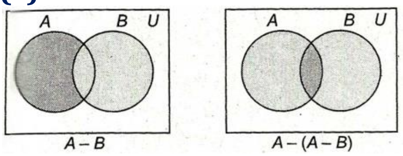
133 **(c)**
We have,
$R = \{(1,3), (1,5), (2,3), (2,5), (3,5), (4,5)\}$
$\Rightarrow R^{-1} = \{(3,1), (5,1), (3,2), (5,2), (5,3), (5,4)\}$
Hence, $RoR^{-1} = \{(3,3), (3,5), (5,3), (5,5)\}$

134 **(b)**
Let $(a, b) \in R$. Then,
a and b are born in different months $\Rightarrow (b, a) \in R$

So, R is symmetric

Clearly, R is neither reflexive nor transitive

136 **(c)**



From the venn diagram

$A - (A - B) = A \cap B$

137 **(b)**

Required number of subsets is equal to the number of subsets containing 2 and any number of elements from the remaining elements 1 and 4

So, required number of elements $= 2^2 = 4$

140 **(b)**

Clearly, 2 is a factor of 6 but 6 is not a factor of 2. So, the relation 'is factor of' is not symmetric. However, it is reflexive and transitive

142 **(d)**

Clearly, R is neither reflexive, nor symmetric and not transitive

143 **(d)**

Clearly, given relation is an equivalence relation

145 **(c)**

Each subset will contain 3 and any number of elements from the remaining 3 elements 1,2 and 4

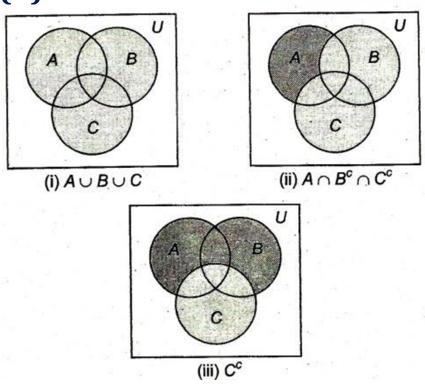
So, required number of elements $= 2^2 = 8$

146 **(a)**

Since $(1,1), (2,2), (3,3) \in R$. Therefore, R is reflexive. We observe that $(1,2) \in R$ but $(2,1) \notin R$, therefore R is not symmetric.

It can be easily seen that R is transitive

147 **(b)**



From figures (i), (ii) and (iii), we get

$(A \cup B \cup C) \cap (A \cap B^C \cap C^C) \cap C^C = (B^C \cap C^C)$

148 **(d)**

A relation on set A is a subset of $A \times A$

Let $A = \{a_1, a_2, \dots, a_n\}$. Then, a reflexive relation on A must contain at least n elements $(a_1, a_1), (a_2, a_2), \dots, (a_n, a_n)$

$\therefore$ Number of reflexive relations on A is 2^{n^2-n}

Clearly, $n^2 - n = n, n^2 - n = n - 1, n^2 - n = n^2 - 1$ have solutions in N but $n^2 - n = n + 1$ is not solvable in N.

So, 2^{n+1} cannot be the number of reflexive relations on A

149 **(a)**

We have,

$A \Delta B = (A \cup B) - (A \cup B)$

$\Rightarrow n(A \Delta B) = n(A) + n(B) - 2\, n(A \cap B)$

So, $n(A \Delta B)$ is greatest when $n(A \cap B)$ is least

It is given that $A \cap B \neq \phi$. So, least number of elements in $A \cap B$ can be one

$\therefore$ Greatest possible value of $n(A \Delta B)$ is $7 + 6 - 2 \times 1 = 11$

150 **(d)**

Let $R = \{(x, y): y = ax + b\}$. Then,

$(-2, -7), (-1, -4) \in R$

$\Rightarrow -7 = -2a + b$ and $-4 = -a + b$

$\Rightarrow a = 3, b = -1$

$\therefore y = 3x - 1$

Hence, $R = \{(x, y): y = 3x - 1, -2 \leq x < 3, x \in Z\}$