# TRANSIENT AND ALTERNATING CURRENT CIRCUIT

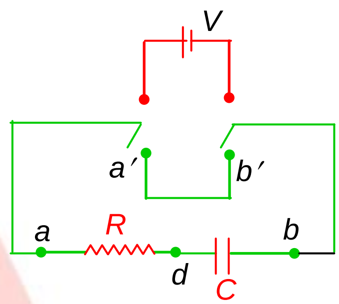
## 1 INTRODUCTION

In case of circuit having pure resistance only with direct current source, the current reaches its steady state value almost instantaneously. But when capacitor and inductor are part of the circuit, it takes some time. As capacitor is charged, it oppose the flow of current and similarly any change in current is opposed by the inductor.

## 2 R – C CIRCUIT

### 2.1 CHARGING OF A CAPACITOR

The circuit diagram for charging and discharging a capacitor is given here. A resistor *R* and a capacitor *C* are connected with a double throw switch, by which the battery *V* can be connected in the circuit.




Initially, the capacitor is uncharged. When the switch is thrown to include the battery in the circuit, charge flows to the capacitor through the resistance *R*. This is called the charging current. The current continues till the voltage $V_{db}$ across the capacitor is equal to the voltage *V* of the battery. If during the charging process the instantaneous current is *I* at an instant as shown, and the potential difference between *a* and *d*, and that between *d* and *b* are $V_{ad}$ and $V_{db}$ respectively, then

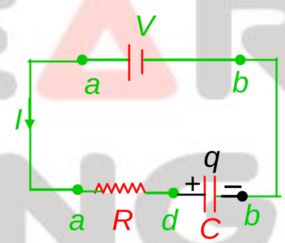
$$V_{ad} = iR \text{ and } V_{db} = \frac{q}{C} \quad \text{… (i)}$$

where *q* is the charge on the capacitor at that instant

$$V_{ab} = V = V_{ad} + V_{db} = iR + \frac{q}{C} \quad \text{… (ii)}$$

where *V* is the constant voltage of the battery.

$$i = \frac{V}{R} - \frac{q}{RC} \quad \text{… (ii)}$$




Initially, as soon as the connection is made there is a current $I_0 = \frac{V}{R}$, since the charge on the capacitance is zero

As the charging continues, *q* increases and *i* decreases and finally becomes zero. At that time,

$$\frac{V}{R} - \frac{q}{RC} = 0$$

$q = CV = Q_0$, where $Q_0$ is the final charge on the capacitor.

In equation (iii), *i* may be written as $\frac{dq}{dt}$, so

$$\frac{dq}{dt} = \frac{V}{R} - \frac{q}{RC}$$

$$\frac{dq}{VC - q} = \frac{dt}{RC}$$

Integrating both sides,
we have

$$\int_0^q \frac{dq}{VC - q} = \int_0^t \frac{dt}{RC}$$

$$-\left[\ln(VC-q)\right]_0^q = \frac{1}{RC}\left[t\right]_0^t$$

or, $$-\ln\left(\frac{VC-q}{VC}\right)=\frac{t}{RC}$$

or, $$\left(1-\frac{q}{VC}\right)=e^{\frac{-t}{RC}}$$

or, $$\frac{q}{CV}=\left(1-e^{\frac{-t}{RC}}\right)$$

or, $$q = CV\left(1-e^{-\frac{t}{RC}}\right)$$

**or,** $$q = Q_0\left(1-e^{\frac{-t}{RC}}\right) \qquad \dots (1)$$

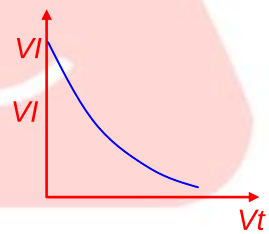
Differentiating with respect to time.

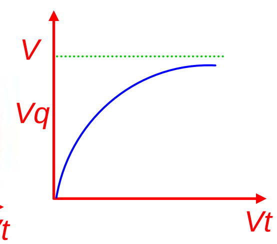
$$i=\frac{dq}{dt}=\frac{d}{dt}[VC(1-e^{-t/RC})]=\frac{V}{R}e^{-t/RC}=I_0e^{-t/RC}$$

We see that the charge and current both follow the exponential law.

When time $t=RC, q=Q_0\left(1-\frac{1}{e}\right)$ and $i=\frac{I_0}{e}$

This means that at $t = RC$, the charge has increased to $1-\frac{1}{e}$ = 63% of its final value. The current has decreased to $\frac{1}{e}$ = 37% of its initial value. The time $t = RC$ is called the **time constant** of the circuit.




The half-life of the circuit $t_h$ is the time at which the current is half its initial value and the charge on the capacitor is half its final value $t_h=RC\ln 2=0.693RC$

## 2.2 DISCHARGING OF A CAPACITOR

If in the circuit, the switch is thrown to the down position, the battery is removed from the circuit and the capacitor is discharged through resistance $R$.

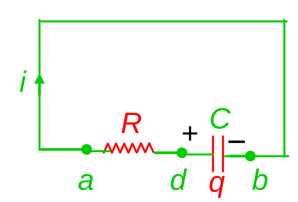
When the switch is thrown, the charge on the capacitor is $Q_0$ and the potential difference $V_{ab}$ is zero.

Thus $V_{ad}+V_{db}=0$

$$V_{ad}=-V_{db}=I_0R$$

$$I_0=\frac{Q_0}{RC}=\frac{V_0}{R}$$

when $t=0, q=Q_0$ and $I_0=\frac{Q_0}{RC}=\frac{V_0}{R}$, where $V_0$ is the initial potential difference across the capacitor.




When the capacitor is discharging both the charge and the current decrease.

Let $i$ be the current and $q$ be the charge on the capacitor at any instant. Then

$$\frac{q}{C}-iR=0$$

$$\Rightarrow \quad \frac{q}{C}-\left(-\frac{dq}{dt}\right)R=0$$

$$\Rightarrow \qquad \frac{dq}{q} = -\frac{dt}{RC}$$

$$\ln q = -\frac{t}{RC} + \text{Constant}$$

At $t = 0, q = Q_0$

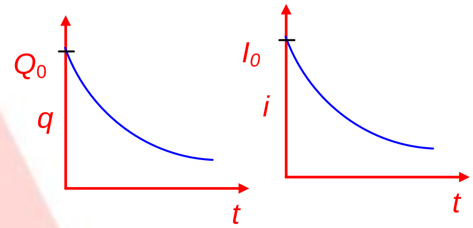
Constant $= lnQ_0$

$$lnq = -\frac{t}{RC} + lnQ_0; \; ln\frac{q}{Q_0} = -\frac{t}{RC}$$

$$q = Q_0 e^{-t/RC} \qquad \dots (2)$$

Current $i = -\dfrac{dq}{dt} = \dfrac{Q_0}{RC}e^{-t/RC}$

$$= I_0 e^{-t/RC}$$



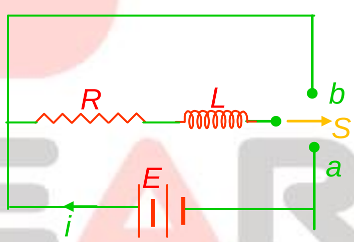

$$i = I_0 e^{-t/RC} \qquad \dots (3)$$

At time $\quad t = CR, q = \dfrac{Q_0}{e} \quad i = \dfrac{I_0}{e}$

## 3 GROWTH AND DECAY OF CURRENT IN L – R CIRCUIT

### 3.1 GROWTH OF CURRENT

Consider a circuit containing a resistance *R*, an inductance *L*, a two way key and a battery of e.m.f. *E* connected in series as shown in figure. When the switch *S* is connected to *a*, the current in the circuit grows from zero value. The inductor opposes the growth of the current. This is due to the fact that when the current grows through inductor, a back e.m.f. is developed which opposes the growth of current in the circuit. So the rate of growth of current is reduced. During the growth of current in the circuit, let *i* be the current in the circuit at any instant *t*. Using Kirchoffs voltage law in the circuit we obtain




$$E - L\frac{di}{dt} = R\,i$$

or $\qquad E - R\,i = \dfrac{Ldi}{dt}$

or $\qquad \dfrac{di}{E - R\,i} = \dfrac{dt}{L}$

Multiplying by – *R* on both the sides, we get $\dfrac{-R\,di}{E - Ri} = \dfrac{-Rdt}{L}$

Integrating the above equation, we have

$\log_e (E - Ri) = -\dfrac{R}{L}t + A \qquad \dots$ (i)

Where *A* is integration constant. The value of this constant can be obtained by applying the condition that current *i* is zero just at start i.e. at $t = 0$. Hence

$\log_e E = 0 + A$

or $\quad A = \log_e E \qquad \dots$ (ii)

Substituting the value of $A$ from equation (ii) in equation (i) we get

$$\log_e (E - Ri) = -\frac{R}{L}t + \log_e E$$

or $$\log_e\left(\frac{E - Ri}{E}\right) = -\frac{R}{L}t$$

or $$\left(\frac{E - Ri}{E}\right) = \exp\left(-\frac{R}{L}t\right)$$

or $$1 - \frac{Ri}{E} = \exp\left(-\frac{R}{L}t\right)$$

or $$\frac{Ri}{E} = \left\{1 - \exp\left(-\frac{R}{L}t\right)\right\}$$

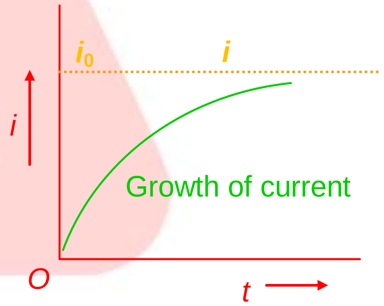
$\therefore$ $$i = \frac{E}{R}\left\{1 - \exp\left(-\frac{R}{L}t\right)\right\}$$

The maximum current in the circuit $i_0 = E/R$. So $i = i_0\left\{1 - \exp\left(-\frac{R}{L}t\right)\right\}$ *… (4)*

Equation (4) gives the current in the circuit at any instant $t$. It is obvious from equation (4) that $i = i_0$, when

$$\exp.\left(-\frac{R}{L}t\right) = 0 \text{ i.e., at } t = \infty$$

Hence the current never attains the value $i_0$ but it approaches it asymptotically. A graph between current and time is shown in figure.




☞ We observe the following points

(i) When $t = (L/R)$ then

$$i = i_0\left\{1 - \exp.\left(-\frac{R}{L} \times \frac{L}{R}\right)\right\}$$

$$= i_0\{1 - \exp.(-1)\} = i_0\left(1 - \frac{1}{e}\right)$$

$$= 0.63\, i_0$$

Thus after an interval of $(L/R)$ second, the current reaches to a value which is 63% of the maximum current. The value of $(L/R)$ is known as **time constant** of the circuit and is represented by $\tau$. Thus the time constant of a circuit may be defined as the time in which the current rises from zero to 63% of its final value. In terms of $\tau$,

$$i = i_0\,(1 - e^{-t/\tau})$$

**(ii)** The rate of growth of current $(di/dt)$ is given by

$$\frac{di}{dt} = \frac{d}{dt}\left[i_0\left\{1 - \exp\left(-\frac{R}{L}t\right)\right\}\right]$$

$$\Rightarrow \frac{di}{dt} = i_0\left(\frac{R}{L}\right)\exp\left(-\frac{R}{L}t\right) \qquad \text{… (5)}$$

From equation (4), exp. $\left(-\frac{R}{L}t\right)=\frac{i_0-i}{i_0}$

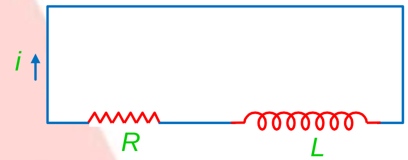
$\therefore \qquad \frac{di}{dt}=i_0\left(\frac{R}{L}\right)\left(\frac{i_0-i}{i_0}\right)=\frac{R}{L}\,(i_0-i)$ **... *(6)***

This shows that the rate of growth of the current decreases as $i$ tends to $i_0$. For any other value of current, it depends upon the value of $R/L$. Thus greater is the value of time constant, smaller will be the rate of growth of current.

## 3.2 DECAY OF CURRENT

Let the circuit be disconnected from battery and switch $S$ is thrown to point $b$ in the figure. The current now begins to fall. In the absence of inductance, the current would have fallen from maximum $i_0$ to zero almost instantaneously. But due to the presence of inductance, which opposes the decay of current, the rate of decay of current is reduced.

Suppose during the decay of current, $i$ be the value of current at any instant $t$. Using Kirchhoff's voltage law in the circuit we get




$$-L\frac{di}{dt}=Ri$$

or $$\frac{di}{dt}=-\frac{R}{L}i$$

Integrating this expression, we get

$$\log_e i=-\frac{R}{L}t+B$$

Where $B$ is constant of integration. The value of $B$ can be obtained by applying the condition that when $t = 0$, $i = i_0$

$\therefore \qquad \log_e i_0 = B$

Substituting the value of $B$, we get

$$\log_e\ i=-\frac{R}{L}t+\log_e i_0$$

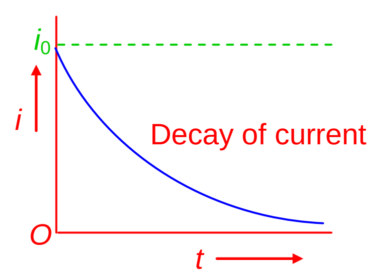
or $$\log_e \frac{i}{i_0}=-\frac{R}{L}t$$

or $$(i/i_0)=\exp\left(-\frac{R}{L}t\right)$$

***or*** $$\boldsymbol{i=i_0\exp\left(-\frac{R}{L}t\right)=i_0\exp(-t/\tau)}$$ **... *(7)***

where $\tau = L / R$ = inductive time constant of the circuit.

It is obvious from equation that the current in the circuit decays exponentially as shown in figure.




☞ We observe the following points

**(i)** After $t = L/R$ , the current in the circuit is given by

$$i=i_0\exp\left(-\frac{R}{L}\times\frac{L}{R}\right)=i_0\exp.(-1)$$

$= (i_0/e) = i_0 /2.718 = 0.37\ i_0$

So after a time $(L/R)$ second, the current reduces to 37% of the maximum current $i_0$. $(L/R)$ is known as time constant $\tau$ . This is defined as the time during which the current decays to 37% of the maximum current during decay.

**(ii)** The rate of decay of current is given by

$$\frac{di}{dt}=\frac{d}{dt}\left\{i_0 \exp\left(-\frac{R}{L}t\right)\right\}$$

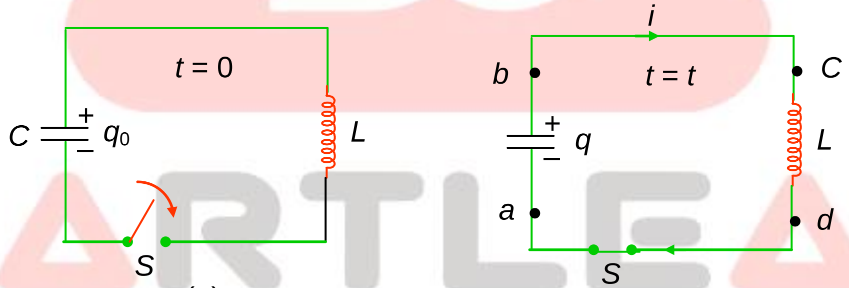
$$\Rightarrow \quad \frac{di}{dt} = \frac{R}{L} i_0 \exp\left(-\frac{R}{L}t\right)=-\frac{R}{L}i \qquad \ldots (8)$$

*or* $$-\frac{di}{dt}=\frac{R}{L}i$$

This equation shows that when $L$ is small, the rate of decay of current will be large i.e., the current will decay out more rapidly.

## 4 OSCILLATION IN L – C CIRCUIT

If a charged capacitor $C$ is short-circuited though an inductor $L$, the charge and current in the circuit start oscillating simple harmonically. If the resistance of the circuit is zero, no energy is dissipated as heat. Assume an ideal situation in which energy is not radiated away from the circuit. With these idealizations-zero resistance and no radiation, the oscillations in the circuit persist indefinitely and the energy is transferred from the capacitor's electric field to the inductor' s magnetic field back and forth. The total energy associated with the circuit is constant. This is analogous to the transfer of energy in an oscillating mechanical system from potential energy to kinetic energy and back, with constant total energy. Such an analogous mechanical system is an example of spring mass system.




(a) (b)

Let us now derive an equation for the oscillations of charge and current in an L-C circuit.

Refer figure (a): The capacitor is pre charged to a potential difference V such that charge on capacitor $q_0 = CV$

Here $q_0$ is the maximum charge on the capacitor. At time $t$ = 0, it is connected to an inductor through a switch $S$. At time $t$ = 0, switch S is closed.

Refer figure (b): When the switch is closed, the capacitor starts discharging. Let at time $t$ charge on the capacitor is $q$ ($< q_0$) and since, it is further decreasing, there is a current $i$ in the circuit in the direction shown in figure.

The potential difference across capacitor = potential difference across inductor, or

$$V_b - V_a = V_c - V_d$$

$$\therefore \quad \frac{q}{C}=L\left(\frac{di}{dt}\right) \qquad \ldots \text{(i)}$$

Now, as the charge is decreasing, $i = \left(\frac{-dq}{dt}\right)$ or $\frac{di}{dt}=-\frac{d^2q}{dt^2}$

Substituting in equation (i), we get $\frac{q}{C} = -L\left(\frac{d^2q}{dt^2}\right)$

or $\frac{d^2q}{dt^2} = -\left(\frac{1}{LC}\right)q$ … (ii)

This is the standard equation of simple harmonic motion $\left(\frac{d^2x}{dt^2} = -\omega^2 x\right)$

Here $\omega = \frac{1}{\sqrt{LC}}$ … (iii)

The general solution of equation (ii), is

***$q = q_0 \cos(\omega t \pm \phi)$*** … (9)

In our case $\phi = 0$ as $q = q_0$ at $t = 0$.

Thus, we can say that in the circuit, charge oscillates with angular frequency given by equation **(iii).** Thus,

☞ In $L - C$ oscillations, $q$, $i$ and $\frac{di}{dt}$ all oscillate simple harmonically with same angular frequency $\omega$, but the phase difference between $q$ and $i$ or between $i$ and $\frac{di}{dt}$ is $\frac{\pi}{2}$. Their amplitudes are $q_0$, $q_0\omega$ and $\omega^2 q_0$ respectively. So

***$q = q_0 \cos \omega t$, then*** … (10)

$i = \frac{dq}{dt} = -q_0\omega \sin\omega t$ … (11)

***and*** $\frac{di}{dt} = -q_0\omega^2 \cos\omega t$ … (12)

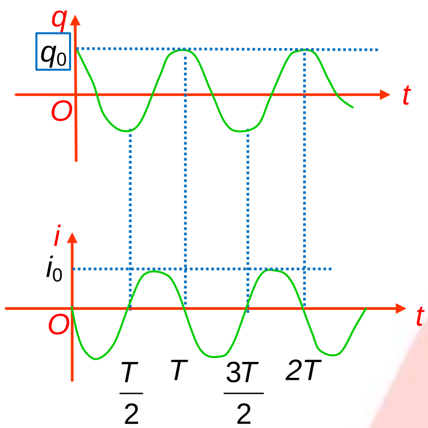
Potential energy in the capacitor

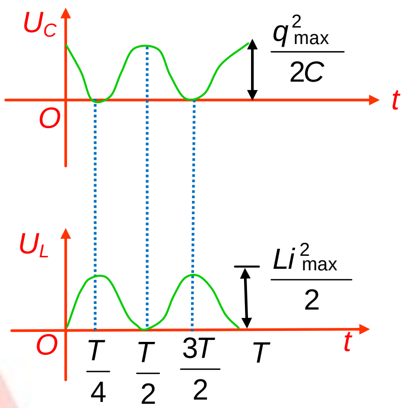
$$U_C = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{q_0^2}{C}\cos^2\omega t = \frac{q_0^2}{4C}(1+\cos 2\omega t) \quad \text{… (13)}$$

Potential energy in the inductor

$$U_L = \frac{1}{2}LI^2 = \frac{1}{2}\frac{q_0^2}{C}\sin^2\omega t = \frac{q_0^2}{4C}(1-\cos 2\omega t) \quad \text{… (14)}$$

Thus potential energy stored in the capacitor and that in the inductor also oscillates between maximum value and zero with double the frequency. All these quantities are shown in the figures that follows




## 5 INTRODUCTION: ON ALTERNATING CURRENT

Until now, we have studied only circuits with direct current (dc)- which flows only in one direction. The primary source of emf in such circuit is a battery. When a resistance is connected across the terminals of the battery, a current is established in the circuit, which flows in a unique direction from the positive terminal to the negative terminal via the external resistance.

But most of the electric power generated and used in the world is in the form of **alternating current (ac)**, the magnitude of which changes continuously with time and direction is reversed periodically as shown in figure and it is given by

$$i = \frac{e}{R} = \frac{e_0}{R}\ Sin\ wt = i_0\ Sin\ wt \qquad \dots (15)$$

Here $i$ is instantaneous value of current i.e. magnitude of current at any instant of time and $i_0$ is the maximum value of current which is called **peak current** or the **current amplitude** and the current repeats its value after each time interval

$T = \frac{2\pi}{\omega}$ as shown in figure. This time interval is called the **time period**.

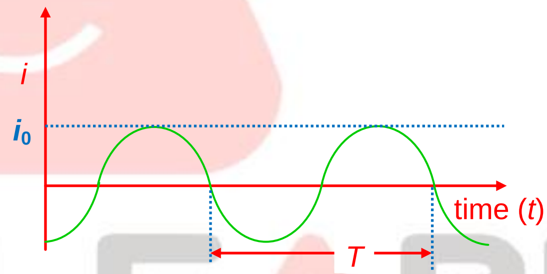



Fig. (a)

The current is positive for half the time period and negative for remaining half period. It means direction of current is reversed after each half time period. The frequency of ac in India is 50 Hz.

**An alternating voltage** is given by

$V = V_0/\sqrt{2}$ $\qquad \dots (16)$

It also varies alternatively as shown in the figure (b), where $V$ is instantaneous voltage and $V_0$ is peak voltage. It is produced by ac generator also called as ac dynamo.

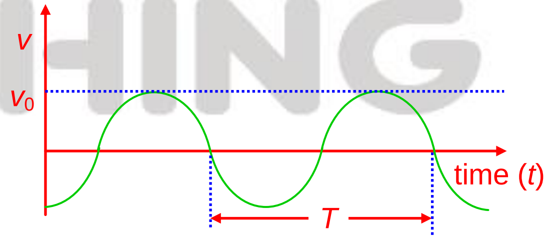
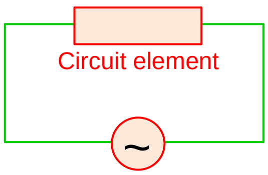



Fig. (b)

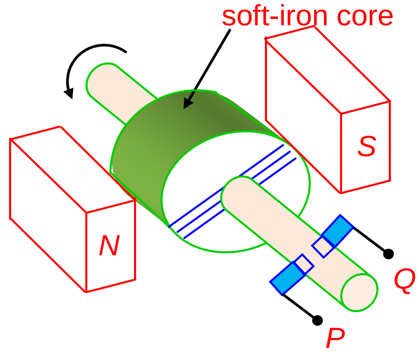
**AC Circuit:** An ac circuit consists of circuit element i.e., resistor, capacitor, inductor or any combination of these and a generator that provides the alternating current as shown in figure. The ac source is represented by symbol —(~)— in the circuit.




## 6 AC GENERATOR

The basic principle of the ac generator is a direct consequence of **Faraday's laws of electro magnetic induction**. When a coil of $N$ turns and area of cross section $A$ is rotated in a uniform magnetic field $B$ with constant angular velocity as shown in figure, a sinusoidal voltage (emf) is induced in the coil.

The armature rotates about its axis between the pole – pieces of a magnet, known as field magnet (N, S).
Generally, this is an electromagnet. When a permanent magnet is used, the machine is called magneto. However a higher peak e.m.f. needs larger magnetic field and hence necessitates the use of electromagnet. The current necessary for the field magnet (when it is electromagnet) is supplied by the generator itself through a feedbac system.




## 7 AVERAGE AND RMS VALUE OF ALTERNATING CURRENT

### 7.1 AVERAGE CURRENT (MEAN CURRENT)

As we know an alternating current is given by

$$i = i_0 \sin(\omega t + \phi) \qquad \ldots \text{(i)}$$

The mean or average value of ac over any time $t$ is given by

$$i_{avg} = \frac{\int_0^t i\,dt}{\int_0^t dt}$$

Using equation (i)

$$i_{avg} = \frac{\int_0^t i_0 \sin(\omega t + \phi)}{\int_0^t dt}$$

In one complete cycle average current

$$i_{avg} = -\frac{i_0}{T}\left[\frac{\cos(\omega t + \phi)}{\omega}\right]_0^T$$

$$= -\frac{i_0}{T}\left[\frac{\cos(\omega T + \phi) - \cos\phi}{\omega}\right]$$

$$= -\frac{i_0}{T}\left[\frac{\cos(2\pi + \phi) - \cos\phi}{\omega}\right] = 0 \qquad (\text{as } \omega T = 2\pi)$$

Since ac is positive during the first half cycle and negative during the other half cycle so $i_{avg}$ will be zero for long time also. Hence the dc instrument will indicate zero deflection when connected to a branch carrying ac current. So it is defined for either positive half cycle or negative half cycle. Now to find mean value of current $i = i_0 \sin\omega t$ for positive half cycle. i.e. from $t = 0$ to $t = T/2$

$$i_{avg} = \frac{\int_0^{T/2} i_0 \sin \omega t}{\int_0^{T/2} dt} = \frac{2i_0}{\pi} \approx 0.637\, i_0 \qquad \dots (19)$$

*Similarly* $$v_{avg} = \frac{2v_0}{\pi} \approx 0.637\, v_0 \qquad \dots (20)$$

## 7.2 R.M.S. VALUE OF ALTERNATING CURRENT

The notation rms refers to root mean square, which is given by square root of mean of square current.

i.e., $i_{rms} = \sqrt{i^2_{avg}}$

$$i^2_{avg} = \frac{\int_0^T i^2 dt}{\int_0^T dt} = \frac{1}{T}\int_0^T i_0^2 \sin^2(\omega t + \phi) dt = \frac{i_0^2}{2T}\int_0^T [1 - \cos 2(\omega t + \phi)] dt$$

$$= \frac{i_0^2}{2T}\left[t - \frac{\sin 2(\omega t + \phi)}{2\omega}\right]_0^T$$

$$= \frac{i_0^2}{2T}\left[T - \frac{\sin(4\pi + 2\phi) - \sin 2\phi}{2\omega}\right] = \frac{i_0^2}{2}$$

$$i_{rms} = \frac{i_0}{\sqrt{2}} \approx 0.707\, i_0 \qquad \dots (21)$$

*Similarly the rms voltage is given by* $V_{rms} = \frac{V_0}{\sqrt{2}} \approx 0.707\, v_0$ ... (22)

The significance of rms current and rms voltage may be shown by considering a resistance $R$ carrying a current $i = i_0 \sin(\omega t + \phi)$

The voltage across the resistor will be

$$V = Ri = (i_0 R) \sin(\omega t + \phi)$$

The thermal energy developed in the resistor during the time $t$ to $t + dt$ is

$$i^2 R\, dt = i_0^2 R \sin^2(\omega t + \phi)\, dt$$

The thermal energy developed in one time period is

$$U = \int_0^T i^2 R dt = R\int_0^T i_0^2 \sin^2(\omega t + \phi)\, dt$$

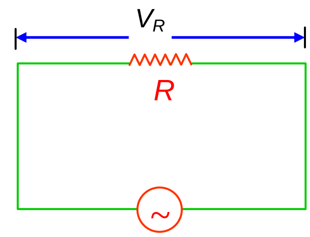
$$= RT\left[\frac{1}{T}\int_0^T i_0^2 \sin^2(\omega t + \phi) dt\right] = i^2_{rms} RT \qquad \dots (23)$$

***It means the root mean square value of ac is that value of steady current, which would generate the same amount of heat in a given resistance in a given time.***

So in ac circuits, current and ac voltage are measured in terms of their rms values. Like when we say that the house hold supply is 220 V ac it means the rms value is 220 V and peak value is $220\sqrt{2}$ = 311 V.

# 8 SERIES AC CIRCUIT

## 8.1 WHEN ONLY RESISTANCE IS IN AC CIRCUIT




$\varepsilon = \varepsilon_0 \sin\omega t$

Consider a simple ac circuit consisting of a resistor of resistance $R$ and an ac generator, as shown in the figure.
According to Kirchhoff's loop law at any instant, the algebraic sum of the potential difference around a closed loop in a circuit must be zero.

$$\varepsilon - V_R = 0$$
$$\varepsilon - i_R R = 0$$

$$\varepsilon_0 \sin\omega t - i_R R = 0$$

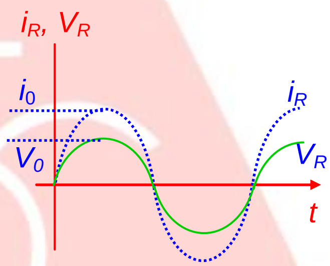
$$i_R = \frac{\varepsilon_0}{R} \sin \omega t = i_0 \sin \omega t \qquad \text{.... (i)}$$

where $i_0$ is the maximum current. $i_0 = \dfrac{\varepsilon_0}{R}$

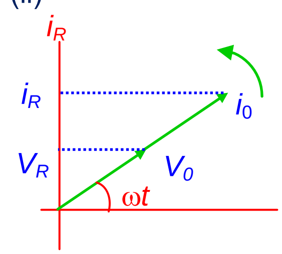
From above equations, we see that the instantaneous voltage drop across the resistor is

$$V_R = i_0 R \sin \omega t \qquad \text{... (ii)}$$

We see in equation (i) & (ii), $i_R$ *and* $V_R$ both vary as sin $\omega t$ and reach their maximum values at the same time as shown in figure (a), they are said to be in phase. A phasor diagram is used to represent phase relationships. The length of the arrows correspond to $V_0$ and $i_0$. The projections of the arrows onto the vertical axis give $V_R$ and $i_R$. In case of the single-loop resistive circuit, the current and voltage phasors lie along the same line, as shown in figure (b), because **$i_R$ and $V_R$ are in phase**.




Wave diagram
Fig. (a)

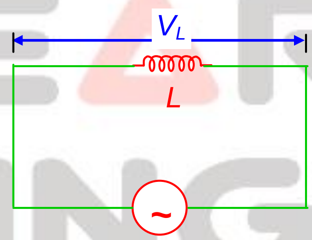



Phasor diagram
Fig. (b)

## 8.2 WHEN ONLY INDUCTOR IS IN AN AC CIRCUIT

Now consider an ac circuit consisting only of an inductor of inductance $L$ connected to the terminals of an ac generator, as shown in the figure. The induced emf across the inductor is given by $Ldi/dt$. On applying Kirchhoff's loop rule to the circuit

$$\varepsilon - V_L = 0 \Rightarrow \varepsilon - L\frac{di}{dt} = 0$$




$\varepsilon = \varepsilon_0 \sin \omega t$

When we rearrange this equation and substitute $\varepsilon = \varepsilon_0 \sin \omega t$, we get

$$L\frac{di}{dt} = \varepsilon_0 \sin \omega t \qquad \text{... (iii)}$$

Integration of this expression gives the current as a function of time

$$i_L = \frac{\varepsilon_0}{L}\int \sin\omega t\, dt = -\frac{\varepsilon_0}{\omega L}\cos\omega t + C$$

For average value of current over one time period to be zero, $C = 0$

$$\therefore \qquad i_L = -\frac{\varepsilon_0}{\omega L} \cos \omega t$$

When we use the trigonometric identity $\cos \omega t = -\sin(\omega t - \pi/2)$, we can express equation as

$$i_L = \frac{\varepsilon_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \qquad \text{... (iv)}$$

From equation (iv) , we see that the current reaches its maximum values when $\cos \omega t = 1$.

$$i_0 = \frac{\varepsilon_0}{\omega L} = \frac{\varepsilon_0}{X_L} \qquad \text{... (v)}$$

where the quantity $X_L$, called the inductive reactance, is

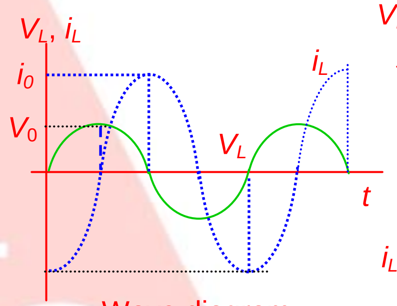
$$X_L = \omega L \qquad \text{... (24)}$$

The expression for the rms current is similar to equation (v), with $\varepsilon_0$ replaced by $\varepsilon_{rms}$. Inductive reactance, like resistance, has unit of ohm.

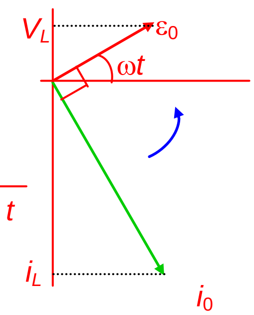
$$V_L = L\frac{di}{dt} = \varepsilon_0 \sin \omega t = I_0 X_L \sin \omega t \qquad \text{... (25)}$$

We can think of equation (v) as Ohm's law for an inductive circuit.

On comparing result of equation (iv) with equation (iii), we can see that **the current and voltage are out of phase with each other by $\pi/2$ rad, or 90°**. A plot of voltage and current versus time is given in figure (a). The voltage reaches its maximum value one quarter of an oscillation period before the current reaches its maximum value. The corresponding phasor diagram for this circuit is shown in figure (b). Thus, we see that for a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90°.



Wave diagram
Fig. (a)



Phasor diagram
Fig. (b)

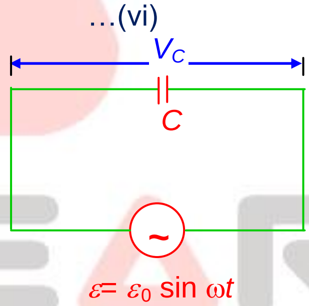
## 8.3 WHEN ONLY CAPACITOR IS IN AN AC CIRCUIT

Figure shows an ac circuit consisting of a capacitor of capacitance $C$ connected across the terminals of an ac generator. On applying Kirchhoff's loop rule to this circuit we get

$$\varepsilon - V_C = 0$$

$$V_C = \varepsilon = \varepsilon_0 \sin \omega t \qquad \text{...(vi)}$$

where $V_C$ is the instantaneous voltage drop across the capacitor. From the definition of capacitance, $V_C = Q/C$, and this value for $V_C$ substituted into equation gives




$$Q = C\varepsilon_0 \sin \omega t$$

Since $i = dQ/dt$, on differentiating above equation gives the instantaneous current in the circuit.

$$i_C = \frac{dQ}{dt} = C\varepsilon_0\omega \cos\omega t$$

Here again we see that the current is not in phase with the voltage drop across the capacitor, given by equation (vi). Using the trigonometric identity $\cos \omega t = \sin(\omega t + \pi/2)$, we can express this equation in the alternative from

$$i_C = \omega C\varepsilon_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \text{... (vii)}$$

From equation (vii), we see that the current in the circuit reaches its maximum value when $\cos \omega t = 1$.

$$i_0 = \omega C\varepsilon_0 = \frac{\varepsilon_0}{X_C}$$

where $X_C$ is called the capacitive reactance.

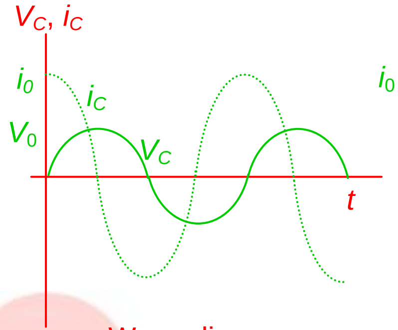
$$X_C = \frac{1}{\omega C} \qquad \text{... (26)}$$

The SI unit of $X_C$ is also ohm. The rms current is given by an expression similar to equation with $V_0$ replaced by $V_{rms}$.

Combining equation (vi) & (vii), we can express the instantaneous voltage drop across the capacitor as

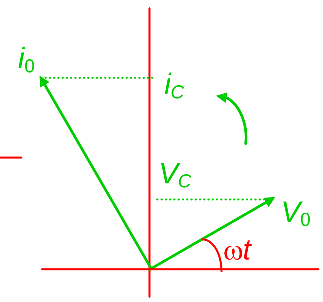
$$V_C = V_0 \sin \omega t = I_0 X_C \sin \omega t \qquad \text{... (27)}$$

Comparing the result of equation (vii) with equation (vi), we see that the **current is $\pi/2$ rad = 90° out of phase with the voltage** across the capacitor. A plot of current and voltage versus time, shows that the current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value. The corresponding phasor diagram is shown in the figure (b).Thus we see that for a sinusoidally applied emf, the current always leads the voltage across a capacitor by 90°.



Wave diagram

Fig. (a)



Phasor diagram

Fig. (b)

## 8.4 VECTOR ANALYSIS (PHASOR ALGEBRA)

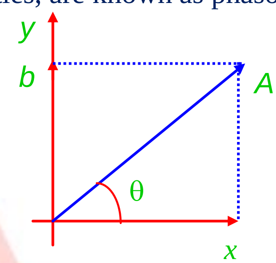
The complex quantities normally employed in ac circuit analysis, can be added and subtracted like coplanar vectors. Such coplanar vectors, which represent sinusoidally time varying quantities, are known as phasors.

In cartesian form, a phasor $A$ can be written as,

$$A = a + jb$$

where a is the $x$-component and $b$ is the $y$ component of phasor $A$.

The magnitude of $A$ is, $|A| = \sqrt{a^2 + b^2}$




and the angle between the direction of phasor $A$ and the positive $x$-axis is,

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

When a given phasor $A$, the direction of which is along the $x$-axis is multiplied by the operator $j$, a new phasor $j\,A$ is obtained which will be 90° anticlockwise from $A$, i.e., along y-axis. If the operator $j$ is multiplied now to the phasor $jA$, a new phasor $j^2A$ is obtained which is along $x$-axis and having same magnitude as of $A$. Thus,

$$j^2 A = -A$$

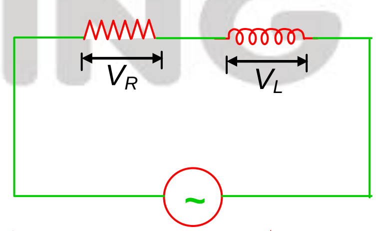
$$j^2 = -1 \qquad \text{or} \quad j = \sqrt{-1}$$

Now using the $j$ operator, let us discuss different circuits of an ac.

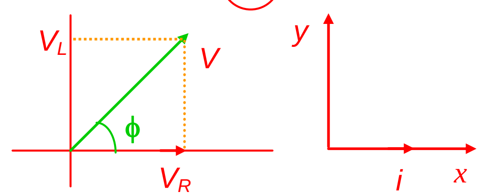
## 8.5 SERIES L-R CIRCUIT

Now consider an ac circuit consisting of a resistor of resistance $R$ and an inductor of inductance $L$ in series with an ac source generator.

Suppose in phasor diagram, current is taken along positive $x$-direction. The $V_R$ is also along positive $x$-direction and $V_L$ along positive $y$-direction as we know potential difference across a resistance in ac is in phase with current and it leads in phase by 90° with current across the inductor, so we can write




$$\begin{aligned} V &= V_R + jV_L = iR + j(iX_L) \\ &= iR + j(i\omega L) \\ &= iZ \end{aligned}$$




Here, $Z = R + jX_L = R + j\,(\omega L)$ is called as impedance of the circuit. Impedance plays the same role in ac circuits as the ohmic resistance does in dc circuits. The modulus of impedance is,

$$|Z| = \sqrt{R^2 + (\omega L)^2} \qquad \ldots (28)$$

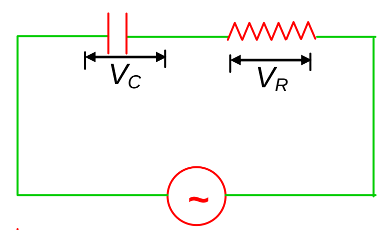
The potential difference leads the current by an angle,

$$\phi = \tan^{-1}\left|\frac{V_L}{V_R}\right| = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \text{... (29)}$$

## 8.6 SERIES C-R CIRCUIT

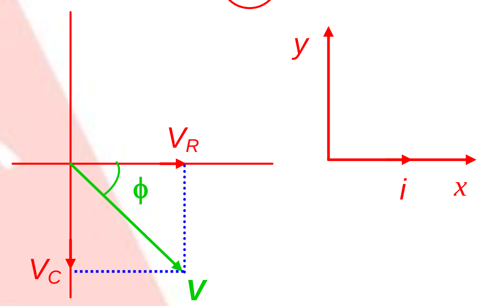
Now consider an ac circuit consisting of a resistor of resistance $R$ and a capacitor of capacitance $C$ in series with an ac source generator.




Suppose in phasor diagram current is taken along positive $x$-direction. Then $V_R$ is also along positive $x$-direction but $V_C$ is along negative $y$-direction as potential difference across a capacitor in ac lags in phase by 90° with the current in the circuit. So we can write.

$$V = V_R - jV_C = iR - j\,(iX_C)$$

$$= iR - j\left(\frac{i}{\omega C}\right) = iZ$$




Here, impedance is, $Z = R - j\left(\frac{1}{\omega C}\right)$

The modulus of impedance is,

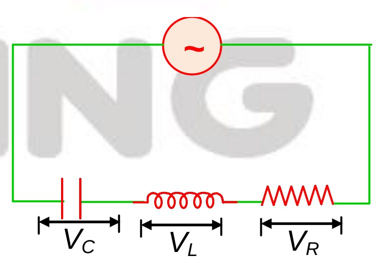
$$|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \qquad \text{... (30)}$$

and the potential difference lags the current by an angle,

$$\phi = \tan^{-1}\left|\frac{V_C}{V_R}\right| = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\left(\frac{1/\omega C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right) \qquad \text{... (31)}$$

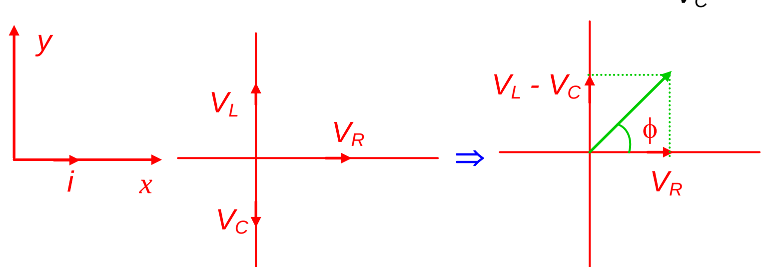
## .7 SERIES L-C-R CIRCUIT

Now consider an ac circuit consisting of a resistor of resistance $R$, a capacitor of capacitance $C$ and an inductor of inductance $L$ in series with an ac source generator.




Suppose in a phasor diagram current is taken along positive $x$-direction. Then $V_R$ is along positive $x$-direction, $V_L$ along positive $y$-direction and $V_C$ along negative $y$-direction, as potential difference across an inductor leads the current by 90° in phase while that across a capacitor, lags by 90°.

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$




So, we can write, $V = V_R + jV_L - jV_C = iR + j(iX_L) - j(iX_C)$

$iR + j\,[i\,(X_L - X_C)] = iZ$

Here impedance is,

$$Z = R + j\,(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

***The modulus of impedance is,*** $|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ ***… (32)***

and the potential difference leads the current by an angle,.

$$\phi = \tan^{-1}\left|\frac{V_L - V_C}{V_R}\right| = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) \quad \text{… (33)}$$

The steady current in the circuit is given by $i = \dfrac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t + \phi)$

where $\phi$ is given from equation (19)

The peak current is $i_0 = \dfrac{V_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

It depends on angular frequency $\omega$ of ac source and it will be maximum when

$$\omega L = \frac{1}{\omega C} \quad \Rightarrow \quad \omega = \sqrt{\frac{1}{LC}}$$

and corresponding frequency is

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \quad \text{… (34)}$$

This frequency is known as **resonant frequency** of the given circuit. At this frequency peak current will be $i_0 = \dfrac{V_0}{R}$

If the resistance $R$ in the $LCR$ circuit is zero, the peak current at resonance is $i_0 = \dfrac{V_0}{0}$

It means, there can be a finite current in pure $LC$ circuit even without any applied emf, when a charged capacitor is connected to pure inductor.

This current in the circuit is at frequency, $\nu = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}}$

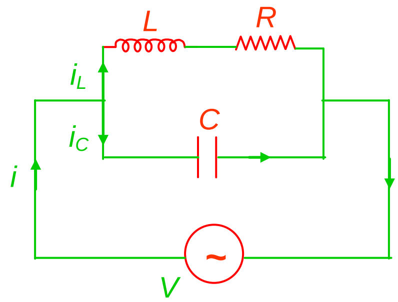
## 9 PARALLEL AC CIRCUIT

Let us consider an alternating source connected across an inductance $L$ in parallel with a capacitor $C$.

The resistance in series with the inductance is $R$.

Let the instantaneous value of emf applied be $V$ and the corresponding current is $i$, $i_L$ and $i_C$. Then,

$$i = i_L + i_C$$



or, $$\frac{V}{Z} = \frac{V}{R + j\omega L} - \frac{V}{j/\omega C}$$

$$= \frac{V}{R + j\omega L} + j(\omega C)V \quad (\text{as } j^2 = -1)$$

$$\frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$\frac{1}{Z}$ is known as ***admittance (Y)***. Therefore,

$$Y = \frac{1}{Z} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C = \frac{R + j(\omega CR^2 + \omega^3 L^2 C - \omega L)}{R^2 + \omega^2 L^2}$$

$\therefore$ The magnitude of the admittance,

***Y = |Y| =*** $$\frac{\sqrt{R^2 + (\omega CR^2 + \omega^3 L^2 C - \omega L)^2}}{R^2 + \omega^2 L^2} \quad \text{... (35)}$$

The admittance will be minimum, when

$\omega CR^2 + \omega^3 L^2 C - \omega L = 0$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

It gives the condition of resonance and the corresponding frequency,

***f =*** $$\frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{... (36)}$$

is known as **resonance frequency**. At resonance frequency, admittance is minimum or the impedance is maximum. Thus, the parallel circuit does not allow this frequency from the source to pass in the circuit. Due to this reason the circuit with such a frequency is known as **rejector circuit**.

☞ ***If R = 0, resonance frequency is*** $\frac{1}{2\pi\sqrt{LC}}$ ***same as resonance frequency in series circuit.***

At resonance, the reactive component of $Y$ is real. The reciprocal of the admittance is called the **parallel resistor** or the **dynamic resistance**. The dynamic resistance is thus, reciprocal of the real part of the admittance.

Dynamic resistance = $\frac{R^2 + \omega^2 L^2}{R}$

Substituting $\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$

we have, dynamic resistance = $\frac{L}{CR}$

$\therefore$ peak current through the supply = $\frac{V_0}{L/CR} = \frac{V_0 CR}{L}$

The peak current through capacitor = $\frac{V_0}{1/\omega C} = \omega C V_0$. The ratio of the peak current through capacitor and through the supply is known as **Q-factor**.

***Thus, Q-factor =*** $$\frac{V_0 \omega C}{V_0 CR/L} = \frac{\omega L}{R} \quad \text{... (37)}$$

This is basically the measure of current magnification. The rejector circuit at resonance exhibits current magnification of $\frac{\omega L}{R}$, similar to the voltage magnification of the same ratio exhibited by the series acceptor circuit at resonance.

☞ ***At resonance the current through the supply and voltage are in phase, while the current*** *through the capacitor leads the voltage by 90°.*

## 10 POWER IN AN AC CIRCUIT

In case of a steady current the rate of doing work is given by,

$P = Vi$

In an alternating circuit, current and voltage both vary with time, so the work done by the soruce in time interval $dt$ is given by

$dW = Vidt$

Suppose in an ac, the current is leading the voltage by an angle $\phi$. Then we can write,

$V = V_0 \sin \omega t$

and $i = i_0 \sin(\omega t + \phi)$

$dW = V_0 i_0 \sin \omega t \sin(\omega t + \phi)\, dt$

$= V_0 i_0 (\sin^2\omega t \cos\phi + \sin\omega t \cos\omega t \sin\phi)\, dt.$

The total work done in a complete cycle is

$$W = V_0 i_0 \cos\phi \int_0^T \sin^2 \omega t\, dt + V_0 i_0 \sin\phi \int_0^T \sin\omega t \cos\omega t\, dt$$

$$= \frac{1}{2} V_0 i_0 \cos\phi \int_0^T (1 - \cos 2\omega t)\, dt + \frac{1}{2} V_0 i_0 \sin\phi \int_0^T \sin 2\omega t\, dt = \frac{1}{2} V_0 i_0 T \cos\phi$$

The average power delivered by the soruce is, therefore,

$$P = \frac{W}{T} = \frac{1}{2} V_0 i_0 \cos\phi = \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{i_0}{\sqrt{2}}\right)(\cos\phi) = V_{rms}\, i_{rms} \cos\phi$$

*or* $\langle P \rangle_{one\ cycle} = V_{rms}\, i_{rms} \cos\phi$ ... (38)

Here, the term $\cos\phi$ is known as **power factor**.

It is said to be leading if current leads voltage, lagging if current lags voltage. Thus, a power factor of 0.5 lagging means current lags the voltage by 60° (as $\cos^{-1} 0.5 = 60°$). The product of $V_{rms}$ and $i_{rms}$ gives the apparent power. While the true power is obtained by multiplying the apparent power by the power factor $\cos\phi$. Thus,

and apparent power = $V_{rms} \times i_{rms}$

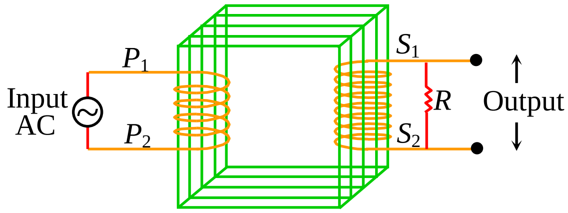
True power = apparent power × power factor

For $\phi = 0°$, the current and voltage are in phase. The power is thus, maximum ($V_{rms} \times i_{rms}$). For $\phi = 90°$, the power is zero. The current is then stated **wattless**. Such a case will arise when resistance in the circuit is zero. The circuit is purely inductive or capacitive.

## 11 TRANSFORMERS

One of the great advantage of *ac* over *dc* for electric-power distribution is that it is much easier to step voltage level up and down with ac than with dc. The necessary conversion is accomplished by a static device called transformer using the principle of mutual induction.

The figure shows an idealised transformer which consists of two coils or windings, electrically insulated from each other but wound on the same core. The winding to which power is supplied is called primary, the winding from which power is delivered is called the secondary.



Laminated core

The *ac* source causes an alternating current in the primary which sets up an alternating flux in the core and this induces an emf in each winding of secondary in accordance with Faraday's law. For ideal transformer we assume that primary has negligible resistance and all the flux in core links both primary and secondary. The primary winding has $N_1$ turns and secondary has $N_2$ turns. When the magnetic flux changes because of changing currents in the two coils, the resulting induced emf are

$$e_1 = -N_1 \frac{d\phi_B}{dt} \text{ and } e_2 = -N_2 \frac{d\phi_B}{dt}$$

The flux per turn $\phi_B$ is same in both primary and the secondary so that the emf per turn is same in each. The ratio of secondary emf $\sum_2$ to the primary emf $\sum_1$ is therefore equal at any instant to the ratio of secondary to primary turns.

$$\frac{e_2}{e_1} = \frac{N_2}{N_1}$$

If the windings have zero resistance, the induced emf $e_1$ and $e_2$ are equal to the terminal voltage across the primary and the secondary respectively, hence

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

If $N_2 > N_1$ then $V_2 > V_1$ and we have **step up** transformer, if $N_2 < N_1$ then $V_2 < V_1$ and we have a **step down** transformer.

If the transformer is assumed to be 100% efficient (no energy losses) the power input is equal to the power output i.e.

$$I_2V_2 = I_1V_1 \qquad \therefore \qquad \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

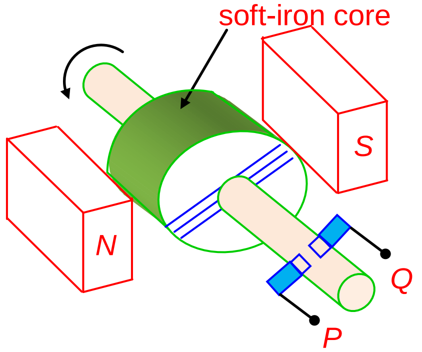
All the currents and voltages derived above have same frequency as that of source. The equations obtained above apply to ideal transformers, although some energy is lost but well designed transformers have efficiency more than 95%, this is a good approximation. The causes of energy losses and their rectification is given below:

| ***Cause*** | ***Rectification*** |
|---|---|
| ***1. Due to poor design and air gaps in the core, all the flux due to primary does not pass through the secondary.*** | ***1. By winding the primary and secondary coil one over the other.*** |
| ***2. Resistance of windings causes $I^2R$ loss.*** | ***2. In high current, low voltage, these are minimised by using thick wire.*** |
| ***3. The alternating magnetic flux induces eddy currents in the core and causes heating.*** | ***3. By using laminated core it can be reduced.*** |
| ***4. Alternating magnetisation of core causes hystersis loss.*** | ***4. It is kept minimum by using a magnetic material having low hystersis loss. (e.g. soft iron)*** |

## AC GENERATOR

A device which converts mechanical energy into electrical energy using principle of electromagnetic induction is called ac generator. A schematic design and simplified diagram is shown in figure.

It consists of three main parts, a magnet, an armature with slip rings and brushes. The armature contains coil of $N$ loops which rotates in the magnetic field generated by the magnets.




Suppose the plane of coil having area $A$ is perpendicular to the magnetic field and rotating with constant angular velocity $\omega$, at $t = 0$. The flux through each turn of coil at time $t$ is $\phi = BA\cos\theta$, where $\theta = \omega t$ is the angle by which coil rotates in time $t$. The total emf induced in the coil is,

$$e = -\frac{Nd\phi}{dt}$$
$$= NBA\omega \sin \omega t$$
$$= e_0 \sin \omega t$$

Here, $e_0 = NBA\omega$ is the maximum generated emf is known as peak emf. This emf is delivered to external circuits from two graphite brushes $B_1$ and $B_2$ which touches permanently the slip rings $C_1$ and $C_2$. As the armature rotates the slip rings $C_1$ and $C_2$ slip against the brushes so that the contact is maintained all the time.

The mechanical energy which gets converted into electrical energy required for rotation is provided by falling water from height in hydro-electric generator. The frequency of rotation is 50Hz in India.

# MIND MAP

## TRANSIENT CURRENT CIRCUIT

### 1. *RC* circuit

**(i) Charging of capacitor**

$$q = q_0\left(1-e^{\frac{-t}{RC}}\right),\ i = \frac{E}{R}e^{\frac{-t}{RC}}$$

**(ii) Discharging of capacitor**

$$q = q_0 e^{\frac{-t}{RC}},\ i = \frac{q_0}{RC}e^{\frac{-t}{RC}}$$

### 2. *L-R* circuit

**(i) Growth of current**

$$i_R = \frac{E}{R}\left(1-e^{\frac{-Rt}{L}}\right)$$

**(ii) Decay of current**

$$i_R = \frac{I_0}{R}e^{\frac{-Rt}{L}}$$

### 3. *L-C* oscillation

When a capcitor is charged up to $Q_0$ and then connected to an inductor

(i) $\frac{Ldi}{dt}+\frac{Q}{C}=0$

(ii) $Q = Q_0\cos\omega t$

(iii) $i = -Q_0\omega\sin\omega t$

(iv) $\omega = \frac{1}{\sqrt{LC}}$