# DPP
**DAILY PRACTICE PROBLEMS**

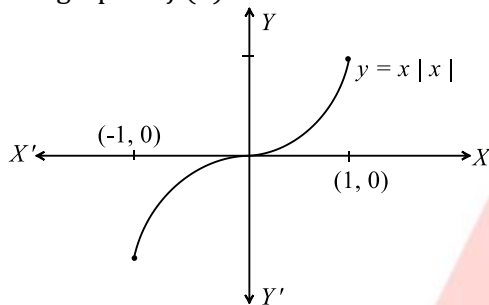| CLASS : XIIth | SOLUTIONS | SUBJECT : MATHS |
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| DATE : | | DPP NO. :1 |

## Topic :-RELATIONS AND FUNCTIONS

**1    (a)**

We have,

$$f(x) = x|x| = \begin{cases} x^2, & 0 \le x \le 1 \\ -x^2, & -1 \le x < 0 \end{cases}$$

The graph of $f(x)$ is as shown below. Clearly, it is a bijection



**2    (b)**

Foe domain of given function

$$-1 \le \log_2 \frac{x^2}{2} \le 1$$

$$\Rightarrow \quad 2^{-1} \le \frac{x^2}{2} \le 2 \Rightarrow 1 \le x^2 \le 4$$

$$\Rightarrow \quad |x| \le 2 \text{ and } |x| \ge 1$$

$$\Rightarrow \quad x \in [-2, 2] - (-1, 1)$$

**3    (c)**

Given,  $f(x) = ax + b, \quad g(x) = cx + d$

$\because \quad f\{g(x)\} = g\{f(x)\}$

$\Rightarrow \quad f(cx + d) = g(ax + b)$

$\Rightarrow \quad a(cx + d) + b = c(ax + b) + d$

$\Rightarrow \quad ad + b = bc + d$

$\Rightarrow \quad f(d) = g(b)$

**4    (c)**

Since $\phi(x) = \sin^4 x + \cos^4 x$ is periodic with period $\pi/2$

$\therefore f(x) = \sin^4 3x + \cos^4 3x$ is periodic with period $\frac{1}{3}\left(\frac{\pi}{2}\right) = \frac{\pi}{6}$

**5    (b)**

We have,

$$f(x) = \log\left(\frac{1+x}{1-x}\right) \text{ and } g(x) = \frac{3x + x^3}{1 + 3x^2}$$

$\therefore fog\,(x) = f(g(x))$

$$\Rightarrow fog\,(x) = f\left(\frac{3x + x^3}{1 + 3x^2}\right) = \log\left(\frac{1 + \frac{3x + x^3}{1 + 3x^2}}{1 - \frac{3x + x^3}{1 + 3x^2}}\right) = \log\left\{\frac{(1+x)^3}{(1-x)^3}\right\}$$

$$\Rightarrow fog\ (x) = \log\left(\frac{1+x}{1-x}\right)^3 = 3\log\left(\frac{1+x}{1-x}\right) = 3f(x)$$

**6**     **(b)**

For choice (a), we have

$f(x) = f(y); x, y \in [-1, \infty)$

$\Rightarrow |x+1| = |y+1| \Rightarrow x+1 = y+1 \Rightarrow x = y$

So, $f$ is an injection

For choice (b), we obtain

$g(2) = \dfrac{5}{2}$ and $g\left(\dfrac{1}{2}\right) = \dfrac{5}{2}$

So, $g(x)$ is not injective

It can be easily seen that the functions in choices in options (c) and (d) are injective maps

**7**     **(b)**

Given, $f(x) = x - [x], g(x) = [x]$ for $x \in R$.

$$\therefore \quad f\big(g(x)\big) = f([x])$$
$$= [x] - [x]$$
$$= 0$$

**8**     **(a)**

We have,

$$f(x) = \sqrt{\frac{\log_{0.3}|x-2|}{|x|}}$$

We observe that $f(x)$ assumes real values, if

$$\frac{\log_{0.3}|x-2|}{|x|} \geq 0 \text{ and } |x-2| > 0$$

$\Rightarrow \log_{0.3}|x-2| \geq 0$ and $x \neq 0, 2$

$\Rightarrow |x-2| \leq 1$ and $x \neq 0, 2$

$\Rightarrow x \in [1, 3]$ and $x \neq 2 \Rightarrow x \in [1, 2) \cup (2, 3]$

**9**     **(d)**

Since $g(x) = 3\sin x$ is a many-one function. Therefore, $f(x) - 3\sin x$ is many-one

Also, $-1 \leq \sin x \leq 1$

$\Rightarrow -3 \leq -3\sin x + 3$

$\Rightarrow 2 \leq 5 - 3\sin x \leq 8$

$\Rightarrow 2 \leq f(x) \leq 8 \Rightarrow$ Range of $f(x) = [2, 8] \neq R$

So, $f(x)$ is not onto

Hence, $f(x)$ is neither one-one nor onto

**10**     **(a)**

We have,

$f(x + 2y, x - 2y) = xy$     ....(i)

Let $x + 2y = u$ and $x - 2y = v$. Then,

$x = \dfrac{u+v}{2}$ and $y = \dfrac{u-v}{4}$

Substituting the values of $x$ and $y$ in (i), we obtain

$$f(u, v) = \frac{u^2 - v^2}{2} \text{ and } f(x, y) = \frac{x^2 - y^2}{8}$$

**11**     **(c)**

Given, $f(x) = y = (1-x)^{1/3}$

$\Rightarrow \quad y^3 = 1 - x$

$\Rightarrow \quad x = 1 - y^3$

$\therefore \quad f^{-1}(x) = 1 - x^3$

**12**     **(a)**

We have, $f(x + 2y, x - 2y) = xy$                ...(i)

Let $x + 2y = u$ and $x - 2y = v$

Then, $x = \dfrac{u+v}{2}$ and $y = \dfrac{u-v}{4}$

Subtracting the values of $x$ and $y$ in Eq. (i), we obtain

$$f(u, v) = \frac{u^2 - v^2}{8} \Rightarrow f(x, y) = \frac{x^2 - y^2}{8}$$

**13    (d)**

Given, $f(x) = 5^{x(x-4)}$ for $f : [4, \infty[ \to [4, \infty[$

At $x = 4$

$\qquad f(x) = 5^{4(4-4)} = 1$

Which is not lie in the interval $[4, \infty[$

$\therefore$ Function is not bijective.

Hence, $f^{-1}(x)$ is not defined.

**14    (b)**

Given, $f(x) = x^3 + 3x - 2$

On differentiating w.r.t. $x$, we get

$\qquad f'(x) = 3x^2 + 3$

Put $f'(x) = 0 \Rightarrow 3x^2 + 3 = 0$

$\Rightarrow \qquad\qquad x^2 = -1$

$\therefore$ $f(x)$ is either increasing or decreasing.

At $x = 2, f(2) = 2^3 + 3(2) - 2 = 12$

At $x = 3, f(3) = 3^3 + 3(3) - 2 = 34$

$\therefore f(x) \in [12, 34]$

**15    (b)**

We have,

$$f(\theta) = \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$\therefore f(\theta)$ is periodic with period $\dfrac{2\pi}{2} = \pi$

**16    (c)**

Since, period of $\cos nx = \dfrac{2\pi}{n}$

And period of $\sin\left(\dfrac{x}{n}\right) = 2n\pi$

$\therefore$ Period of $\dfrac{\cos nx}{\sin\left(\frac{x}{n}\right)}$ is $2n\pi$

$\Rightarrow \ 2n\pi = 4\pi \ \Rightarrow \ n = 2$

**17    (c)**

Given, $f(x) = x^3 + 5x + 1$

Now, $f'(x) = 3x^2 + 5 > 0, \forall x \in R$

$\therefore f(x)$ is strictly increasing function.

$\therefore f(x)$ is one-one function.

Clearly, $f(x)$ is a continous function and also increasing on $R$,

$\lim\limits_{x \to -\infty} f(x) = -\infty \ and \ \lim\limits_{x \to \infty} = \infty$

$\therefore f(x)$ takes every value between $-\infty$ and $\infty$

Thus, $f(x)$ is onto function.

**18    (c)**

The function $f(x) = \dfrac{1}{2 - \cos 3x}$ is defined for all $x \in R$. Therefore, domain of $f(x)$ is $R$

Let $f(x) = y$. Then,

$\dfrac{1}{2 - \cos 3x} = y$ and $y > 0$

$\Rightarrow 2 - \cos 3x = \dfrac{1}{y}$

$\Rightarrow \cos 3x = \dfrac{2y-1}{y} \Rightarrow x = \dfrac{1}{3}\cos^{-1}\left(\dfrac{2y-1}{y}\right)$

Now,

$x \in R$, if

$-1 \le \dfrac{2y-1}{y} \le 1$

$\Rightarrow -1 \le 2 - \dfrac{1}{y} \le 1$

$\Rightarrow -3 \le -\dfrac{1}{y} \le -1$

$\Rightarrow 3 \ge \dfrac{1}{y} \ge 1 \Rightarrow \dfrac{1}{3} \le y \le 1 \Rightarrow y \in [1/3, 1]$

**19      (c)**

Given, $A = \{2, 3, 4, 5, \ldots, 16, 17, 18\}$

And $(a, b) = (c, d)$

$\therefore$ Equivalence class of $(3, 2)$ is

$\{(a, b) \in A \times A : (a, b) R\ (3, 2)\}$

$= \{(a, b) \in A \times A : 2a = 3b\}$

$= \left\{(a, b) \in A \times A : b = \dfrac{2}{3}a\right\}$

$\left\{\left(a, \dfrac{2}{3}a\right) : a \in A \times A\right\}$

$= \{(3, 2), (6, 4), (9, 6), (12, 8), (15, 10), (18, 12)\}$

$\therefore$ Number of ordered pairs of the equivalence class$=6$.

**20      (c)**

Given function is $f(n) = 8 - {}^{n}P_{n-4}, 4 \le n \le 6$. It is defined, if

1. $8 - n > 0 \Rightarrow n < 8$ ... (i)

2. $n - 4 \ge 0 \Rightarrow n \ge 4$ ... (ii)

3. $n - 4 \le 8 - n \Rightarrow n \le 6$ ... (iii)

From Eqs. (i), (ii) and (iii), we get $n = 4, 5, 6$

Hence, range of $f(n) = \{ {}^{4}P_0,\ {}^{3}P_1,\ {}^{2}P_2\} = \{1, 3, 2\}$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| A. | A | B | C | C | B | B | B | A | D | A |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| A. | C | A | D | B | B | C | C | C | C | C |
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