

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth

DATE :

SOLUTION

SUBJECT : MATHS

DPP NO. :1

Topic :- INVERSE TRIGONOMETRIC FUNCTIONS

1 (a)

We have,

$$\begin{aligned} & \sin^{-1} \frac{3}{5} + \tan^{-1} \frac{1}{7} \\ &= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7} \\ &= \tan^{-1} \left\{ \frac{3/4 + 1/7}{1 - 3/4 \times 1/7} \right\} = \tan^{-1} \left(\frac{25}{25} \right) = \tan^{-1} 1 = \frac{\pi}{4} \end{aligned}$$

3 (b)

Given that, $x^2 + y^2 + z^2 = r^2$

Now, $\tan^{-1} \left(\frac{xy}{zr} \right) + \tan^{-1} \left(\frac{yz}{xr} \right) + \tan^{-1} \left(\frac{xz}{yr} \right)$

$$\begin{aligned} &= \tan^{-1} \left[\frac{\frac{xy}{zr} + \frac{yz}{xr} + \frac{xz}{yr} - \frac{xyz}{r^3}}{1 - \left(\frac{x^2 + y^2 + z^2}{r^2} \right)} \right] \\ &= \tan^{-1} \left[\frac{\frac{xy}{zr} + \frac{yz}{xr} + \frac{xz}{yr} - \frac{xyz}{r^3}}{1 - \frac{r^2}{r^2}} \right] \\ &= \tan^{-1} \infty = \frac{\pi}{2} \end{aligned}$$

4 (b)

Put $x = \sin \theta$, we get

$$f(x) = \sin^{-1} \left\{ \sin \left(\theta - \frac{\pi}{6} \right) \right\}$$

For, $-\frac{1}{2} \leq x \leq 1$

$$\Rightarrow -\frac{1}{2} \leq \sin \theta \leq 1$$

$$\Rightarrow -\frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

is in the fourth on the first quadrant

$$\therefore f(x) = \theta - \frac{\pi}{6} = \sin^{-1} x - \frac{\pi}{6}$$

5 (c)

$$\begin{aligned} & \cos^{-1} \left(\frac{1}{2} \right) + 2 \sin^{-1} \left(\frac{1}{2} \right) \\ &= \frac{\pi}{3} + 2 \cdot \frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3} \end{aligned}$$

6 (c)

Given, $\tan^{-1} 2\theta + \tan^{-1} 3\theta = \frac{\pi}{4}$



$$\therefore \tan^{-1} \left(\frac{2\theta + 3\theta}{1 - 2\theta \times 3\theta} \right) = \tan^{-1} 1$$

$$\Rightarrow 6\theta^2 + 5\theta - 1 = 0$$

$$\Rightarrow \theta = \frac{-5 \pm \sqrt{25 + 24}}{2 \times 6}$$

$$= \frac{-5 \pm 7}{12} = -1, \frac{1}{6}$$

$$\Rightarrow \theta = \frac{1}{6}$$

7 (b)

$$\text{Let } \theta = \cos^{-1} \left(-\frac{1}{2} \right)$$

$$\Rightarrow \cos \theta = -\frac{1}{2} = -\cos \left(\frac{\pi}{3} \right)$$

$$= \cos \left(\pi - \frac{\pi}{3} \right) = \cos \left(\frac{2\pi}{3} \right)$$

$$\Rightarrow \theta = 2n\pi \pm \frac{2\pi}{3}$$

$$\Rightarrow \theta = \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \dots$$

8 (c)

$$\tan \theta + \tan \left(\frac{\pi}{3} + \theta \right) + \tan \left(-\frac{\pi}{3} + \theta \right) = a \tan 3\theta$$

$$\Rightarrow \tan \theta + \frac{\sqrt{3} + \tan \theta}{1 - \sqrt{3} \tan \theta} + \frac{\tan \theta - \sqrt{3}}{1 + \sqrt{3} \tan \theta} = a \tan 3\theta$$

$$\Rightarrow \tan \theta + \frac{8 \tan \theta}{1 - 3 \tan^2 \theta} = a \tan 3\theta$$

$$\Rightarrow \frac{3(3 \tan \theta - \tan^3 \theta)}{1 - 3 \tan^2 \theta} = a \tan 3\theta$$

$$\Rightarrow 3 \tan 3\theta = a \tan 3\theta$$

$$\Rightarrow a = 3$$

9 (a)

$$\text{Let } \cot^{-1} \frac{3}{4} = \theta \Rightarrow \cot \theta = \frac{3}{4}$$

$$\text{And } \sin \theta = \frac{1}{\sqrt{1 + \cot^2 \theta}} = \frac{1}{\sqrt{1 + \left(\frac{9}{16}\right)}} = \frac{4}{5}$$

$$\therefore \cot^{-1} \frac{3}{4} + \sin^{-1} \frac{5}{13} = \sin^{-1} \frac{4}{5} + \sin^{-1} \frac{5}{13}$$

$$= \sin^{-1} \left[\frac{4}{5} \sqrt{1 - \frac{25}{169}} + \frac{5}{13} \sqrt{1 - \frac{16}{25}} \right]$$

$$= \sin^{-1} \left[\frac{4}{5} \cdot \frac{12}{13} + \frac{5}{13} \cdot \frac{3}{5} \right]$$

$$= \sin^{-1} \left[\frac{48 + 15}{65} \right] = \sin^{-1} \frac{63}{65}$$

10 (d)

$$\text{Now, } \cos^{-1}(\cos 4) = \cos^{-1}\{\cos(2\pi - 4)\} = 2\pi - 4$$

$$\Rightarrow 2\pi - 4 > 3x^2 - 4x$$

$$\Rightarrow 3x^2 - 4x - (2\pi - 4) < 0$$

$$\Rightarrow \frac{2 - \sqrt{6\pi - 8}}{3} < x < \frac{2 + \sqrt{6\pi - 8}}{3}$$

11 (c)

$$\text{Let } \tan^{-1} x = \theta. \text{ Then, } x = \tan \theta$$



Also,

$$x \in (-\infty, -1)$$

$$\Rightarrow -\infty < x < 1 \Rightarrow -\infty < \tan \theta < -1 \Rightarrow -\frac{\pi}{2} < \theta < -\frac{\pi}{4}$$

Now,

$$\begin{aligned} \tan^{-1}\left(\frac{2x}{1-x^2}\right) &= \tan^{-1}(\tan 2\theta) = \tan^{-1}(\tan(\pi + 2\theta)) \\ &= \pi + 2\theta \quad \left[\because -\frac{\pi}{2} < \theta < -\frac{\pi}{4} \Rightarrow 0 < \pi + 2\theta < \frac{\pi}{2} \right] \\ &= \pi + 2 \tan^{-1} x \end{aligned}$$

12 (b)

Let $\sin^{-1} x = \theta$. Then, $x = \sin \theta$ and $\sqrt{1-x^2} = \cos \theta$

Also,

$$\frac{1}{\sqrt{2}} \leq x \leq 1 \Rightarrow \frac{1}{\sqrt{2}} \leq \sin \theta \leq 1 \Rightarrow \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$$

$$\begin{aligned} \therefore \sin^{-1}\left(2x\sqrt{1-x^2}\right) &= \sin^{-1}(\sin 2\theta) \\ &= \sin^{-1}(\sin(\pi - 2\theta)) \\ &= \pi - 2\theta \quad \left[\because \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2} \Rightarrow 0 \leq \pi - 2\theta \leq \frac{\pi}{2} \right] \\ &= \pi - 2 \sin^{-1} x \end{aligned}$$

14 (a)

$$\text{Since, } -\frac{\pi}{2} < \sin^{-1} x \leq \frac{\pi}{2}$$

$$\begin{aligned} \therefore \sin^{-1} x_i &= \frac{\pi}{2}, 1 \leq i \leq 20 \\ \Rightarrow x_i &= 1, 1 \leq i \leq 20 \end{aligned}$$

Thus, $\sum_{i=1}^{20} x_i = 20$

15 (a)

$$1 \text{ rad} > 45^\circ$$

$$\Rightarrow \tan 1^\circ > \tan 45^\circ \Rightarrow \tan 1 > 1$$

$$\text{Also, } \tan^{-1} 1 = \frac{\pi}{4} < 1$$

$$\text{Hence, } \tan 1 > \tan^{-1} 1$$

16 (c)

$$\alpha + \beta = \sin^{-1} \frac{\sqrt{3}}{2} + \cos^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} \frac{1}{3} + \cos^{-1} \frac{1}{3}$$

$$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

$$\text{Also, } \alpha = \frac{\pi}{3} + \sin^{-1} \frac{1}{3} < \frac{\pi}{3} + \sin^{-1} \frac{1}{2}$$

As $\sin \theta$ is increasing in $\left[0, \frac{\pi}{2}\right]$

$$\therefore \alpha < \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$$

$$\Rightarrow \beta > \frac{\pi}{2} > \alpha$$

$$\Rightarrow \alpha < \beta$$

17 (d)

$$\begin{aligned} &2 \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) \\ &= \tan^{-1}\left[\frac{2\left(\frac{1}{3}\right)}{1-\frac{1}{9}}\right] + \tan^{-1}\left(\frac{1}{7}\right) \end{aligned}$$

$$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \times \frac{1}{7}}\right)$$

$$= \tan^{-1}\left(\frac{25}{25}\right) = \frac{\pi}{4}$$

18 (c)

$$\tan\left[\frac{1}{2} \sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2} \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$$

$$= \tan\left[\frac{1}{2} \cdot 2 \tan^{-1} a + \frac{1}{2} \cdot 2 \tan^{-1} a\right]$$

$$= \tan(2 \tan^{-1} a)$$

$$= \tan\left[\tan^{-1}\left(\frac{2a}{1-a^2}\right)\right]$$

$$= \frac{2a}{1-a^2}$$

19 (c)

Let $S_{\infty} = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \dots$

$$\therefore T_n \cot^{-1} 2n^2$$

$$= \tan^{-1} \frac{1}{2n^2}$$

$$= \tan^{-1}\left(\frac{2}{4n^2}\right) = \tan^{-1}\left(\frac{(2n+1) - (2n-1)}{1 + (2n+1)(2n-1)}\right)$$

$$\therefore S_n = \sum_{n=1}^{\infty} \{\tan^{-1}(2n+1) - \tan^{-1}(2n-1)\}$$

$$= \tan^{-1} \infty - \tan^{-1} 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

20 (a)

Given, $\tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{a}{x} + \frac{b}{x}}{1 - \frac{ab}{x^2}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{(a+b)x}{x^2 - ab} = \frac{\pi}{2}$$

$$= \tan \frac{\pi}{2}$$

$$\Rightarrow \frac{(a+b)x}{x^2 - ab} = 0$$

$$\Rightarrow x^2 - ab = 0$$

$$\Rightarrow x = \sqrt{ab}$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	B	B	C	C	B	C	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	C	A	A	C	D	C	C	A



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