

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :1

Topic :-MATRICES

2 (b)

$$\left[\frac{1}{2}(A - A')\right]' = \frac{1}{2}(A - A')'$$

$$= \frac{1}{2}(A' - A)$$

$$= -\frac{1}{2}(A - A')$$

Hence, it is a skew-symmetric matrix.

3 (b)

$$\therefore \text{adj } A = \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$$

$$\text{and } |A| = \begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} = 10$$

$$\therefore A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$$

5 (b)

$$\text{Let } A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\therefore \text{adj } A = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

6 (d)

We have,

$$3A^3 + 2A^2 + 5A + I = 0$$

$$\Rightarrow I = -3A^3 - 2A^2 - 5A$$

$$\Rightarrow IA^{-1} = (-3A^3 - 2A^2 - 5A)A^{-1}$$

$$\Rightarrow A^{-1} = -3A^2 - 2A - 5I$$

8 (b)

The given system of equations are

$$x + y + z = 0,$$

$$2x + 3y + z = 0 \text{ and } x + 2y = 0$$

$$\text{Here, } \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 1(0 - 2) - 1(0 - 1) + (4 - 3)$$

$$= -2 + 1 + 1 = 0$$

\therefore This system has infinite solutions

9 (c)

$$2X - \begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix}$$



$$\Rightarrow 2X = \begin{bmatrix} 4 & 4 \\ 7 & 2 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 2 & 2 \\ 7/2 & 1 \end{bmatrix}$$

11 (b)

Given, $A = A', B = B'$

Now, $(AB - BA)' = (AB)' - (BA)'$

$$= B'A' - A'B'$$

$$= BA - AB$$

$$= -(AB - BA)$$

$\therefore AB - BA$ is a skew-symmetric matrix.

13 (a)

Given that, p is a non-singular matrix such that

$$1 + p + p^2 + \dots + p^n = 0$$

$$\Rightarrow (1 + p)(1 + p + p^2 + \dots + p^n) = 0$$

$$\Rightarrow 1 - p^{n+1} = 0$$

$$\Rightarrow p^{n+1} = 1$$

$$\Rightarrow p^n \times p^1 = 1$$

$$\Rightarrow p^n = 1/p$$

$$\therefore p^{-1} = p^n$$

14 (a)

$$\text{Given, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 25 + 0 - 25 \\ -25 + 0 + 65 \\ 50 + 0 + 30 \end{bmatrix}$$

$$= \frac{1}{40} \begin{bmatrix} 0 \\ 40 \\ 80 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow x = 0, y = 1, z = 2$$

$$\therefore x + y + z = 0 + 1 + 2 = 3$$

15 (d)

$$\therefore \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

Then, $X = y$ and $Y = x$

ie, $y = x$

16 (c)

$$\text{Given } \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} \cdot \frac{1}{1 + \tan^2 \theta} \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \frac{1}{1 + \tan^2 \theta} \begin{bmatrix} 1 - \tan^2 \theta & -2 \tan \theta \\ 2 \tan \theta & 1 - \tan^2 \theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} & \frac{2 \tan \theta}{1 + \tan^2 \theta} \\ \frac{2 \tan \theta}{1 + \tan^2 \theta} & \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow a = \cos 2\theta, \quad b = \sin 2\theta$$



17 (d)

$$\text{Given, } A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}$$

$$\Rightarrow \text{adj } A = (B)' = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$= 3 \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} = 3A'$$

18 (d) Given equations are

$$px + y + z = 0, x + qy + z = 0, x + y + rz = 0$$

Since, the system have a non-zero solution, then

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_2$

$$\Rightarrow \begin{vmatrix} p & 1-p & 0 \\ 1 & q-1 & 1-q \\ 1 & 0 & r-1 \end{vmatrix} = 0$$

$$\Rightarrow (1-p)(1-q)(1-r) \begin{vmatrix} \frac{p}{1-p} & 1 & 0 \\ 1 & -1 & 1 \\ \frac{1}{1-r} & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (1-p)(1-q)(1-r)$$

$$\left[\frac{p}{1-p}(1) - 1 \left(-\frac{1}{1-q} - \frac{1}{1-r} \right) \right] = 0$$

Since, $p, q, r \neq 1$

$$\therefore \frac{p}{1-p} + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

$$\Rightarrow \frac{1}{1-p} - 1 + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

$$\Rightarrow \frac{1}{1-p} + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

19 (b) Given, $AB = I \Rightarrow B = A^{-1}$

$$\text{Now, } A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{\begin{bmatrix} 1 & -\tan \frac{\theta}{2} \\ \tan \frac{\theta}{2} & 1 \end{bmatrix}}{1 + \tan^2 \frac{\theta}{2}}$$

$$= \frac{A^T}{\sec^2 \frac{\theta}{2}} = \cos^2 \frac{\theta}{2} A^T$$

20 (d)

Given equations are

$$3x + y + 2z = 3 \quad \dots(i)$$

$$2x - 3y - z = -3 \quad \dots(ii)$$

and $x + 2y + z = 4 \quad \dots(iii)$



Let $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\begin{aligned} \therefore |A| &= \begin{vmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{vmatrix} \\ &= 3 \begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} + 2 \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} \\ &= 3(-3+2) - 1(2+1) + 2(4+3) \\ &= -3 - 3 + 14 = 8 \end{aligned}$$

$$\begin{aligned} \text{adj. } A &= \begin{bmatrix} -1 & -3 & 7 \\ 3 & 1 & -5 \\ 5 & 7 & -11 \end{bmatrix}^T \\ &= \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix} \end{aligned}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{8} \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\begin{aligned} &= \frac{1}{8} \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix} \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix} \\ &= \frac{1}{8} \begin{bmatrix} -3 - 9 + 20 \\ -9 - 3 + 28 \\ 21 + 15 - 44 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 8 \\ 16 \\ -8 \end{bmatrix} \end{aligned}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$\Rightarrow x = 1, y = 2, z = -1$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	B	A	B	D	B	B	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	A	A	D	C	D	D	B	D