



## DPP

DAILY PRACTICE PROBLEMS

CLASS : XII<sup>th</sup>

DATE :

**SOLUTION**

SUBJECT : MATHS

DPP NO. :1

### Topic :- DETERMINANTS

1 (c)

$$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$$

On multiplying  $R_1, R_2, R_3$  by  $a, b, c$  respectively and divide the whole by  $abc$

$$= \frac{1}{abc} \begin{vmatrix} ab^2c^2 & abc & a(b+c) \\ bc^2a^2 & bca & b(c+a) \\ a^2b^2c & abc & c(a+b) \end{vmatrix}$$

On taking common  $abc$  from  $C_1$  and  $C_2$ , we get

$$= \frac{(abc)(abc)}{abc} \begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ca+bc \end{vmatrix}$$

Now,  $C_1 \rightarrow C_1 + C_3$

$$= abc \begin{vmatrix} ab+bc+ca & 1 & ab+ac \\ ca+bc+ab & 1 & bc+ab \\ ab+bc+ca & 1 & ca+bc \end{vmatrix}$$

$$= (abc)(ab+bc+ca) \begin{vmatrix} 1 & 1 & ab+ac \\ 1 & 1 & bc+ab \\ 1 & 1 & ca+bc \end{vmatrix}$$

$$= 0 \quad [\because \text{two columns are identical}]$$

2 (b)

$$\text{We have, } \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$$

Applying  $C_2 \rightarrow C_2 - C_1$  and  $C_3 \rightarrow C_3 - C_1$

$$\Rightarrow \begin{vmatrix} 1+a & -a & -a \\ 1 & b & 0 \\ 1 & 0 & c \end{vmatrix} = \lambda$$

On expanding w.r.t.  $R_3$ , we get

$$ab + bc + ca + abc = \lambda \quad \dots(i)$$

$$\text{Given } a^{-1} + b^{-1} + c^{-1} = 0$$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

$$\Rightarrow ab + bc + ca = 0$$

$$\text{From Eq. (i), } \lambda = abc$$

3 (c)

We have,

$$\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} x+1 & x+2 & x+a \\ 2x+4 & 2x+6 & 2x+2b \\ x+2 & x+4 & x+c \end{vmatrix} \quad [\text{Applying } R_2 \rightarrow 2R_2]$$



$$= \frac{1}{2} \begin{vmatrix} x+1 & x+2 & x+a \\ 0 & 0 & 0 \\ x+2 & x+4 & x+c \end{vmatrix} \quad [\text{Applying } R_2 - (R_1 + R_3)]$$

$$= 0$$

4 (b)

$$\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & p-r & r-p \end{vmatrix} = \begin{vmatrix} 0 & b-c & c-a \\ 0 & y-z & z-x \\ 0 & q-r & r-p \end{vmatrix} = 0 \quad (C_1 \rightarrow C_1 + C_2 + C_3)$$

5 (c)

$$\begin{vmatrix} a-b+c & -a-b+c & 1 \\ a+b+2c & -a+b+2c & 2 \\ 3c & 3c & 3 \\ 2a & -2a & 0 \\ a+b+2c & -a+b+2c & 2 \\ 3c & 3c & 3 \end{vmatrix}$$

$$[\text{using } R_1 \rightarrow R_1 + R_2 - R_3]$$

$$= 2a(-3a + 3b + 6c - 6c) + 2a(3a + 3b + 6c - 6c)$$

$$= 12ab$$

6 (a)

Ratio of cofactor to its minor of the element  $-3$ , which is in the 3rd row and 2nd column  $= (-1)^{3+2} = -1$

7 (d)

We have,

$$\Delta = \begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x+1+\omega+\omega^2 & x+\omega+\omega^2+1 & x+1+\omega+\omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

[Applying  $R_1 \rightarrow R_1 + R_2 + R_3$ ]

$$\Rightarrow \Delta = (x+1+\omega+\omega^2) \begin{vmatrix} 1 & 1 & 1 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$$

$$\Rightarrow \Delta = x \begin{vmatrix} 1 & 0 & 0 \\ \omega & x+\omega^2-\omega & 1-\omega \\ \omega^2 & 1-\omega^2 & x+\omega-\omega^2 \end{vmatrix}$$

$$\Rightarrow \Delta = x[(x+\omega^2-\omega)(x+\omega-\omega^2) - (1-\omega)(1-\omega^2)]$$

$$\therefore \Delta = 0 \Rightarrow x = 0$$

9 (d)

Applying  $C_2 \rightarrow C_2 - C_1$  and  $C_3 \rightarrow C_3 - C_1$  to the given determinant and expanding it along first row, we get

$$\Rightarrow (\sin B - \sin A)(\sin C - \sin A)$$

$$\times \begin{vmatrix} 1 & 1 \\ 1 + \sin B + \sin A & 1 + \sin C + \sin A \end{vmatrix} = 0$$

$$\Rightarrow (\sin B - \sin A)(\sin C - \sin A)(\sin C - \sin B) = 0$$

$$\Rightarrow \sin B = \sin A \text{ or } \sin C = \sin A \text{ or } \sin C = \sin B$$

$$\Rightarrow A = B \text{ or } B = C \text{ or } C = A$$

$$\Rightarrow \Delta ABC \text{ is isosceles}$$

10 (c)

$$\text{We have, } D_r = \begin{vmatrix} 2^{r-1} & 3^{r-1} & 4^{r-1} \\ x & y & z \\ 2^n - 1 & (3^n - 1)/2 & (4^n - 1)/3 \end{vmatrix}$$



$$\Rightarrow \sum_{r=1}^n D_r = \begin{vmatrix} \sum_{r=1}^n 2^{r-1} & \sum_{r=1}^n 3^{r-1} & \sum_{r=1}^n 4^{r-1} \\ x & y & z \\ 2^n - 1 & (3^n - 1)/2 & (4^n - 1)/3 \end{vmatrix}$$

$$\Rightarrow \sum_{r=1}^n D_r = \begin{vmatrix} 2^n - 1 & (3^n - 1)/2 & (4^n - 1)/3 \\ x & y & z \\ 2^n - 1 & (3^n - 1)/2 & (4^n - 1)/3 \end{vmatrix}$$

$$\sum_{r=1}^n D_r = 0 \quad (\because \text{two rows are same})$$

11 (b)

We have,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \sin A & 1 + \sin B & 1 + \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \sin A & \sin B & \sin C \\ \sin A + \sin^2 A & \sin B + \sin^2 B & \sin C + \sin^2 C \end{vmatrix} = 0$$

Applying  $R_2 \rightarrow R_2 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ \sin A & \sin B & \sin C \\ \sin^2 A & \sin^2 B & \sin^2 C \end{vmatrix} = 0 \quad \text{Applying } R_3 \rightarrow R_3 - R_2$$

$$\Rightarrow (\sin A - \sin B)(\sin B - \sin C)(\sin C - \sin A) = 0$$

$$\Rightarrow \sin A = \sin B \text{ or, } \sin B = \sin C \text{ or, } \sin C = \sin A$$

$\Rightarrow \Delta ABC$  is isosceles

12 (c)

We have,

$$\det(A) = \begin{vmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{vmatrix} = 2(1 + \sin^2 \theta)$$

Now,

$$0 \leq \sin^2 \theta \leq 1 \text{ for all } \theta \in [0, 2\pi)$$

$$\Rightarrow 2 \leq 2 + 2 \sin^2 \theta \leq 4 \text{ for all } \theta \in [0, 2\pi)$$

$$\Rightarrow \det(A) \in [2, 4]$$

13 (d)

$$\text{Let } \Delta = \begin{vmatrix} 1 & 5 & \pi \\ \log_e e & 5 & \sqrt{5} \\ \log_{10} 10 & 5 & e \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 5 & \pi \\ 1 & 5 & \sqrt{5} \\ 1 & 5 & e \end{vmatrix} = 5 \begin{vmatrix} 1 & 1 & \pi \\ 1 & 1 & \sqrt{5} \\ 1 & 1 & e \end{vmatrix} \quad (\because \log_a a = 1)$$

$$= 0 \quad (\because \text{two columns are identical})$$

14 (a)

Applying  $C_1 \rightarrow C_1 + C_2 + C_3$ , we get

$$f(x) = \begin{vmatrix} 1 + a^2x + x + xb^2 + x + c^2x & (1 + b^2)x & (1 + c^2)x \\ x + a^2x + 1 + b^2x + x + c^2x & (1 + b^2)x & (1 + c^2)x \\ x + a^2x + x + b^2x + 1 + c^2x & (1 + b^2)x & (1 + c^2)x \end{vmatrix}$$

$$= \begin{vmatrix} 1 & (1 + b^2)x & (1 + c^2)x \\ 1 & 1 + b^2x & (1 + c^2)x \\ 1 & (1 + b^2)x & 1 + c^2x \end{vmatrix}$$

$$[\because a^2 + b^2 + c^2 + 2 = 0]$$

Applying  $R_1 \rightarrow R_1 - R_3$ ,  $R_2 \rightarrow R_2 - R_3$



$$= \begin{vmatrix} 0 & 0 & x-1 \\ 0 & 1-x & x-1 \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix} = 1[0 - (x-1)(1-x)]$$

$$= (x-1)^2$$

$\Rightarrow f(x)$  is a polynomial of degree 2

15 (c)

Since system of equations is consistent.

$$\therefore \begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & -c \\ -b & 3b & -c \end{vmatrix} = 0$$

$$\Rightarrow c + bc - 6b + b + 2c + 3bc = 0$$

$$\Rightarrow 3c + 4bc - 5b = 0$$

$$\Rightarrow c = \frac{5b}{3+4b}$$

$$\text{But } c < 1 \Rightarrow \frac{5b}{3+4b} < 1$$

$$\Rightarrow \frac{b-3}{3+4b} < 0$$

$$\Rightarrow b \in \left(-\frac{3}{4}, 3\right)$$

16 (a)

Applying  $R_2 \rightarrow R_2 - R_3$ , we get

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 4 & 4 \\ (2^x - 2^{-x})^2 & (3^x - 3^{-x})^2 & (5^x - 5^{-x})^2 \end{vmatrix}$$

$$= 4 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ (2^x - 2^{-x})^2 & (3^x - 3^{-x})^2 & (5^x - 5^{-x})^2 \end{vmatrix}$$

$$= 4 \times 0 = 0 \quad [\because \text{two rows are identical}]$$

17 (b)

We have,

$$AA^{-1} = I$$

$$\Rightarrow \det(AA^{-1}) = \det(I)$$

$$\Rightarrow \det(A) \det(A^{-1}) = 1 \quad \left[ \because \det(AB) = \det(A) \det(B) \right]$$

$$\text{and, } \det(I) = 1$$

$$\Rightarrow \det(A^{-1}) = \frac{1}{\det(A)}$$

18 (b)

We have,

$$\begin{vmatrix} 1+ax & 1+bx & 1+cx \\ 1+a_1x & 1+b_1x & 1+c_1x \\ 1+a_2x & 1+b_2x & 1+c_2x \end{vmatrix}$$

$$= \begin{vmatrix} 1+ax & (b-a)x & (c-a)x \\ 1+a_1x & (b_1-a_1)x & (c_1-a_1)x \\ 1+a_2x & (b_2-a_2)x & (c_2-a_2)x \end{vmatrix}$$

Applying  $C_2 \rightarrow C_2 - C_1$  and  $C_3 \rightarrow C_3 - C_1$

$$= x^2 \begin{vmatrix} 1+ax & b-a & c-a \\ 1+a_1x & b_1-a_1 & c_1-a_1 \\ 1+a_2x & b_2-a_2 & c_2-a_2 \end{vmatrix}$$

$$= x^2 \left[ (1+ax) \{ (b_1-a_1)(c_2-a_2) - (b_2-a_2)(c_1-a_1) \} - (1+a_1x) \{ (b-a)(c_2-a_2) - (c-a)(b_2-a_2) \} \right. \\ \left. + (1+a_2x) \{ (b-a)(c_1-a_1) - (c-a)(b_1-a_1) \} \right]$$

$$= x^2(\lambda x + \mu), \text{ where } \lambda \text{ and } \mu \text{ are constants}$$

$$= \mu x^2 + \lambda x^3$$

$$\text{Hence, } A_0 = A_1 = 0$$



19 (b)

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

Applying  $R_3 \rightarrow R_3 - xR_2$

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ 0 & 0 & a+x \end{vmatrix} = (a+x)(a^2 + ax)$$

$$\Rightarrow f(x) = a(a+x)^2$$

$$\therefore f(2x) = a(a+2x)^2$$

$$\Rightarrow f(2x) - f(x) = ax(2a+3x)$$

20 (c)

$$\begin{vmatrix} -12 & 0 & \lambda \\ 0 & 2 & -1 \\ 2 & 1 & 15 \end{vmatrix} = -360$$

$$\Rightarrow -12(30+1) - 4\lambda = -360$$

$$\Rightarrow -372 + 360 = 4\lambda \Rightarrow \lambda = -\frac{12}{4} = -3$$

### ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	C	B	C	A	D	D	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	D	A	C	A	B	B	B	C

**SMARTLEARN  
COACHING**