DPP
DAILY PRACTICE PRODUCTS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. :1

Topic :-INTEGRALS

1 **(a)**

We have,

$$f(x)=\int_0^x |x-2|\,dx \Rightarrow f'(x)=|x-2|$$

Clearly, $f'(x)$ is everywhere continuous and differentiable except at $x=2$

2 **(b)**

If $f(t)$ is an odd function, then $\int_0^x f(t)dt$ is an even function

3 **(c)**

$$\int \frac{\sec x \operatorname{cosec} x}{2\cot x-\sec x \operatorname{cosec} x}\,dx$$

$$=\int \frac{\frac{1}{\cos x \sin x}}{\frac{2\cos x}{\sin x}-\frac{1}{\sin x \cos x}}\,dx$$

$$=\int \frac{dx}{2\cos^2 x-1}$$

$$=\int \frac{dx}{\cos 2x}=\int \sec 2x\ \ dx$$

$$=\frac{1}{2}\log|\sec 2x+\tan 2x|+c$$

4 **(d)**

Let $I=\int_0^\pi \frac{\theta \sin\theta}{1+\cos^2\theta}d\theta$ ...(i)

$$\Rightarrow I=\int_0^\pi \frac{(\pi-\theta)\sin\theta}{1+\cos^2\theta}d\theta \ \ ...\text{(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I=\int_0^\pi \frac{\pi\sin\theta}{1+\cos^2\theta}\,d\theta$$

Put $\cos\theta=t \Rightarrow -\sin\theta d\theta=dt$

$$\therefore 2I=-\pi\int_1^{-1}\frac{1}{1+t^2}dt=2\pi\int_0^1\frac{1}{1+t^2}dt$$

$$=2\pi[\tan^{-1}t\ \ ]_0^1=2\pi.\frac{\pi}{4}$$

$$\Rightarrow\ I=\frac{\pi^2}{4}$$

5 **(d)**

Let $f(\theta)=\left[\int_0^{\sin^2\theta}\sin^{-1}\sqrt{\phi}\ \ d\phi+\int_0^{\cos^2\theta}\cos^{-1}\sqrt{\phi}\ \ d\phi\right]$

$$f'(\theta)=\left(\frac{d}{d\theta}\sin^2\theta\right)\left[\sin^{-1}\sqrt{\sin^2\theta}\right]+\left(\frac{d}{d\theta}\cos^2\theta\right)\left[\cos^{-1}\sqrt{\cos^2\theta}\right]$$

$(2\sin\theta\cos\theta)\theta - (2\sin\theta\cos\theta)\theta = 0$

$\therefore\ f(\theta) = \text{constant} = a\quad$ [say]

$\therefore\ f\left(\dfrac{\pi}{4}\right) = a$

$\Rightarrow \displaystyle\int_0^{1/2} (\sin^{-1}\sqrt{\phi} + \cos^{-1}\sqrt{\phi})d\phi = a$

$\Rightarrow \dfrac{\pi}{2}[\phi]_0^{1/2} = a \ \Rightarrow\ \dfrac{\pi}{4} = a$

6 **(c)**

Let $I = \int_0^{\pi/2} \frac{dx}{1+\tan^3 x} = \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x+\cos^3 x} dx \ldots$(i)

$\Rightarrow I = \displaystyle\int_0^{\pi/2} \frac{\cos^3\left(\frac{\pi}{2}-x\right)}{\sin^3\left(\frac{\pi}{2}-x\right)+\cos^3\left(\frac{\pi}{2}-x\right)}dx$

$I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x} dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$2I = \displaystyle\int_0^{\pi/2} \frac{\sin^3 x+\cos^3 x}{\sin^3 x+\cos^3 x}dx$

$= \displaystyle\int_0^{\pi/2} 1dx = \frac{\pi}{2}$

$\Rightarrow I = \dfrac{\pi}{4}$

7 **(c)**

Since, $I = \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \int_0^1 \frac{x}{\sqrt{x}} dx$

Because in $x\epsilon(0,1), x > \sin x$

$I < \displaystyle\int_0^1 \sqrt{x}\, dx = \frac{2}{3}\left[x^{3/2}\right]_0^1 \Rightarrow I < \frac{2}{3}$

For $x\epsilon(0,1), \frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$

And $J = \int_0^1 \frac{\cos x}{\sqrt{x}} dx < \int_0^1 x^{-\frac{1}{2}} dx = 2 \Rightarrow J < 2$

8 **(c)**

$\displaystyle\int_{-1}^{1} |x|dx + \int_1^3 |x-2|dx$

$= 2\displaystyle\int_0^1 x\,dx + \int_1^2 (2-x)dx + \int_2^3 (x-2)dx$

$= 2\times\dfrac{1}{2} + \left[2x - \dfrac{x^2}{2}\right]_1^2 + \left[\dfrac{x^2}{2} - 2x\right]_2^3$

$= 1 + \dfrac{1}{2} + \dfrac{1}{2} = 2$

9 **(a)**

Putting $x^n + 1 = t$ and $n\,x^{n-1}dx = dt$, we get

$I = \displaystyle\int \frac{1}{x\,(x^n+1)}dx = \frac{1}{n}\int \frac{1}{t(t-1)}dt = \frac{1}{n}\int\left(\frac{1}{t-1} - \frac{1}{t}\right)dt$

$\Rightarrow I = \dfrac{1}{n}\log\left(\dfrac{t-1}{t}\right) + C = \dfrac{1}{n}\log\left(\dfrac{x^n}{x^n+1}\right) + C$

10 **(d)**

We have,

$$I=\int_0^{1/2}|\sin\pi x|\,dx=\frac{1}{\pi}\int_0^{\pi/2}|\sin t|\,dt, \text{where } t=\pi x$$

$$\Rightarrow I=\frac{1}{\pi}\int_0^{\pi/2}\sin t\,dt=\frac{1}{\pi}$$

12 **(c)**

We know that,

$$\left|\int_a^b f(x)g(x)dx\right|\le\sqrt{\int_a^b f^2(x)\,dx}.\sqrt{\int_a^b g^2(x)\,dx}$$

Putting $f(x)=1$ and $g(x)=\sqrt{1+x^3}$, we get

$$I\int_0^1\sqrt{1+x^3}\,dx\le\sqrt{\int_0^1 1+x^3\,dx}=\sqrt{1+\frac{1}{4}}=\frac{\sqrt{5}}{2}$$

$$\Rightarrow I<2 \text{ and } I<\frac{\sqrt{7}}{2}$$

13 **(b)**

We have,

$$I=\frac{1}{c}\int_{ac}^{bc}f\left(\frac{x}{c}\right)dx$$

Putting $\frac{x}{c}=t$ and $dx=c\,dt$, we get

$$I=\frac{1}{c}\int_a^b f(t)\,c\,dt=\int_a^b f(t)\,dt=\int_a^b f(x)\,dx$$

14 **(c)**

$$\int e^x\left(\frac{1-x}{1+x^2}\right)^2dx=\int e^x\frac{(1+x^2-2x)}{(1+x^2)^2}dx$$

$$=\int e^x\left(\frac{1}{1+x^2}-\frac{2x}{(1+x^2)^2}\right)dx$$

$$=e^x.\frac{1}{1+x^2}+\int\frac{2xe^x}{(1+x^2)^2}dx-\int\frac{2xe^x}{(1+x^2)^2}dx$$

$$=\frac{e^x}{1+x^2}+c$$

15 **(a)**

We have,

$$\frac{d}{dx}(f(x))=\frac{e^{\sin x}}{x}\Rightarrow\int\frac{e^{\sin x}}{x}dx=F(x)$$

$$\therefore\int_1^4\frac{3}{x}e^{\sin x^3}dx=\int_1^4\frac{e^{\sin x^3}}{x^3}.3x^2dx=\int_1^{64}\frac{e^{\sin x^3}}{x^3}d(x^3)$$

$$=F(64)-F(1)$$

Hence, $k=64$

17 **(a)**

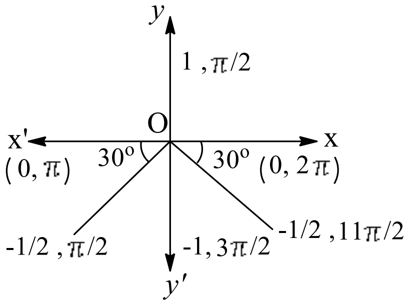
We have,

$$I=\int\frac{\log(x+1)-\log x}{x(x+1)}dx=\int\frac{\log(1+1/x)}{(x^2+x)}dx$$

$$\Rightarrow I=\int \frac{\log(1+1/x)}{(x^2(1+1/x)}dx=-\int \log\left(1+\frac{1}{x}\right)d\left\{\log\left(1+\frac{1}{x}\right)\right\}$$
$$\Rightarrow I=-\frac{1}{2}\left\{\log\left(1+\frac{1}{x}\right)\right\}^2+C$$
$$\Rightarrow I=-\frac{1}{2}[\log(x+1)-\log x]^2+C$$
$$\Rightarrow I=-\frac{1}{2}[\{\log(x+1)\}^2+(\log x)^2-2\log(x+1)\cdot\log x]+C$$
$$\Rightarrow I=-\frac{1}{2}\{\log(x+1)\}^2-\frac{1}{2}(\log x)^2+\log(x+1)\cdot\log x+C$$

18 **(a)**

It is a question of greatest integer function. We have, subdivide the interval π to 2πas under keeping in view that we have to evaluate $[2\sin x]$



We know that, $\sin\frac{\pi}{6}=\frac{1}{2}$

$\sin\left(\pi+\frac{\pi}{6}\right)=\sin\frac{7\pi}{6}=-\frac{1}{2}$

$\sin\frac{11\pi}{6}=\sin\left(2\pi-\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$

$\sin\frac{9\pi}{6}=\sin\frac{3\pi}{6}=-1$

Hence, we divide the intervalπto 2π as $\left(\pi,\frac{7\pi}{6}\right),\left(\frac{7\pi}{6},\frac{11\pi}{6}\right),\left(\frac{11\pi}{6},2\pi\right)$

$\sin x=\left(0,-\frac{1}{2}\right),\left(-1,-\frac{1}{2}\right),\left(0,-\frac{1}{2}\right)$

$2\sin x=(0,-1),(-2,-1),(0,-1)$

$[2\sin x]=-1$

$\therefore\ I=I_1+I_2+I_3$

$=\int -1\,dx+\int -2\,dx\int -1\,dx$

Between proper limits

$=-\frac{\pi}{6}-2\left(\frac{4\pi}{6}\right)-\frac{\pi}{6}=-\frac{10\pi}{6}=-\frac{5\pi}{3}$

19 **(c)**

$$\int_0^1\frac{d}{dx}\left[\sin^{-1}\left(\frac{2x'}{1+x^2}\right)\right]dx$$
$$=\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]_0^1=\sin^{-1}(1)-\sin^{-1}(0)=\frac{\pi}{2}$$

20 **(c)**

Let $F(x)=e^x$

Also, $f(x)+g(x)=x^2\ \Rightarrow\ g(x)=x^2-e^x$

Now, $\int_0^1 f(x)g(x)dx=\int_0^1 e^x(x^2-e^x)dx$

$$= \int_0^1 x^2 e^2\, dx - \int_0^1 e^{2x} dx$$

$$= [x^2 e^x - \int 2x e^x dx \quad]_0^1 - \frac{1}{2}[e^{2x}]_0^1$$

$$= [x^2 e^x - 2x e^x + 2e^x]_0^1 - \frac{1}{2}(e^2 - 1)$$

$$= [(1-2+2)e^1 - (0-0+2)e^0] - \frac{1}{2}e^2 + \frac{1}{2}$$

$$= e - \frac{e^2}{2} - \frac{3}{2}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	C	D	D	C	C	C	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	B	C	A	A	A	A	C	C