DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

Solutio

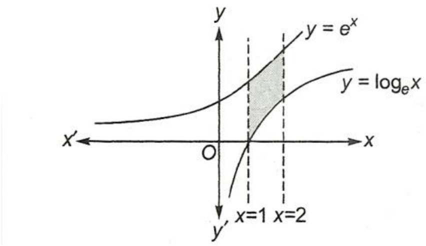
Subject : Maths
DPP No. : 1

Topic :-Applications of integrales

2 **(c)**

Required area

$A = \int_1^2 (e^x - \log_e x)dx$



$= [e^x]_1^2 - \left[x \log_e x - \int 1dx\right]_1^2$

$= e^2 - e - [x\log_e x - x]_1^2$

$= e^2 - e - [2\log_e 2 - 2 - (0 - 1)]$

$= e^2 - e - 2\log_e 2 + 1$

3 **(b)**

We have,

$\int_1^k (8\,x^2 - x^5)\,dx = \frac{16}{3}$

$\Rightarrow \left[\frac{8\,x^3}{3} - \frac{x^6}{6}\right]_1^k = \frac{16}{3}$

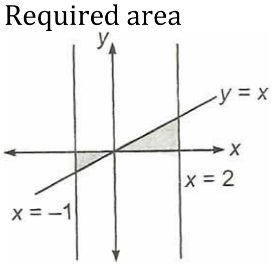
$\Rightarrow \left(\frac{8\,k^3}{3} - \frac{k^6}{6}\right) - \left(\frac{8}{3} - \frac{1}{6}\right) = \frac{16}{3}$

$\Rightarrow 16\,k^3 - k^6 - 16 + 1 = 32$

$\Rightarrow k^6 - 16k^3 + 47 = 0 \Rightarrow k^3 = 8 \pm \sqrt{17} \Rightarrow k = \left(8 \pm \sqrt{17}\right)^{1/3}$

4 **(d)**

Required area



$\left|\int_{-1}^0 x\,dx\right| + \left|\int_0^2 x\,dx\right|$

$= \left|\left[\frac{x^2}{2}\right]_{-1}^{0}\right| + \left|\left[\frac{x^2}{2}\right]_{0}^{2}\right|$

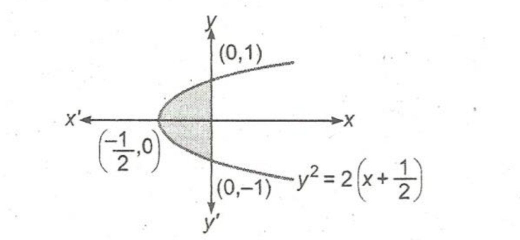
$= \left|-\frac{1}{2}\right| + |2|$

$= 2 + \frac{1}{2} = \frac{5}{2}$ sq unit

5 **(b)**

Given curve can be rewritten as

$y^2 = 2\left(x + \frac{1}{2}\right)$



$\therefore$ Required area $= \int_{-1}^{1} x \ dy$

$= 2\int_0^1 \frac{y^2 - 1}{2} dy$

$= \left|\left[\frac{y^3}{3} - y\right]_0^1\right|$

$= \frac{2}{3}$ sq unit

7 **(a)**

On the solving the given equations of curves, we get $x = 0,2$

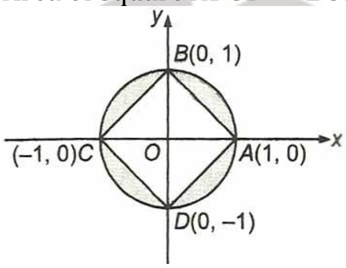
$\therefore$ Required volume $= \pi \int_0^2 [(2x+1)^2 - (x^2+1)^2]dx$

$= \pi \int_0^2 (-x^4 + 2x^2 + 4x)dx$

$= \pi\left[-\frac{x^5}{5} + \frac{2x^3}{3} + \frac{4x^2}{2}\right]_0^2 = \frac{104\pi}{15}$ sq units

8 **(c)**

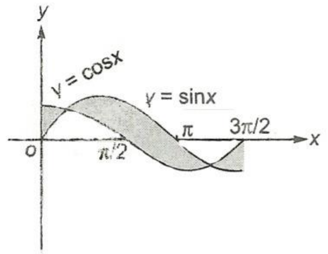
Area of square $ABCD = 2$ sq unit



Area of circle $= \pi$ sq unit

$\Rightarrow$ Required area $= (\pi - 2)$ sq unit

9 **(a)**



Required area

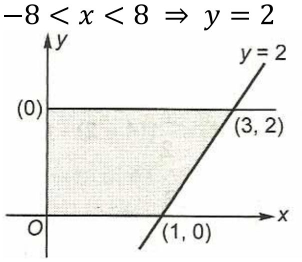
$$= \int_0^{x/4} (\cos x - \sin x) dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx + \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x) dx$$

$$= (\sin x + \cos x)_0^{\pi/4} + (-\cos x - \sin x)_{\pi/4}^{5\pi/4} + (\sin x + \cos x)_{5\pi/4}^{3\pi/2}$$

$= (4\sqrt{2} - 2)$sq units

10 **(c)**

$-8 < x < 8 \Rightarrow y = 2$



$\therefore$ Required area $= \frac{1}{2}(1+3) \times 2$

$= 4$ sq unit

12 **(d)**

Required area, $A = \int_{1-e}^{0} \log_e (x+e) dx$

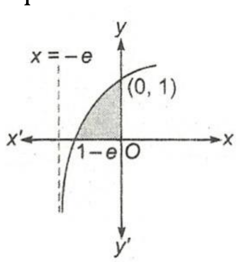
Put $x + e = t \Longrightarrow dx = dt$

$\therefore A = \int_1^e \log_e t \, dt$

$= [t \log_e t - t]_1^e$

$= (e - e - 0 + 1)$

$= 1$ sq unit



13 **(a)**

The two curves $y^2 = 4\,ax$ and $y = mx$ intersect at $(4a/m^2, 4a/m)$ and the area enclosed by the two curves is given by $\int_0^{4a/m^2} (\sqrt{4\,ax} - mx)\,dx$

$$\therefore \int_{300}^{4a/m^2} \left(\sqrt{4\,ax} - mx\right) dx = \frac{a^2}{3} \Rightarrow \frac{8}{3}\frac{a^2}{m^3} = \frac{a^2}{3} \Rightarrow m^3 = 8 \Rightarrow m = 2$$

14 **(c)**

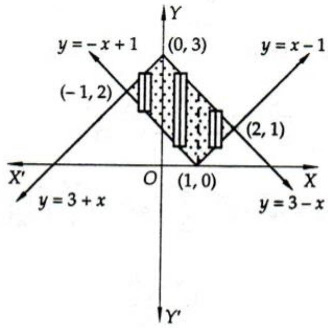
Let A be the required area. Then,

$$A = \int_{-1}^{0} \{(3+x) - (-x+1)\}$$

$$+ \int_{0}^{1} \{(3-x) - (-x+1)\}\, dx + \int_{1}^{2} \{(3-x) - (x-1)\}\, dx$$

$$\Rightarrow A = \int_{-1}^{0} (2+2x)dx + \int_{0}^{1} 2dx + \int_{1}^{2} (4-2x)\, dx$$

$$\Rightarrow A = [2x + x^2]_{-1}^{0} + [2x]_0^1 + [4x - x^2]_1^2 = 4$$

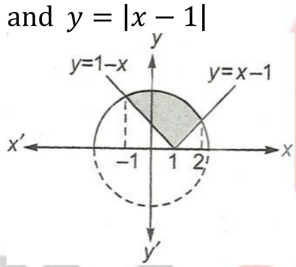


15 **(b)**

Given, $y = \sqrt{5 - x^2}$ and $y = |x - 1|$

or $y^2 + x^2 = 5$

and $y = |x - 1|$



∴ Required area

$$= \int_{-1}^{2} \sqrt{5 - x^2}\, dx - \int_{-1}^{1} (1-x)\, dx - \int_{1}^{2} (x-1)\, dx$$

$$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} - \left[x - \frac{x^2}{2}\right]_{-1}^{1} - \left[\frac{x^2}{2} - x\right]_{1}^{2}$$

$$= \left[1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 + \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right] - \left[1 - \frac{1}{2} - \left(-1 - \frac{1}{2}\right)\right] - \left[2 - 2 - \left(\frac{1}{2} - 1\right)\right]$$

$$= 2 + \frac{5}{2}\sin^{-1}\left(\frac{2}{\sqrt{5}}\sqrt{1 - \frac{1}{5}} + \frac{1}{\sqrt{5}}\sqrt{1 - \frac{4}{5}}\right) - \frac{5}{2}$$

$$= \frac{5}{2}\sin^{-1}(1) = \frac{5\pi}{4} - \frac{1}{2} = \left(\frac{5\pi - 2}{4}\right) sq\ unit$$

19 **(a)**

Volume of generated solid

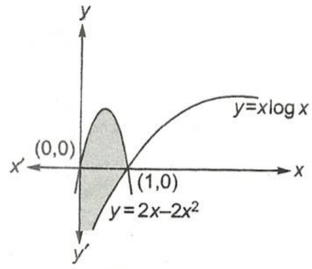
$$= \pi \int_0^2 x^2\ dy = \pi \int_0^2 (9 - y^2)dy = \pi \left[9y - \frac{1}{3}y^3\right]_0^2$$

$$\pi = \left[18 - \frac{8}{3}\right] = \frac{46}{3}\pi \text{ cu units}$$

20 **(c)**

∴ Required area

$$= \int_0^1 [(2x - 2x^2) - (x \log x)]dx$$



$$= \left[x^2 - \frac{2x^3}{3} - \left(\frac{x^2}{2}\log x - \frac{x^2}{4}\right)\right]_0^1$$

$$= \left[1 - \frac{2}{3} - \left(0 - \frac{1}{4}\right)\right] = \frac{7}{12} \text{ sq unit}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	C	B	D	B	C	A	C	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	D	A	C	B	B	A	C	A	C