DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. :1

Topic :-DIFFERENTIAL EQUATIONS

1 **(a)**

The given differential equation can be rewritten as

$$y+\frac{d^2y}{dx^2}=\left[a+\left(\frac{dy}{dx}\right)^{3/2}\right]^2$$

$$\Rightarrow \quad y+\frac{d^2y}{dx^2}=x^2+\left(\frac{dy}{dx}\right)^3+2x\left(\frac{dy}{dx}\right)^{3/2}$$

$$\Rightarrow \quad \left[y+\frac{d^2y}{dx^2}-x^2-\left(\frac{dy}{dx}\right)^3\right]^2=\left[2x\left(\frac{dy}{dx}\right)^{3/2}\right]^2$$

∴ Order and degree of the given differential equation is 2 and 2 respectively.

2 **(c)**

Given differential equation is $\frac{dy}{dx}+y=e^x$

$$\therefore \quad \text{IF}=e^{\int P\,dx}=e^{\int 1\,dx}=e^x$$

Now, solution is

$$ye^x=\int e^{2x}\ dx$$

$$\Rightarrow \quad ye^x=\frac{e^{2x}}{2}+\frac{c}{2}$$

$$\Rightarrow \quad 2ye^x=e^{2x}+c$$

3 **(b)**

We have,

$\phi(x)=\phi'(x)$

$\Rightarrow \frac{\phi'(x)}{\phi(x)}=1$

$\Rightarrow \log\phi(x)=x+\log C \Rightarrow \phi(x)=C\,e^x$

Putting $x=1, \phi(1)=2$, we get $C=\frac{2}{e}$

$\therefore \phi(x)=2e^{x-1} \Rightarrow \phi(3)=2\,e^2$

4 **(b)**

Given equation

$$\frac{dy}{dx}+\sin\left(\frac{x+y}{2}\right)=\sin\left(\frac{x-y}{2}\right)$$

$$\Rightarrow \frac{dy}{dx}=\sin\left(\frac{x-y}{2}\right)-\sin\left(\frac{x+y}{2}\right)$$

$$\Rightarrow \frac{dy}{dx}=-2\sin\left(\frac{y}{2}\right)\cos\left(\frac{x}{2}\right)$$

$$\Rightarrow \text{cosec}\left(\frac{y}{2}\right)dy=-2\cos\left(\frac{x}{2}\right)dx$$

On integrating both sides, we get

$$\int \text{cosec}\left(\frac{y}{2}\right)dy=-\int 2\cos\left(\frac{x}{2}\right)dx+c$$

$\Rightarrow \dfrac{\log(\tan\frac{y}{4})}{\frac{1}{2}} = -\dfrac{2\sin\left(\frac{x}{2}\right)}{\frac{1}{2}} + c$

$\Rightarrow \log(\tan\frac{y}{4}) = c - 2\sin\left(\frac{x}{2}\right)$

5 **(a)**

The family of curves is

$x^2 + y^2 - 2ax = 0$ ...(i)

Differentiating w.r.t. to x, we get

$2x + 2y\,\dfrac{dy}{dx} - 2a = 0 \Rightarrow a = x + y\,\dfrac{dy}{dx}$

Substituting the value of a in (i), we obtain

$x^2 + y^2 - 2x\left(x + y\frac{dy}{dx}\right) = 0$ or, $y^2 - x^2 - 2xy\frac{dy}{dx} = 0$

6 **(b)**

Given, $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x+c_5}$

$\Rightarrow \quad y = (c_1\cos c_3 + c_2\cos c_3)\cos x$

$-(c_1\sin c_3 + c_2\sin c_3)\sin x - c_4 e^{c_5} e^x$

$\Rightarrow \quad y = A\cos x - B\sin x + Ce^x$

Where, $A = c_1\cos c_3 + c_2\cos c_3$

$B = c_1\sin c_3 + c_2\sin c_3$

And $C = -c_4 e^{c_5}$

Which is an equation containing three arbitrary constant. Hence, the order of the differential equation is 3.

7 **(c)**

Given equation is $e^x + \sin\left(\frac{dy}{dx}\right) = 3$

Since, the given differential equation cannot be written as a polynomial in all the differential coefficients, the degree of the equation is not defined.

8 **(d)**

Given, $x = \sin t\,,\ y = \cos pt$

$\frac{dx}{dt} = \cos t\,,\ \frac{dy}{dt} = -p\sin pt$

$\therefore \quad \frac{dy}{dx} = -\frac{p\sin pt}{\cos t}$

$\Rightarrow \quad y_1 = \frac{-p\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\Rightarrow \quad y_1\sqrt{1-x^2} = -p\sqrt{1-y^2}$

$\Rightarrow \quad y_1^2(1-x^2) = p^2(1-y^2)$

$\Rightarrow \quad 2y_1y_2(1-x^2) - 2xy_1^2 = -2yy_1p^2$ [differentiating]

$\Rightarrow \quad (1-x^2)y_2 - xy_1 + p^2y = 0$

9 **(c)**

Given, $y = xe^{cx}$ ...(i)

$\Rightarrow \quad \frac{dy}{dx} = e^{cx} + xe^{cx}.c = \frac{y}{x} + y.c$ ...(ii)

From Eq. (ii),

Log $y = \log x + cx$

$\Rightarrow c\dfrac{1}{x}\log\dfrac{y}{x}$

$\frac{dy}{dx} = \frac{y}{x} + \frac{y}{x}\log\frac{y}{x}$

$= \frac{y}{x}\left(1 + \log\frac{y}{x}\right)$

10 **(a)**
Given differential equation is

$$y = x\frac{dy}{dx} + \left(a^2\left(\frac{dy}{dx}\right)^2 + b^2\right)^{\frac{1}{3}}$$

$$\Rightarrow \quad \left(y - x\frac{dy}{dx}\right)^3 = a^2\left(\frac{dy}{dx}\right)^2 + b^2$$

$\therefore$ Order and degree of the above differential equations are 1 and 3 respectively.

11 **(b)**
We have,
$y_3^{2/3} + 2 + 3y_2 + y_1 = 0$
$\Rightarrow y_3^{2/3} = -(3y_2 + y_1 + 2)$
$\Rightarrow y_2^3 = -(3y_2 + y_1 + 2)^3$
Clearly, it is differential equation of third order and second degree

12 **(b)**
Given, $x^2 + y^2 = 1$ ....(i)
On differentiating w. r. t. x, we get
$2x + 2yy' = 0 \Rightarrow x + yy' = 0$
Again, on differentiating w. r. t. x, we get
$1 + (y')^2 + yy'' = 0$

13 **(a)**
We have,
$$\frac{dy}{dx} = \frac{x\log x^2 + x}{\sin y + y\cos y}$$
$$\Rightarrow \int(\sin y + y\cos y)dy = 2\int x\log x\ dx + \int x\,dx$$
$$\Rightarrow y\sin y = x^2\log x + C$$

14 **(b)**
The given differential equation is
$$\frac{dy}{dx} + P(x)y = Q(x).y^n$$
$$\Rightarrow \quad \frac{1}{y^n}.\frac{dy}{dx} + y^{-n+1}P(x) = Q(x)$$
Put $\frac{1}{y^{n-1}} = v$
$$\Rightarrow \quad (-n+1)y^{-n}\frac{dy}{dx} = \frac{dv}{dx}$$
$$\therefore \quad \frac{1}{(-n+1)}.\frac{dv}{dx} + P(x).v = Q(x)$$
$$\Rightarrow \quad \frac{dv}{dx} + (1-n)P(x)v = (1-n)Q(x)$$
Hence, required substitution is $v = \frac{1}{y^{n-1}}$

15 **(b)**
Since, length of subnormal $= a$
$\Rightarrow \quad y\frac{dy}{dx} = a \quad \Rightarrow y\,dy = a\,dx$
On integrating both sides, we get
$$\frac{y^2}{2} = ax + b$$
Where b is a constant of integration
$\Rightarrow \quad y^2 = 2ax + 2b$

16 **(c)**

Given, $\frac{dx}{dt} = \cos^2 \pi x$
On differentiating w. r. t. x, we get
$\frac{d^2x}{dt^2} = -2\pi \sin 2\pi x = \text{negative}$
The particle never reaches the point, it means
$\frac{d^2x}{dt^2} = 0 \Rightarrow -2\pi \sin 2\pi x = 0$
$\Rightarrow \sin 2\pi x = \sin \pi$
$\Rightarrow 2\pi x = \pi \Rightarrow x = \frac{1}{2}$
The particle never reaches at $x = \frac{1}{2}$

17 **(b)**

Given, $\frac{dy}{dx} = \frac{x+y}{x-y}$
Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$
$\therefore \quad v + x\frac{dv}{dx} = \frac{1+v}{1-v}$
$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v^2}{1-v}$
$\Rightarrow \quad \frac{1}{x}dx = \left(\frac{1}{1+v^2} - \frac{v}{1+v^2}\right)dv$
$\Rightarrow \quad \log_e x = \tan^{-1} v - \frac{1}{2}\log_e(1+v^2) - \log_e c$ [integrating]
$\Rightarrow \quad \log_e x = \tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log_e\left[1 + \left(\frac{y}{x}\right)^2\right] - \log_e c$
$\Rightarrow \quad c(x^2+y^2)^{1/2} = e^{\tan^{-1}(y/x)}$

18 **(b)**

Given, $\frac{d^2y}{dx^2} = e^{-2x}$
$\Rightarrow \quad \frac{dy}{dx} = \frac{e^{-2x}}{-2} + c$ [integrating]
$\Rightarrow \quad y = \frac{e^{-2x}}{4} + cx + d$ [integrating]

19 **(b)**

Given, $x^2 + y^2 = 1$
On differentiating w. r. t. x, we get
$2x + 2yy' = 0$
$\Rightarrow \quad x + yy' = 0$
Again , differentiating, we get
$1 + yy'' + (y')^2 = 0$

20 **(a)**

We have,
$\frac{dy}{dx} = \frac{y-1}{x^2+x}$
$\Rightarrow \frac{1}{x^2+x}dx = \frac{1}{y-1}dy$
$\Rightarrow \int \frac{1}{x(x+1)}dx = \int \frac{1}{y-1}dy$
$\Rightarrow \int \frac{1}{x(x+1)}dx = \int \frac{1}{y-1}dy$
$\Rightarrow \int \left(\frac{1}{x} - \frac{1}{x+1}\right)dx = \int \frac{1}{y-1}dy$
$\Rightarrow \log x - \log(x+1) = \log(y-1) + \log C$

$\Rightarrow \frac{x}{x+1} = C(y-1)$ ...(i)

This passes through (1, 0)

$\therefore \frac{1}{2} = -C$

Substituting the value of C in (i), we get

$\frac{x}{x+1} = -\frac{1}{2}(y-1)$

$\Rightarrow (x+1)(y-1) = -2x \Rightarrow xy + x + y - 1 = 0$

This is the required curve

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	B	B	A	B	C	D	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	A	B	B	C	B	B	B	A