DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. :1

Topic :-VECTOR ALGEBRA

1 **(d)**

We have,

$|\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$

Let θ be the angle between $\vec{a}$ and $\vec{b}$

Now, $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow |\vec{c}|^2 = |\vec{a} + \vec{b}|$

$\Rightarrow |\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b}$

$\Rightarrow |\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 = 2|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 49 = 9 + 25 + 2 \times 3 \times 5 \cos\theta$

$\Rightarrow 15 = 30\cos\theta \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = \pi/3$

2 **(c)**

$\therefore \ [\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] = \vec{\mathbf{a}} \cdot \left(|\vec{\mathbf{b}}||\vec{\mathbf{c}}| \sin\frac{2\pi}{3}\hat{\mathbf{n}}\right)$

$= |\vec{\mathbf{a}}||\vec{\mathbf{b}}||\vec{\mathbf{c}}| \left(\sin\frac{2\pi}{3}\right)$

$[\because \ \vec{\mathbf{a}} \cdot \hat{\mathbf{n}} = |\vec{\mathbf{a}}||\hat{\mathbf{n}}| \cos 0° = |\vec{\mathbf{a}}|]$

$= 2 \times 3 \times 4 \times \frac{\sqrt{3}}{2} = 12\sqrt{3}$

3 **(a)**

Given that, $\overrightarrow{\mathbf{OA}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}, \overrightarrow{\mathbf{OB}} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\overrightarrow{\mathbf{OC}} = \hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

$\overrightarrow{\mathbf{AB}} = \overrightarrow{\mathbf{OB}} - \overrightarrow{\mathbf{OA}}$

$= (3-2)\hat{\mathbf{i}} + (-2-1)\hat{\mathbf{j}} + (1+1)\hat{\mathbf{k}}$

$= \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$|\overrightarrow{\mathbf{AB}}| = \sqrt{1^2 + (-3)^2 + 2^2}$

$= \sqrt{1+9+4} = \sqrt{14}$

$\overrightarrow{\mathbf{BC}} = \overrightarrow{\mathbf{OC}} - \overrightarrow{\mathbf{OB}}$

$= (1-3)\hat{\mathbf{i}} + (4+2)\hat{\mathbf{j}} + (-3-1)\hat{\mathbf{k}}$

$= -2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

$|\overrightarrow{\mathbf{BC}}| = \sqrt{(-2)^2 + 6^2 + (-4)^2}$

$= \sqrt{4+36+16} = \sqrt{56}$

$\overrightarrow{\mathbf{CA}} = \overrightarrow{\mathbf{OA}} - \overrightarrow{\mathbf{OC}}$

$= (2-1)\hat{\mathbf{i}} + (1-4)\hat{\mathbf{j}} + (-1+3)\hat{\mathbf{k}}$

$= \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$|\overrightarrow{\mathbf{CA}}| = \sqrt{1^2 + (-3)^2 + (2)^2}$

$= \sqrt{1+9+4} = \sqrt{14}$

It is clear that two sides of a triangle are equal.

$\therefore$ Points A, B, C from an isosceles triangle.

4 **(b)**

The component of $\vec{a}$ along $\vec{b}$ is given by

$$\left\{\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right\} = \frac{18}{25}(3\hat{j}+4\hat{k})$$

5 **(a)**

It is given that $\vec{c}$ and $\vec{d}$ are collinear vectors

$\therefore \vec{c} = \lambda\,\vec{d}$ for some scalar λ

$\Rightarrow (x-2)\vec{a}+\vec{b} = \lambda\{(2x+1)\vec{a}-\vec{b}\}$

$\Rightarrow \{x-2-\lambda(2x+1)\}\vec{a}+(\lambda+1)\vec{b} = \vec{0}$

$\Rightarrow \lambda+1=0$ and $x-2-\lambda(2x+1)=0$ [$\because \vec{a},\vec{b}$ are non-collinear]

$\Rightarrow \lambda=-1$ and $x=\frac{1}{3}$

6 **(a)**

Equation of plane is $\vec{\mathbf{r}}\cdot\hat{\mathbf{n}} = d$,

where d is the perpendicular distance of the plane from origin

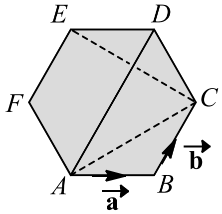
$\therefore$ Required plane is $(\alpha x+\beta y+\gamma z) = p$

7 **(c)**

In ΔABC, $\overrightarrow{\mathbf{AB}}+\overrightarrow{\mathbf{BC}}+\overrightarrow{\mathbf{AC}}$

$\Longrightarrow \overrightarrow{\mathbf{AC}} = \vec{\mathbf{a}}+\vec{\mathbf{b}}$

AD is parallel to BC and $AD = 2\,BC$



$\therefore \overrightarrow{\mathbf{AD}} = 2\vec{\mathbf{b}}$

In ΔACD, $\overrightarrow{\mathbf{AC}}+\overrightarrow{\mathbf{CD}} = \overrightarrow{\mathbf{AD}}$

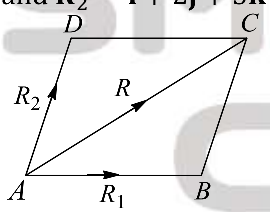
$\Longrightarrow \overrightarrow{\mathbf{CD}} = 2\vec{\mathbf{b}}-(\vec{\mathbf{a}}+\vec{\mathbf{b}}) = \vec{\mathbf{b}}-\vec{\mathbf{a}}$

Now, $\overrightarrow{\mathbf{CE}} = \overrightarrow{\mathbf{CD}}+\overrightarrow{\mathbf{DE}} = \vec{\mathbf{b}}-2\vec{\mathbf{a}}$

9 **(d)**

Let $\vec{\mathbf{R}}_1 = 2\hat{\mathbf{i}}+4\hat{\mathbf{j}}-5\hat{\mathbf{k}}$

and $\vec{\mathbf{R}}_2 = \hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$



$\therefore \vec{\mathbf{R}}$ (along $\overrightarrow{\mathbf{AC}}$) $= \vec{\mathbf{R}}_1+\vec{\mathbf{R}}_2 = 3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-2\hat{\mathbf{k}}$

$\therefore \vec{\mathbf{a}}$ (unit vector angle $\overrightarrow{\mathrm{AC}}$) $= \dfrac{\vec{\mathbf{R}}}{|\vec{\mathbf{R}}|} = \dfrac{3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-2\hat{\mathbf{k}}}{\sqrt{9+36+4}}$

$= \frac{1}{7}(3\hat{\mathbf{i}}+6\hat{\mathbf{j}}-2\hat{\mathbf{k}})$

11 **(b)**

Since $\vec{a},\vec{b},\vec{c}$ are non-coplanar vectors. Therefore, $\vec{a},\vec{b},\vec{c}$ are linearly independent vectors

$\therefore x\vec{a}+y\,\vec{b}+z\,\vec{c} = \vec{0} \Rightarrow x=y=z=0$

12 **(a)**

Suppose point $\hat{\imath}+2\hat{\jmath}+3\hat{k}$ divides the join of points $-2\hat{\imath}+3\hat{\jmath}+5\hat{k}$ and $7\hat{\imath}-\hat{k}$ in the ratio $\lambda:1$. Then,

$\hat{\imath}+2\hat{\jmath}+3\hat{k}=\dfrac{\lambda(7\hat{\imath}-\hat{k})+(-2\hat{\imath}+3\hat{\jmath}+5\hat{k})}{\lambda+1}$
$\Rightarrow (\lambda+1)\hat{\imath}+2(\lambda+1)\hat{\jmath}+3(\lambda+1)\hat{k}=(7\lambda-2)\hat{\imath}+3\hat{\jmath}+(-\lambda+5)\hat{k}$
$\Rightarrow \lambda+1=7\lambda-2, 2(\lambda+1)=3$ and $3(\lambda+1)=-\lambda+5$
$\Rightarrow \lambda=\dfrac{1}{2}$
Hence, required ratio is 1 : 2
13 **(d)**
Clearly,
$\vec{a}-\vec{b}+\vec{b}-\vec{c}+\vec{c}-\vec{a}=\vec{0}$
$\therefore \vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}$ are coplanar
$\Rightarrow (\vec{a}-\vec{b})\cdot\{(\vec{b}\cdot\vec{c})\times(\vec{c}-\vec{a})\}=0$
314 **(d)**
Two given lines intersect, if
$7\hat{\mathbf{i}}+10\hat{\mathbf{j}}+13\hat{\mathbf{k}}+s(2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}})$
$=3\hat{\mathbf{i}}+5\hat{\mathbf{j}}+7\hat{\mathbf{k}}+t(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})$
$\Longrightarrow (7+2s)\hat{\mathbf{i}}+(10+3s)\hat{\mathbf{j}}+(13+4s)\hat{\mathbf{k}}$
$=(3+t)\hat{\mathbf{i}}+(5+2t)\hat{\mathbf{j}}+(7+3t)\hat{\mathbf{k}}$
$\Longrightarrow 7+2s=3+t$
$\Longrightarrow 2s-t=-4$ ...(i)
$10+3s=5+2t$
$\Longrightarrow 3s-2t=-5$ ...(ii)
and $13+4s=7+3t$
$\Longrightarrow 4s-3t=-6$ ...(iii)
On solving Eqs. (i) and (iii), we get
$s=-3, t=-2$
$\therefore$ Required line is
$7\hat{\mathbf{i}}+10\hat{\mathbf{j}}+13\hat{\mathbf{k}}+(-3)[2\hat{\mathbf{i}}+3\hat{\mathbf{j}}+4\hat{\mathbf{k}}]$
$\Longrightarrow\ \hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$ is the required line.
16 **(c)**
Given that, $\vec{\mathbf{a}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$ and $\vec{\mathbf{b}}=2\hat{\mathbf{i}}-\hat{\mathbf{k}}$
Let $\vec{\mathbf{r}}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}$, then
$\vec{\mathbf{r}}\times\vec{\mathbf{a}}=\vec{\mathbf{b}}\times\vec{\mathbf{a}} \Rightarrow (\vec{\mathbf{r}}-\vec{\mathbf{b}})\times\vec{\mathbf{a}}=\vec{\mathbf{0}}$
Now, $\vec{\mathbf{r}}-\vec{\mathbf{b}}=(x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}})-(2\hat{\mathbf{i}}-\hat{\mathbf{k}})$
$=(x-2)\hat{\mathbf{i}}+y\hat{\mathbf{j}}+(z+1)\hat{\mathbf{k}}$
$\therefore\ (\vec{\mathbf{r}}-\vec{\mathbf{b}})\times\vec{\mathbf{a}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ x-2 & y & z+1\\ 1 & 1 & 0\end{vmatrix}=\vec{\mathbf{0}}$
$\Rightarrow\ -(z+1)\hat{\mathbf{i}}+(z+1)\hat{\mathbf{j}}+(x-2-y)\hat{\mathbf{k}}=\vec{\mathbf{0}}$
On equating the coefficient of $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$, we get
$z=-1, x-y=2$ ....(i)
Now, $\vec{\mathbf{r}}\times\vec{\mathbf{b}}=\vec{\mathbf{a}}\times\vec{\mathbf{b}} \Rightarrow (\vec{\mathbf{r}}-\vec{\mathbf{a}})\times\vec{\mathbf{b}}=\vec{\mathbf{0}}$
And $\vec{\mathbf{r}}-\vec{\mathbf{a}}=(x-1)\hat{\mathbf{i}}+(y-1)\hat{\mathbf{j}}+z\hat{\mathbf{k}}$
$\therefore (\vec{\mathbf{r}}-\vec{\mathbf{a}})\times\vec{\mathbf{b}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ x-1 & y-1 & z\\ 2 & 0 & -1\end{vmatrix}=\vec{\mathbf{0}}$
$\Rightarrow (-y+1)\hat{\mathbf{i}}-\hat{\mathbf{j}}(-x+1-2z)+(-2y+2)\hat{\mathbf{k}}=\vec{\mathbf{0}}$
$\Rightarrow y=1, x+2z=1$ ...(ii)
From Eqs. (i) and (ii), we get
$x=3, y=1\ z=-1$

$\therefore \vec{\mathbf{r}} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

17 **(a)**

Given, $\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = \vec{\mathbf{B}} \times (\vec{\mathbf{C}} \times \vec{\mathbf{A}})$ ....(i)

Also, $[\vec{\mathbf{A}}\,\vec{\mathbf{B}}\,\vec{\mathbf{C}}] \neq 0 ie. \vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{C}}$ are not coplanar.

∴From Eq. (i)

$(\vec{\mathbf{A}} \cdot \vec{\mathbf{C}}) - (\vec{\mathbf{A}} \cdot \vec{\mathbf{B}})\vec{\mathbf{C}} = (\vec{\mathbf{B}} \cdot \vec{\mathbf{A}})\vec{\mathbf{C}} - (\vec{\mathbf{B}} \cdot \vec{\mathbf{C}})\vec{\mathbf{A}}$

$\Rightarrow (\vec{\mathbf{B}} \cdot \vec{\mathbf{C}})\vec{\mathbf{A}} + (\vec{\mathbf{A}} \cdot \vec{\mathbf{C}})\vec{\mathbf{B}} - [(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}) + (\vec{\mathbf{B}} \cdot \vec{\mathbf{C}})]\vec{\mathbf{C}} = \vec{\mathbf{0}}$

$\Rightarrow \vec{\mathbf{B}} \cdot \vec{\mathbf{C}} = \vec{\mathbf{A}} \cdot \vec{\mathbf{C}} = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{0}}$

$[\because [\vec{\mathbf{A}}\,\vec{\mathbf{B}}\,\vec{\mathbf{C}}] \neq 0]$

Now, consider

$\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = (\vec{\mathbf{A}} \cdot \vec{\mathbf{C}})\vec{\mathbf{B}} - (\vec{\mathbf{A}} \cdot \vec{\mathbf{B}})\vec{\mathbf{C}}$

$= 0 \cdot \vec{\mathbf{B}} - 0 \cdot \vec{\mathbf{C}} = \vec{\mathbf{0}}$

319 **(a)**

$$[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

Applying $C_3 \to C_3 + C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ y & x & 1+x \end{vmatrix} = 1[1 + x - x] = 1$$

Hence, $[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]$ does not depend upon neither x nor y.

20 **(b)**

The required vector is given by

$$\hat{n} = \frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|} = \frac{\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10

A.	D	C	A	B	A	A	C	C	D	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	A	D	D	A	C	A	C	A	B