DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. :1

Topic :-THREE DIMENSIONAL GEOMETRY

1 **(b)**

Given, $\frac{x-1}{2}=\frac{y+1}{3}$
$=\frac{z-1}{4}$
$=\lambda$

and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}=\mu$

$\Longrightarrow x=2\lambda+1, y=3\lambda-1, z=4\lambda+1$

and $x=\mu+3, y=2\mu+k, z=\mu$

As the lines intersect they must have a point in common.

$\therefore 2\lambda+1=\mu+3, 3\lambda-1=2\mu+k, 4\lambda+1=\mu$

$\Longrightarrow \lambda=-\frac{3}{2}$ and $\mu=-5$

$\therefore k=3\lambda-2\mu-1$

$\Longrightarrow k=3\left(-\frac{3}{2}\right)-2(-5)-1$

$\Longrightarrow k=\frac{9}{2}$

2 **(d)**

Let the point on x-axis is $A(x,0,0)$

Given, $B=(1,2,3,)$and $C=(3,5,-2)$

Since, $|AB|=|AC|$

$\Longrightarrow \sqrt{(x-1)^2+(0-2)^2+(0-3)^2}$
$=\sqrt{(x-3)^2+(0-5)^2+(0+2)^2}$

$\Longrightarrow x^2+1-2x+4+9=x^2+9-6x+25+4$

$\Longrightarrow x=6$

$\therefore$ Required point is $(6,0,0)$

3 **(c)**

Angle between two lines given by

$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$\therefore \cos\theta\frac{1\times 3+2\times -2+3\times 1}{\sqrt{1^2+2^2+3^2}\sqrt{3^2+(-2)^2+1^2}}=\frac{2}{\sqrt{14}\sqrt{14}}$

$\therefore \theta=\cos^{-1}\left(\frac{1}{7}\right)$

4 **(a)**

Let the give points are A, B and C respectively

$\therefore$ Direction ratios of AB and BC are $1,-3,-2$ and $2,-6,K-2$ respectively

Since given points are collinear

$\therefore \frac{2}{1}=\frac{-6}{-3}=\frac{K-2}{-2}$

$\Rightarrow K-2=-4$

$\Rightarrow K=-2$

5 **(d)**

Equation of plane through $(2,5,-3)$ is

$a(x-2)+b(y-5)+c(z+3)=0$ ...(i)

Which is perpendicular to

$x+2y+2z=1$

and $x - 2y + 3z = 4$
then $a + 2b + 2c = 0$ ...(ii)
and $a - 2b + 3c = 0$ ...(iii)
On eliminating a, b, c from Eqs.(i), (ii) and (iii), we get

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ 1 & 2 & 2 \\ 1 & -2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 10x - y - 4z - 27 = 0$

6 **(c)**

Given line can be rewritten as

$$\frac{x-1}{\frac{1}{4}} = \frac{y-\frac{1}{3}}{-\frac{1}{3}} = \frac{z-\frac{1}{2}}{\frac{1}{2}}$$

$\therefore$ Direction cosines are

$$\frac{\frac{1}{4}}{\sqrt{\frac{1}{16}+\frac{1}{9}+\frac{1}{4}}}, \frac{\frac{-1}{3}}{\sqrt{\frac{1}{16}+\frac{1}{9}+\frac{1}{4}}}, \frac{\frac{1}{2}}{\sqrt{\frac{1}{16}+\frac{1}{9}+\frac{1}{4}}}$$

$$= \frac{3}{\sqrt{16}}, \frac{-4}{\sqrt{16}}, \frac{6}{\sqrt{16}}$$

7 **(a)**

Equation of required plane is
$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$
Which is passing through $(1, 1, 1)$
$\Longrightarrow -3 + 14\lambda = 0$
$\Longrightarrow \lambda = \frac{3}{14}$
$\therefore$Required plane is $20x + 23y + 26z = 69$

8 **(c)**

Equation of plane through $(0, -4, -6)$ is
$a(x - 0) + b(y + 4) + c(z + 6) = 0$ ...(i)
Point $(-2, 9, 3)$ lies on Eq. (i), then
$-2a + 13b + 9c = 0$ ...(ii)
Also required plane is perpendicular to $x - 4y - 2z = 8$
$\therefore a - 4b - 2c = 0$ ...(iii)
From Eqs. (i), (ii), (iii) we get

$$\begin{vmatrix} x & y+4 & z+6 \\ -2 & 13 & 9 \\ 1 & -4 & -2 \end{vmatrix} = 0$$

$ie, 2x + y - z - 2 = 0$

9 **(b)**

Let α, β, γ be the angles with x-axis, z-axis respectively, then direction cosines are cos α, cos β, and cos γ
Given, $\alpha = \frac{\pi}{3}$, $\beta = \frac{\pi}{4}$
$\therefore l = \cos\frac{\pi}{3} = \frac{1}{2}$, $m = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $n = \cos\gamma$
Using $l^2 + m^2 + n^2 = 1$
$\Longrightarrow \frac{1}{4} + \frac{1}{2} + n^2 = 1 \Longrightarrow n = \frac{1}{2}$
$\therefore \cos\gamma = \frac{1}{2} \Longrightarrow \gamma = \frac{\pi}{3}$

10 **(c)**

Given $(3, 4, -1)$and $(-1, 2, 3)$ are the end points of diameter of sphere
$\therefore$ Radius $= \frac{1}{2}$(length of the diameter)
$= \frac{1}{2}\sqrt{(3+1)^2 + (4-2)^2 + (-1-3)^2}$
$= 3$

11 **(c)**

Let $A(5,-4,2), B(4,-3,1), C(7,-6,4)$ and $D(8,-7,5)$
Then, $AB = \sqrt{(4-5)^2 + (-3+4)^2 + (1-2)^2}$
$= \sqrt{1+1+1} = \sqrt{3}$
$BC\sqrt{(7-4)^2 + (-6+3)^2 + (4-1)^2}$
$= \sqrt{9+9+9} = 3\sqrt{3}$
$CD = \sqrt{(8-7)^2 + (-7+6)^2 + (5-4)^2}$
$= \sqrt{1+1+1} = \sqrt{3}$
$AD = \sqrt{(8-5)^2 + (-7+4)^2 + (5-2)^2}$
$= \sqrt{9+9+9} = 3\sqrt{3}$
Position vector f $\overrightarrow{\mathbf{AB}} = (4-5)\hat{\mathbf{i}} + (-3+4)\hat{\mathbf{j}} + (1-2)\hat{\mathbf{k}}$
$= -\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$
And position vector of $\overrightarrow{\mathbf{BC}} = (7-4)\hat{\mathbf{i}} + (-6+3)\hat{\mathbf{j}} + (4-1)\hat{\mathbf{k}}$
$= 3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$
Now, $\overrightarrow{\mathbf{AB}} \cdot \overrightarrow{\mathbf{BC}} = (-\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}).(3\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$
$= -3 - 3 - 3 \neq 0$
$\therefore \square ABCD$ is parallelogram

12 **(b)**

The required plane passes through the points having position vectors $\overrightarrow{a_1}$ and $\overrightarrow{a_2}$ and is parallel to the vector $\vec{b}$.
Therefore, it is normal to the vector $(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}$
So, the equation of the required plane is
$(\vec{r} - \overrightarrow{a_1}) \cdot \{(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}\} = 0$
$\Rightarrow \vec{r} \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b} - \overrightarrow{a_1} \cdot (\overrightarrow{a_2} \times \vec{b}) = 0$
$\Rightarrow \vec{r} \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b} = [\overrightarrow{a_1}\, \overrightarrow{a_2}\, \vec{b}]$

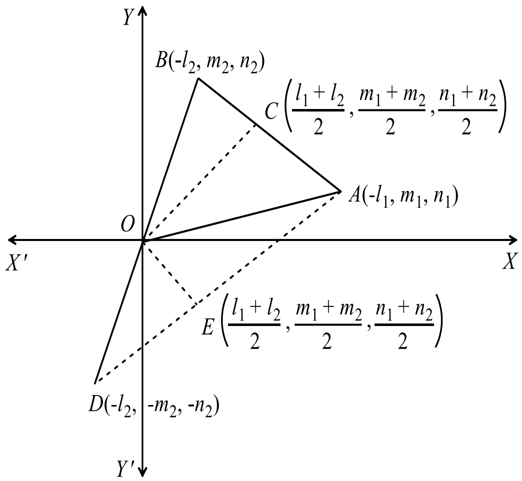
13 **(a)**

If $\left(\frac{1}{2}, \frac{1}{3}, n\right)$ are the DC's of line, then using the relation $l^2 + m^2 + n^2 = 1$, we get
$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 + n^2 = 1$
$\Rightarrow n^2 = 1 - \frac{1}{4} - \frac{1}{9}$
$\Rightarrow n^2 = \frac{23}{36}$
$\Rightarrow n = \frac{\sqrt{23}}{6}$

14 **(b)**

The equation of a plane through the line of intersection of the planes $\vec{r} \cdot \vec{a} = \lambda$ and $\vec{r} \cdot \vec{b} = \mu$ can be written as
$(\vec{r} \cdot \vec{a} - \lambda) + k(\vec{r} \cdot \vec{b} - \mu) = 0$
Or, $\vec{r} \cdot (\vec{a} + k\vec{b}) = \lambda + k\mu \quad \dots \text{(i)}$
This passes through the origin
$\therefore \vec{0} \cdot (\vec{a} + k\vec{b}) = \lambda + \mu\, k \Rightarrow k = \frac{-\lambda}{\mu}$
Putting the value of k in (i), we get the equation of the required plane as
$\vec{r} \cdot (\mu\vec{a} - \lambda\vec{b}) = 0 \Rightarrow \vec{r} \cdot (\lambda\vec{b} - \mu\vec{a}) = 0$

15 **(c)**



In Fig. OE is the external bisector

The co-ordinates of E are $\left(\frac{l_1-l_2}{2}, \frac{m_1-m_2}{2}, \frac{n_1-n_2}{2}\right)$

Therefore, direction ratios of OE are proportional to

$\frac{l_1 - l_2}{2}, \frac{m_1 - m_2}{2}, \frac{n_1 - n_2}{2}$

16 **(d)**

The equation of a plane passing through $(1, -2, 3)$ is

$a(x - 1) + b(y + 2) + c(z - 3) = 0$

It passes through $(-1, 2, -1)$ and is parallel to the given line

$\therefore a(-2) + b(4) + c(-4) = 0$ and, $2a + 3b + 4c = 0$

$\Rightarrow \frac{a}{28} = \frac{b}{0} = \frac{c}{-14} \Rightarrow \frac{a}{2} = \frac{b}{0} = \frac{c}{-1}$

Hence, $a : b : c = 2 : 0 : -1$

<u>ALITER</u> Let $P(1, -2, 3)$ and $Q(-1, 2, -1)$ be the given points

Given line is parallel to the vector $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

$\therefore$ Normal to the plane is parallel to the vector

$\vec{P}Q \times \vec{b} = 28\hat{\imath} - 14\hat{k} = 14(2\hat{\imath} + 0\hat{\jmath} - \hat{k})$

17 **(b)**

The equation of a line passing through the points $A(\hat{\imath} - \hat{\jmath} + 2\,\hat{k})$ and $B(3\hat{\imath} + \hat{\jmath} + \hat{k})$ is given by

$\vec{r} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + \hat{\jmath} + \hat{k})$

The position vector of a variable point P on the line, is $(\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + \hat{\jmath} + \hat{k})$

$\therefore \vec{A}P = \lambda(3\hat{\imath} + \hat{\jmath} + \hat{k}) \Rightarrow |\vec{A}P| = |\lambda|\sqrt{11}$

Now, $|\lambda|\sqrt{11} = 3\sqrt{11}, \Rightarrow \lambda = \pm 3$

Thus, the position vectors of P are

$10\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$ and $-8\hat{\imath} - 4\hat{\jmath} - \hat{k}$

18 **(c)**

The given equation of sphere is

$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$

$\therefore$ Coordinates of centre of sphere= $\left(-\frac{3}{2}, 0, 2\right)$

and radius of sphere = $\sqrt{u^2 + v^2 + w^2 - d}$

$= \sqrt{\frac{9}{4} + 4 - 1} = \frac{\sqrt{21}}{2}$

20 **(c)**

It is given that the line

$$\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{2}$$

Is perpendicular to the required. This means that the normal to the plane is parallel to the line. So, its direction ratios are proportional to $2, -1, 2$

The plane passes through the origin

Hence, its equation is

$2(x-0)-(y-0)+2(z-0)=0 \Rightarrow 2x-y+2z=0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	C	A	D	C	A	C	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	A	B	C	D	B	C	B	C