# DPP
DAILY PRACTICE PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. :1

## Topic :-PROBABILITY

1 **(b)**

The total number of ways in which $n$ persons can sit at a round table is $(n-1)!$. So, total number of cases is $(n-1)!$

Let $A$ and $B$ be two specified persons. Considering these two as one person, the total number of ways in which $n-1$ persons, $n-2$ other persons and one $AB$ can sit at a round table is $(n-2)!$. So, favourable number of cases is $2!\,(n-2)!$ Thus, the required probability is

$$p=\frac{2!\,(n-2)!}{(n-1)!}=\frac{2}{n-1}$$

Hence, the required odds are $(1-p):p$ or $(n-3):2$

2 **(b)**

Here $p=19/20, q=1/20, n=5, r=5$. The required probability is

$$^5C_5\left(\frac{19}{20}\right)^5\left(\frac{1}{20}\right)^6=\left(\frac{19}{20}\right)^5$$

3 **(a)**

Let the number of red and blue balls be $r$ and $b$, respectively. Then, the probability of drawing two red balls is

$$p_1=\frac{^rC_2}{^{r+b}C_2}=\frac{r(r-1)}{(r+b)(r+b-1)}$$

The probability of drawing two blue balls is

$$p_2=\frac{^bC_2}{^{r+b}C_2}=\frac{b(b-1)}{(r+b)(r+b-1)}$$

The probability of drawing one red and one blue ball is

$$p_3=\frac{^rC_1\,^bC_1}{^{r+b}C_2}=\frac{2br}{(r+b)(r+b-1)}$$

By hypothesis, $p_1=5p_2$ and $p_3=6p_2$

$\therefore r(r-1)=5b(b-1)$ and $2br=6b(b-1)$

$\Rightarrow r=6, b=3$

4 **(d)**

Consider two events as follows:

$A_1$: getting number $i$ on first die

$B_1$: getting a number more than $i$ on second die

The required probability is

$$P(A_1\cap B_1)+P(A_2\cap B_2)+P(A_3\cap B_3)+P(A_4\cap B_4)+P(A_5\cap B_5)=\sum_{i=1}^{5}P(A_i\cap B_i)=\sum_{i=1}^{5}P(A_i)P(B_i)$$

[$\because A_i, B_i$ are independent]

$$=\frac{1}{6}\,[P(B_1)+P(B_2)+\ldots+P(B_5)]$$

$$=\frac{1}{6}\left(\frac{5}{6}+\frac{4}{6}+\frac{3}{6}+\frac{2}{6}+\frac{1}{6}\right)=\frac{5}{12}$$

5 **(b)**

Let,

$P(S) = P(1 \text{ or } 2) = 1/3$
$P(F) = P(3 \text{ or } 4 \text{ or } 5 \text{ or } 6) = 2/3$
$P(A \text{ wins}) = P[(S\ S \text{ or } S\ F\ S\ S \text{ or } S\ F\ S\ S\ F\ S\ S \text{ or} \dots)$
Or $(F\ S\ S \text{ or } F\ S\ F\ S\ S \text{ or} \dots)]$

$$= \frac{\frac{1}{9}}{1-\frac{2}{9}} + \frac{\frac{2}{27}}{1-\frac{2}{9}}$$
$$= \frac{1}{9} \times \frac{9}{7} + \frac{2}{27} \times \frac{9}{7}$$
$$= \frac{1}{7} + \frac{2}{21} = \frac{3+2}{21} = \frac{5}{21}$$
$$P(A \text{ winning}) = \frac{5}{21}, P(B \text{ winning}) = \frac{16}{21}$$

6 **(a)**
Let $E_1 = 1, 4, 7, \dots$ ($n$ each)
$E_2 = 2, 5, 8, \dots$ ($n$ each)
$E_3 = 3, 6, 9, \dots$ ($n$ each)
$x$ and $y$ belong to $(E_1, E_2), (E_2, E_1)$ or $(E_3, E_3)$. So, the required probability is
$$\frac{n^2 + {}^nC_2}{{}^{3n}C_2} = \frac{1}{3}$$

7 **(c)**
Let $A$ be the event that 11 is picked and $B$ be the event that sum is even. The number of ways of selecting 11 along with one more-odd number is $n(A \cap B) = {}^7C_1$
The number of ways of selecting either two even numbers or selecting two odd numbers is $n(B) = 1 + {}^8C_2$
$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{7}{29} = 0.24$$

8 **(c)**
18 draws are required for 2 aces means in the first 17 draws, there is one ace and 16 other cards and 18th draw produces an ace. So, the required probability is
$$\frac{{}^{48}C_{16} \times {}^4C!}{{}^{52}C_{17}} \times \frac{3}{35} = \frac{561}{15925}$$

9 **(c)**
Given,
$$7a - 9b = 0 \Rightarrow b = \frac{7}{9}a$$
Hence, number of pairs $(a, b)$ can be (9, 7); (18, 14); (27, 21); (36, 28). Hence, the required probability is $4/{}^{39}C_2 = 4/741$

10 **(b)**
The total number of cases is $11!/2! \times 2!$ The number of favourable cases is $11!/(2! \times 2!) - 9!$ Therefore, required probability is
$$1 - \frac{9! \times 4}{11!} = \frac{53}{55}$$

11 **(a)**
The divisibility of the product of four numbers depends upon the value of the last digit of each number. The last digit of a number can be any one of the 10 digits 0,1, 2, …9. So, the total number of ways of selecting last digits of four numbers is $10 \times 10 \times 10 \times 10 = 10^4$. If the product of the 4 numbers is not divisible by 5 or 10, then the number of choices for the last digit of each number is 8 (excluding 0 or 5). So, favourable number of ways is $8^4$. Therefore, the probability that the product is not divisible by 5 or 10 is $(8/10)^4$. Hence, the required probability is $1 - (8/10)^4 = 369/625$

12 **(c)**

Let $E$ =event when each American man is seated adjacent to his wife
and $A$ =event when Indian man is seated adjacent to his wife.
Now, $n(A \cap E) = (4!) \times (2!)^5$
Even when each American man is seated adjacent to his wife.
Again, $n(E) = (5!) \times (2!)^4$

$\therefore P\left(\frac{A}{E}\right) = \frac{n(A \cap E)}{n(E)}$

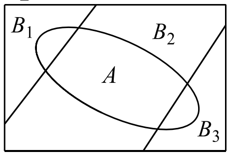
$= \frac{(4!) \times (2!)^3}{(5!) \times (2!)^4} = \frac{2}{5}$

13 **(c)**

Let the probability that a man aged $x$ dies in a year $p$. Thus the probability that a man aged $x$ does not die in a year $= 1 - p$. The probability that all $n$ men aged $x$do not die in a year is $(1-p)^n$. Therefore, the probability that at least one man dies in a year is $1-(1-p)^n$. The probability that out of $n$ men, $A_1$ dies first is $1/n$. Since this event is independent of the event that at least one man dies in a year, hence, the probability that $A_1$ dies in the year and he is first one to die is $1/n[1-(1-p)^n]$

14 **(b)**

Let us consider the following events
$A$: card shows up black
$B_1$: card with both sides black
$B_2$: card with both sides white
$B_1$: card with one side white and one black




$P(B_1) = \frac{2}{10}, P(B_2) = \frac{3}{10}, P(B_3) = \frac{5}{10}$

$P(A/B_1) = 1,\ P(A/B_2) = 0\ \ P(A/B_3) = \frac{1}{2}$

$$P(B_1/A) = \frac{\frac{2}{10} \times 1}{\frac{2}{10} \times 1 + \frac{3}{10} \times (0) + \frac{5}{10} \times \frac{1}{2}} = \frac{4}{4+5} = \frac{4}{9}$$

15 **(c)**

Possibilities of getting 9 are (5, 4), (4, 5), (6, 3), (3, 6)

$\therefore p = \frac{4}{36} = \frac{1}{9}$ and $q = 1 - \frac{1}{9} = \frac{8}{9}$

Therefore, the required probability is

$^3C_2 q^1 p^2 = (3)\left(\frac{8}{9}\right)\left(\frac{1}{9}\right)^2 = \frac{8}{243}$

16 **(d)**

The number of ways of arranging $n$ numbers is $n!$ In each order obtained, we must now arrange the digits $1, 2, \dots k$ as group and the $n-k$ remaining digits. This can be done in $(n-k+1)!$ ways. Therefore, the probability for the required event is $(n-k+1)!/n!$

17 **(a)**

For each toss, there are four choices:

1. A gets head, $B$ gets head

2. $A$ gets tail, $B$ gets head

3. $A$ gets head, $B$ gets tail

4. $A$ gets tail, $B$ gets tail

Thus, exhaustive number of ways is $4^{50}$. Out of the four choices listed above, (iv) is not favourable to the required event in a toss. Therefore, favourable number of cases is $3^{50}$. Hence, the required probability is $(3/4)^{50}$

18 **(c)**

Let $a_n$ be the number5 of strings of $H$ and $T$ of length $n$ with no two adjacent $H$'s. Then $a_1 = 2, a_2 = 3$.
Also,
$a_{n+2} = a_{n+1} + a_n$ (since the string must with $T$ or $HT$)
So, $a_3 = 5, a_4 = 8, a_5 = 8 + 5 = 13$
Therefore, the required probability is $13/2^5 = 13/52$

19 **(a)**

We have ratio of the ships $A, B$ and $C$ for arriving safely are 2:5,3:7 and 6:11, respectively. Therefore, the probability of ship $A$ for arriving safely is 2/(2+5)=2/7
Similarly, for $B$ the probability is 3/(3+7)=3/10 and for $C$ the probability is $C = 6/(6 + 11) = 6/17$
Therefore, the probability of all the ships for arriving safely is $(2/7) \times (3/10) \times (6/17)$ 18/595

20 **(a)**

Out of 9 socks, 2 can be drawn in ${}^9C_2$ ways. Therefore, the total number of cases is ${}^9C_2$. Two socks drawn from the drawer will match if either both are brown or both are blue. Therefore, favourable number of cases is ${}^5C_2 + {}^4C_2$. Hence, the required probability is
$$\frac{{}^5C_2 + {}^4C_2}{{}^9C_2} = \frac{4}{9}$$

**ANSWER-KEY**

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|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | B | B | A | D | B | A | C | C | C | B |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | A | C | C | B | C | D | A | C | A | A |
| | | | | | | | | | | |