

## DPP

f DAILY PRACTICE PROBLEMS

CLASS : XIIth  
DATE :

**SOLUTION**

SUBJECT : MATHS  
DPP NO. : 1

### Topic :- CONTINUITY AND DIFFERENTIABILITY

1 (b)

We have,

$$-\pi/4 < x < \pi/4$$

$$\Rightarrow -1 < \tan x < 1 \Rightarrow 0 \leq \tan^2 x < 1 \Rightarrow [\tan^2 x] = 0$$

$$\therefore f(x) = [\tan^2 x] = 0 \text{ for all } x \in (-\pi/4, \pi/4)$$

Thus,  $f(x)$  is a constant function on  $\in (-\pi/4, \pi/4)$

So, it is continuous on  $\in (-\pi/4, \pi/4)$  and  $f'(x) = 0$  for all  $x \in (-\pi/4, \pi/4)$

2 (d)

Since,  $f(x)$  is continuous at  $x = 0$

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0)$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{-e^x + 2^x}{x} = f(0)$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{-e^x + 2^x \log 2}{1} = f(0) \quad [\text{by L'Hospital's rule}]$$

$$\Rightarrow f(0) = -1 + \log 2$$

3 (b)

Since  $f(x)$  is an even function

$$\therefore f(-x) = f(x) \text{ for all } x$$

$$\Rightarrow -f'(-x) = f'(x) \text{ for all } x$$

$$\Rightarrow f'(-x) = -f'(x) \text{ for all } x$$

$\Rightarrow f'(x)$  is an odd function

4 (c)

We have,

$$f(x) = \begin{cases} [\cos \pi x], & x < 1 \\ |x - 2|, & 1 \leq x < 2 \\ 2 - x, & 1 \leq x < 2 \\ -1, & 1/2 < x < 1 \\ 0, & 0 < x \leq 1/2 \\ 1, & x = 0 \\ 0, & -1/2 \leq x < 0 \\ -1, & -3/2 < x < -1/2 \end{cases}$$

It is evident from the definition that  $f(x)$  is discontinuous at  $x = 1/2$

5 (b)

We have,

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2 - h) = \lim_{h \rightarrow 0} \frac{|-2 - h + 2|}{\tan^{-1}(-2 - h + 2)}$$

$$\Rightarrow \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} \frac{h}{\tan^{-1}(-h)} = \lim_{h \rightarrow 0} \frac{-h}{\tan^{-1} h} = -1$$

and,

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{h \rightarrow 0} f(-2 + h) = \lim_{h \rightarrow 0} \frac{|-2 + h + 2|}{\tan^{-1}(-2 + h + 2)}$$

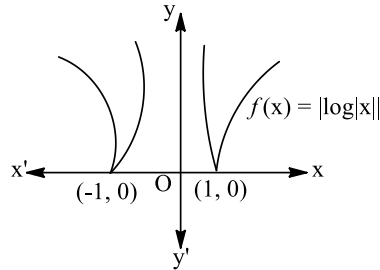


$$\Rightarrow \lim_{x \rightarrow -2^+} f(x) = \lim_{h \rightarrow 0} \frac{h}{\tan^{-1} h} = 1$$

$$\therefore \lim_{x \rightarrow -2^-} f(x) \neq \lim_{x \rightarrow -2^+} f(x)$$

So,  $f(x)$  is neither continuous nor differentiable at  $x = -2$

6 (b)



From the graph of  $f(x) = |\log|x||$  it is clear that  $f(x)$  is everywhere continuous but not differentiable at  $x = \pm 1$ , due to sharp edge

7 (b)

We have,

$$\lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x - a} = \lim_{x \rightarrow a} \frac{xf(a) - af(a) - a(f(x) - f(a))}{x - a}$$

$$\Rightarrow \lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x - a} = \lim_{x \rightarrow a} \frac{f(a)(x - a)}{x - a} - a \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$\Rightarrow \lim_{x \rightarrow a} \frac{xf(a) - af(x)}{x - a} = f(a) - af'(a) = 4 - 2a$$

8 (c)

Given,  $f(x) = x(\sqrt{x} + \sqrt{x+1})$ . At  $x = 0$  LHL of  $\sqrt{x}$  is not defined, therefore it is not continuous at  $x = 0$   
Hence, it is not differentiable at  $x = 0$

9 (a)

$$\text{Here, } f'(x) = \begin{cases} 2ax, & b \neq 0, x \leq 1 \\ 2bx + a, & x > 1 \end{cases}$$

Since,  $f(x)$  is continuous at  $x = 1$

$$\therefore \lim_{h \rightarrow 0} f(x) = \lim_{h \rightarrow 1^+} f(x)$$

$$\Rightarrow a + b = b + a + c \Rightarrow c = 0$$

Also,  $f(x)$  is differentiable at  $x = 1$

$$\therefore (\text{LHD at } x = 1) = (\text{RHD at } x = 1)$$

$$\Rightarrow 2a = 2b(1) + a \Rightarrow a = 2b$$

10 (d)

We have,

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \left\{ \frac{x^2}{4} - \frac{3x}{4} + \frac{13}{4} \right\} = \frac{1}{4} - \frac{3}{2} + \frac{13}{4} = 2$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} |x - 3| = 2$$

$$\text{and, } f(1) = |1 - 3| = 2$$

$$\therefore \lim_{x \rightarrow 1^-} f(x) = f(1) = \lim_{x \rightarrow 1^+} f(x)$$

So,  $f(x)$  is continuous at  $x = 1$

We have,

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} |x - 3| = 0, \quad \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} |x - 3| = 0$$

$$\text{and, } f(3) = 0$$

$$\therefore \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3)$$

So,  $f(x)$  is continuous at  $x = 3$

Now,

(LHD at  $x = 1$ )



$$= \left\{ \frac{d}{dx} \left( \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} \right) \right\}_{x=1} = \left\{ \frac{x}{2} - \frac{3}{2} \right\}_{x=1} = \frac{1}{2} - \frac{3}{2} = -1$$

$$(\text{RHD at } x = 1) = \left\{ \frac{d}{dx} (-(x-3)) \right\}_{x=1} = -1$$

$\therefore$  (LHD at  $x = 1$ ) = (RHD at  $x = 1$ )

So,  $f(x)$  is differentiable at  $x = 1$

11 (d)

$$f(x) = \begin{cases} \frac{2 \sin x - \sin 2x}{2x \cos x}, & \text{if } x \neq 0, \\ a, & \text{if } x = 0 \end{cases}$$

$$\text{Now, } \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{2x \cos x} \quad \left( \frac{0}{0} \text{ form} \right)$$

$$= \lim_{x \rightarrow 0} \frac{2(\cos x - x \sin x)}{2(\cos x - x \sin x)}$$

$$= \lim_{x \rightarrow 0} \frac{2 - 2}{2(1 - 0)} = 0$$

Since,  $f(x)$  is continuous at  $x = 0$

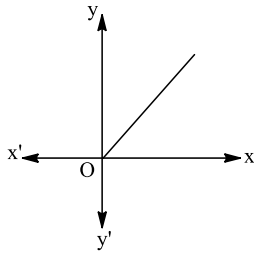
$$\therefore f(0) = \lim_{x \rightarrow 0} f(x)$$

$$\Rightarrow a = 0$$

12 (a)

Given,  $f(x) = x + |x|$

$$\therefore f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$



It is clear from the graph of  $f(x)$  is continuous for every value of  $x$

Alternate

Since,  $x$  and  $|x|$  is continuous for every value of  $x$ , so their sum is also continuous for every value of  $x$

13 (a)

Since  $f(x)$  is continuous at  $x = 0$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = f(0) = \lim_{x \rightarrow 0^+} f(x)$$

$$\Rightarrow \lim_{x \rightarrow 0} \{1 + |\sin x|^{\frac{a}{|\sin x|}}\} = b = \lim_{x \rightarrow 0} e^{\frac{\tan 2x}{\tan 3x}}$$

$$\Rightarrow e^a = b = e^{2/3} \Rightarrow a = \frac{2}{3} \text{ and } a = \log_e b$$

14 (b)

We have,

$$f(x) = \begin{cases} x^2 + \frac{(x^2/1 + x^2)}{1 - (1/1 + x^2)} = x^2 + 1, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\text{Clearly, } \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = 1 \neq f(0)$$

So,  $f(x)$  is discontinuous at  $x = 0$

15 (d)

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{1 - 1}{-h} = 0$$



$$\begin{aligned} \text{RHD} &= \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{1 + \sin(0+h) - 1}{h} = \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \\ &\Rightarrow \text{LHD} \neq \text{RHD} \end{aligned}$$

16 (a)

Given,  $f(x) = x - |x - x^2|$

At  $x = 1$ ,  $f(1) = 1 - |1 - 1| = 1$

$$\begin{aligned} \lim_{x \rightarrow 1^-} f(x) &= \lim_{h \rightarrow 0} [(1-h) - |(1-h) - (1-h)^2|] \\ &= \lim_{h \rightarrow 0} [(1-h) - |h - h^2|] = 1 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 1^+} f(x) &= \lim_{h \rightarrow 0} [(1+h) - |(1+h) - (1+h)^2|] \\ &= \lim_{h \rightarrow 0} [1+h - |-h^2 - h|] = 1 \end{aligned}$$

$$\therefore \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$$

17 (a)

We have,

$$f(x+y+z) = f(x)f(y)f(z) \text{ for all } x, y, z \dots \text{(i)}$$

$$\Rightarrow f(0) = f(0)f(0)f(0) \quad [\text{Putting } x = y = z = 0]$$

$$\Rightarrow f(0)\{1 - f(0)^2\} = 0$$

$$\Rightarrow f(0) = 1 \quad [\because f(0) = 0 \Rightarrow f(x) = 0 \text{ for all } x]$$

Putting  $z = 0$  and  $y = 2$  in (i), we get

$$f(x+2) = f(x)f(2)f(0)$$

$$\Rightarrow f(x+2) = 4f(x) \text{ for all } x$$

$$\Rightarrow f'(2) = 4f'(0) \quad [\text{Putting } x = 0]$$

$$\Rightarrow f'(2) = 4 \times 3 = 12$$

18 (b)

For  $x > 1$ , we have

$$f(x) = |\log|x|| = \log x \Rightarrow f'(x) = \frac{1}{x}$$

For  $x < -1$ , we have

$$f(x) = |\log|x|| = \log(-x) \Rightarrow f'(x) = \frac{1}{x}$$

For  $0 < x < 1$ , we have

$$f(x) = |\log|x|| = -\log x \Rightarrow f'(x) = -\frac{1}{x}$$

For  $-1 < x < 0$ , we have

$$f(x) = -\log(-x) \Rightarrow f'(x) = -\frac{1}{x}$$

$$\text{Hence, } f'(x) = \begin{cases} \frac{1}{x}, & |x| > 1 \\ -\frac{1}{x}, & |x| < 1 \end{cases}$$

19 (c)

Since,  $\lim_{x \rightarrow 0} f(x) = f(0)$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = k$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{-(-\sin x)}{2x} = k \quad [\text{using L'Hospital's rule}]$$

$$\Rightarrow \frac{1}{2} \lim_{x \rightarrow 0} \frac{\sin x}{x} = k \Rightarrow k = \frac{1}{2}$$

20 (b)

Given,  $f(x) = |x - 1| + |x - 2|$



$$= \begin{cases} x-1+x-2, & x \geq 2 \\ x-1+2-x, & 1 \leq x < 2 \\ 1-x+2-x, & x < 1 \end{cases}$$

$$= \begin{cases} 2x-3, & x \geq 2 \\ 1, & 1 \leq x < 2 \\ 3-2x, & x < 1 \end{cases}$$

$$f'(x) = \begin{cases} 2, & x > 2 \\ 0, & 1 < x < 2 \\ -1, & x < 1 \end{cases}$$

Hence, except  $x = 1$  and  $x = 2$ ,  $f(x)$  is differentiable everywhere in  $R$

**ANSWER-KEY**

Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	B	C	B	B	B	C	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	A	B	D	A	A	B	C	B



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