DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :2

Topic :-RELATIONS AND FUNCTIONS

1 **(c)**

Clearly, $X = R^+$ and $Y = R$

2 **(b)**

Given, $f(x).f\left(\frac{1}{2}\right) = f(x) + f\left(\frac{1}{x}\right)$

Let $f(x) = x^n \pm 1$, where $n \in I$.

Now, $f(4) = 65$

Case I

Let $f(x) = x^n + 1$

$\Rightarrow \quad f(4) = 4^n + 1$

$\Rightarrow \quad 65 = 4^n + 1$

$\Rightarrow \quad n = 3$

Case II

Let $f(x) = x^n - 1$

$\Rightarrow \quad f(4) = 4^n - 1 \Rightarrow 65 = 4^n - 1$

$\Rightarrow \quad 4^n = 66$

The quality does not hold true for $n \in Z$.

Therefore, $f(x) = x^3 + 1$

Now, $f(6) = 6^3 + 1 = 216 + 1 = 217$

3 **(b)**

Since, the graph is symmetrical about the line= $x = 2$

$\Rightarrow \quad f(2 + x) = f(2 - x)$

4 **(c)**

We have,

$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases} \text{ and } g(x) = x(1 - x^2)$$

$\therefore fog\,(x) = f\big(g(x)\big)$

$$\Rightarrow fog\,(x) = \begin{cases} -1, & \text{if } g(x) < 0 \\ 0, & \text{if } g(x) = 0 \\ 1, & \text{if } g(x) > 0 \end{cases}$$

$$\Rightarrow fog\,(x) = \begin{cases} -1, & \text{if } x \in (-1,0) \cup (1,\infty) \\ 0, & \text{if } x = 0, \pm 1 \\ 1, & \text{if } x \in (-\infty,-1) \cup (0,1) \end{cases}$$

5 **(b)**

Reflexive xRx

Since, $x^2 = x.x$

$x^2 = xy$

Transitive, $xRy \Rightarrow x^2 = xy$

And $yRz \Rightarrow y^2 = yz$

Now, $x^2y^2 = xy^2z \Rightarrow x^2 = xz$

$\Rightarrow xRz$

$\therefore$ It is transitive.

6 **(c)**

We have,

$f(x)=\sin\left(\frac{\pi x}{n-1}\right)+\cos\left(\frac{\pi x}{n}\right), n\in Z, n>2$

Since $\sin\left(\frac{\pi x}{n-1}\right)$ and $\cos\left(\frac{\pi x}{n}\right)$ are periodic functions with period $2(n-1)$ and $2n$ respectively. Therefore, $f(x)$ is periodic with period equal to LCM of $\big(2n, 2(n-1)\big)=2n(n-1)$

7 **(b)**

Let $g(x)$ be the even extension of $f(x)$ on $[-4,4]$

Then,

$g(x)=\begin{cases} f(x) \text{ for } x\in[-4,0] \\ f(-x) \text{ for } x\in[0,4]\end{cases}$

$\Rightarrow g(x)=\begin{cases} e^x+\sin x \text{ for } x\in[-4,0] \\ e^{-x}+\sin(-x) \text{ for } x\in[0,4]\end{cases}$

$\Rightarrow g(x)=\begin{cases} e^x+\sin x \text{ for } x\in[-4,0] \\ e^{-x}-\sin x \text{ for } x\in[0,4]\end{cases}$

$\Rightarrow g(x)=e^{-|x|}-\sin|x|$ for $x\in[-4,4]$

8 **(d)**

Clearly, $f(x)$ is an even function and $f(x)<0$ for all $x>0$

Therefore, the graph of $f(x)$ lies in the third and fourth quadrants

9 **(d)**

The given function is

$$f(x)=\sqrt{1-2x}+2\sin^{-1}\left(\frac{3x-1}{2}\right)$$

For domain of $f(x)$, $1-2x\geq 0$ and $-1\leq\frac{3x-1}{2}\leq 1$

$\Rightarrow \quad x\leq\frac{1}{2}$ and $-2\leq 3x-1\leq 2$

$\Rightarrow \quad x\leq\frac{1}{2}$ and $-\frac{1}{3}\leq x\leq 1$

$\therefore$ Domain of $f(x)=\left[-\frac{1}{3},\frac{1}{2}\right]$

10 **(c)**

We have,

$f(x)=\log_{(x+3)}(x^2-1)$

Clearly, $f(x)$ is defined for x satisfying the following conditions

(i) $x^2-1>0$ (ii) $x+3>0$ and $x+3\neq 1$

Now, $x^2-1>0\Rightarrow x\in(-\infty,-1)\cup(1,\infty)$

and,

$x+3>0$ and $x+3\neq 1\Rightarrow x>-3$ and $x=-2$

$\Rightarrow x\in(-3,-2)\cup(-2,\infty)$

Hence, the domain of $f(x)$ is $(-3,-2)\cup(-2,-1)\cup(1,\infty)$

11 **(b)**

$x^2-6x+7=(x-3)^2-2$

Obviously, minimum value is -2 and maximum is ∞.

12 **(d)**

We have,

$fof^{-1}(x)=x$

$\Rightarrow f\big(f^{-1}(x)\big)=x$

$\Rightarrow f(y)=x$ where $y=f^{-1}(x)$

$\Rightarrow \frac{e^y-e^{-y}}{e^y+e^{-y}}+2=x\Rightarrow\frac{e^y-e^{-y}}{e^y+e^{-y}}=x-2\Rightarrow\frac{2e^y}{-2e^{-y}}=\frac{x-1}{x-3}$

$\Rightarrow e^{2y}=\frac{x-1}{3-x}$

$\Rightarrow y = \frac{1}{2}\log\left(\frac{x-1}{3-x}\right)$

$\Rightarrow f^{-1}(x) = \frac{1}{2}\log\left(\frac{x-1}{3-x}\right)$

13 **(b)**

$f(x) = \frac{4^x}{4^x + 2}$

$\therefore \quad f(1-x) + f(x) = \frac{4^{1-x}}{4^{1-x}+2} + \frac{4^x}{4^x+2}$

$= \frac{4}{4+2.4^x} + \frac{4^x}{4^x+2} = \frac{2}{2+4^x} + \frac{4^x}{4^x+2} = 1$

By putting $x = \frac{1}{97}, \frac{2}{97}, \frac{3}{97}, \dots . \frac{48}{97}$

And adding, we get

$f\left(\frac{1}{97}\right) + f\left(\frac{2}{97}\right) + \dots + f\left(\frac{96}{97}\right) = 48$

14 **(c)**

Given, $f(x) = \frac{2\sin 8x \cos x - 2\sin 6x \cos 3x}{2\cos 2x \cos x - 2\sin 3x \sin 4x}$

$= \frac{(\sin 9x + \sin 7x) + (\sin 9x + \sin 3x)}{(\cos 3x + \cos x) + (\cos 7x - \cos x)}$

$= \frac{\sin 7x - \sin 3x}{\cos 7x + \cos 3x}$

$= \frac{2\cos 5x \sin 2x}{2\cos 2x \cos 5x} = \tan 2x$

$\therefore$ Period of $f(x) = \frac{\pi}{2}$

15 **(d)**

$gof = g\{f(x)\} = g(x^2) = x^2 + 5$

16 **(b)**

We have,

$f(x) = \log_{2x-5}(x^2 - 3x - 10)$

For $f(x)$ to be defined, we must have

$x^2 - 3x - 10 > 0, 2x - 5 > 0$ and $2x - 5 \neq 1$

$\Rightarrow (x-5)(x+2) > 0, x > \frac{5}{2}$ and $\frac{5}{2}$ and $x \neq 3$

$\Rightarrow x > 5 \Rightarrow x \in (5, \infty)$

17 **(c)**

Since, $f(x)$ is an even function therefore its values is always greater than equal to 0 and we know

$x^2 < x^2 + 1$ or $\frac{x^2}{x^2+1} < 1$

$\therefore$ Required range is $[0, 1)$.

18 **(d)**

We have,

$f(x^2) = |x^2 - 1| \neq |x-1|^2 = [f(x)]^2$

$f(|x|) = \big||x| - 1\big| \neq |x-1| = |f(x)|$

And,

$f(x+y) = |x+y-1| \neq |x-1| + |y-1| = f(x) + f(y)$

Hence, none of the above given option is true

19 **(d)**

We have,

$f(x+2) - 2f(x+1) + f(x)$

$= a^{x+2} - 2\, a^{x+1} + a^x$

$= a^x(a^2 - 2\,a + 1) = a^x(a-1)^2 = (a-1)^2 f(x)$

So, option (a) holds

It can be easily checked that options (b) and (c) are also true but option (d) is not true

20 **(b)**

It can be easily seen that $f: A \to A$ is a bijection. Let $f(x) = y$. Then,

$f(x) = y$

$\Rightarrow x(2 - x) = y$

$\Rightarrow x^2 - 2x + y = 0$

$\Rightarrow x^2 - 2x + y = 0$

$\Rightarrow x = \dfrac{2 \pm \sqrt{4 - 4y}}{2}$

$\Rightarrow x = 1 \pm \sqrt{1 - y}$

$\Rightarrow x = 1 - \sqrt{1 - y}$ $[\because x \leq 1]$

$\Rightarrow f^{-1}(y) = 1 - \sqrt{1 - y}$

Hence, $f^{-1}: A \to A$ is defined as $f^{-1}(x) = 1 - \sqrt{1 - x}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	B	C	B	C	B	D	D	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	D	B	C	D	B	C	D	D	B