

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth

DATE :

SOLUTION

SUBJECT : MATHS

DPP NO. :2

Topic :- INVERSE TRIGONOMETRIC FUNCTIONS

1 (b)

We have, $\sum x_1 =$

$\cos 2\beta, \sum x_1 x_2 x_3 = \cos \beta$ and $x_1 x_2 x_3 x_4 = -\sin \beta$

$\sin 2\beta, \sum x_1 x_2 =$

$$\begin{aligned} &\therefore \tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4 \\ &= \tan^{-1} \left(\frac{\sum x_1 - \sum x_1 x_2 x_3}{1 - \sum x_1 x_2 + x_1 x_2 x_3 x_4} \right) \\ &= \tan^{-1} \left(\frac{\sin 2\beta - \cos \beta}{1 - \cos 2\beta - \sin \beta} \right) \\ &= \tan^{-1} \left(\frac{(2 \sin \beta - 1) \cos \beta}{\sin \beta (2 \sin \beta - 1)} \right) \\ &= \tan^{-1} (\cot \beta) \\ &= \tan^{-1} \left(\tan \left(\frac{\pi}{2} - \beta \right) \right) = \frac{\pi}{2} - \beta \end{aligned}$$

2 (d)

$$\begin{aligned} &\tan^{-1} \left(\frac{\tan x}{4} \right) + \tan^{-1} \left(\frac{3 \sin 2x}{5 + 3 \cos 2x} \right) \\ &= \tan^{-1} \left(\frac{\tan x}{4} \right) + \tan^{-1} \left(\frac{\frac{6 \tan x}{1 + \tan^2 x}}{5 + \frac{3(1 - \tan^2 x)}{1 + \tan^2 x}} \right) \\ &= \tan^{-1} \left(\frac{\tan x}{4} \right) + \tan^{-1} \left(\frac{6 \tan x}{8 + 2 \tan^2 x} \right) \\ &= \tan^{-1} \left(\frac{\tan x}{4} \right) + \tan^{-1} \left(\frac{3 \tan x}{4 + \tan^2 x} \right) \\ &= \tan^{-1} \left(\frac{\frac{\tan x}{4} + \frac{3 \tan x}{4 + \tan^2 x}}{1 - \frac{3 \tan^2 x}{4(4 + \tan^2 x)}} \right) \left(\text{as } \left| \frac{\tan x}{4} \cdot \frac{3 \tan x}{4 + \tan^2 x} \right| < 1 \right) \\ &= \tan^{-1} \left(\frac{16 \tan x + \tan^3 x}{16 + \tan^2 x} \right) \\ &= \tan^{-1} (\tan x) = x \end{aligned}$$

4 (c)

Let $\cos^{-1} x = \theta$. Then, $x = \cos \theta$

Also,

$$-1 \leq x \leq 0 \Rightarrow -1 \leq \cos \theta \leq 0 \Rightarrow \frac{\pi}{2} \leq \theta \leq \pi$$

Now,

$$\cos^{-1}(2x^2 - 1) = \cos^{-1}(\cos 2\theta) = \cos^{-1}(\cos(2\pi - 2\theta))$$

$$= 2\pi - 2\theta \left[\begin{array}{l} \because \frac{\pi}{2} \leq \theta \leq \pi \Rightarrow \pi \leq 2\theta \leq 2\pi \\ \Rightarrow 0 \leq 2\pi - 2\theta \leq \pi \end{array} \right]$$

$$= 2\pi - 2 \cos^{-1} x$$

5 (b)

$$\begin{aligned} \therefore \cos^{-1} \frac{3}{5} - \sin^{-1} \frac{4}{5} &= \cos^{-1} x \\ \Rightarrow \sin^{-1} \frac{4}{5} - \sin^{-1} \frac{4}{5} &= \cos^{-1} x \\ \Rightarrow \cos^{-1} x &= 0 \Rightarrow x = \cos 0 = 1 \\ \therefore x &= 1 \end{aligned}$$

6 (d)

We have,

$$\begin{aligned} \sec^{-1} x &= \operatorname{cosec}^{-1} y \Rightarrow \cos^{-1} \frac{1}{x} = \sin^{-1} \frac{1}{y} \\ \therefore \cos^{-1} \frac{1}{x} + \cos^{-1} \frac{1}{y} &= \sin^{-1} \frac{1}{y} + \cos^{-1} \frac{1}{y} = \frac{\pi}{2} \end{aligned}$$

8 (c)

Let $\sin^{-1} x = \theta$. Then, $x = \sin \theta$

Also,

$$-1 \leq x \leq -\frac{1}{2}$$

$$\Rightarrow -1 \leq \sin \theta \leq -\frac{1}{2} \Rightarrow -\frac{\pi}{2} \leq \theta \leq -\frac{\pi}{6} \Rightarrow -\frac{3\pi}{2} \leq 3\theta \leq -\frac{\pi}{2}$$

Now,

$$\begin{aligned} \sin^{-1}(3x - 4x^3) &= \sin^{-1}(\sin 3\theta) \\ &= \sin^{-1}(-\pi - 3\theta) \end{aligned}$$

$$= -\pi - 3\theta \left[-\frac{3\pi}{2} \leq 3\theta \leq -\frac{\pi}{2} \Rightarrow -\frac{\pi}{2} \leq -\pi - 3\theta \leq \frac{\pi}{2} \right]$$

$$= -\pi - 3 \sin^{-1} x$$

9 (d)

We have,

$$\frac{\tan \frac{6\pi}{15} - \tan \frac{\pi}{15}}{1 + \tan \frac{6\pi}{15} \tan \frac{\pi}{15}} = \tan \frac{\pi}{3}$$

$$\Rightarrow \tan \frac{6\pi}{15} - \tan \frac{\pi}{15} = \sqrt{3} + \sqrt{3} \tan \frac{6\pi}{15} \tan \frac{\pi}{15}$$

$$\Rightarrow \tan \frac{2\pi}{5} - \tan \frac{\pi}{15} - \sqrt{3} \tan \frac{2\pi}{5} \tan \frac{\pi}{15} = \sqrt{3}$$

10 (a)

$$\begin{aligned} \tan \left\{ \cos^{-1} \left(-\frac{2}{7} \right) - \frac{\pi}{2} \right\} &= \tan \left\{ \pi - \cos^{-1} \left(\frac{2}{7} \right) - \frac{\pi}{2} \right\} \\ &= \tan \left\{ \frac{\pi}{2} - \cos^{-1} \left(\frac{2}{7} \right) \right\} \\ &= \tan \left\{ \sin^{-1} \frac{2}{7} \right\} \\ &= \tan \left\{ \tan^{-1} \left(\frac{2}{3\sqrt{5}} \right) \right\} = \frac{2}{3\sqrt{5}} \end{aligned}$$

11 (a)

$$\begin{aligned} \sin \left[\sin^{-1} \left(\frac{1}{3} \right) + \sec^{-1}(3) \right] + \cos \left[\tan^{-1} \left(\frac{1}{2} \right) + \tan^{-1}(2) \right] \\ = \sin \left[\sin^{-1} \left(\frac{1}{3} \right) + \cos^{-1} \left(\frac{1}{3} \right) \right] \end{aligned}$$

$$\begin{aligned} + \cos \left[\tan^{-1} \left(\frac{1}{2} \right) + \cot^{-1} \left(\frac{1}{2} \right) \right] \\ = \sin \frac{\pi}{2} + \cos \frac{\pi}{2} \end{aligned}$$



$$\left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \text{ and } \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \right]$$

$$= 1$$

12 (a)

Let $\tan^{-1} x = \theta$. Then, $x = \tan \theta$

Also,

$$-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} < \tan \theta < \frac{1}{\sqrt{3}} \Rightarrow -\frac{\pi}{6} < \theta < \frac{\pi}{6} \Rightarrow -\frac{\pi}{2} < 3\theta < \frac{\pi}{2}$$

Now,

$$\tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) = \tan^{-1}(\tan 3\theta)$$

$$\Rightarrow \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) = 3\theta \left[\because -\frac{\pi}{2} < 3\theta < \frac{\pi}{2} \right]$$

$$\Rightarrow \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right) = 3 \tan^{-1} x$$

13 (b)

Let $\cos^{-1} \left(\frac{4}{5} \right) = \theta$. Then, $\cos \theta = \frac{4}{5}$

$$\therefore \sin \left(\frac{1}{2} \cos^{-1} \frac{4}{5} \right) = \sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - \frac{4}{5}}{2}} = \frac{1}{\sqrt{10}}$$

14 (a)

$$\begin{aligned} \tan^{-1} 2x + \tan^{-1} 3x &= \frac{\pi}{4} \\ \Rightarrow \frac{3x + 2x}{1 - 6x^2} &= \frac{\pi}{4} \\ \Rightarrow 5x &= 1 - 6x^2 \\ \Rightarrow 6x^2 + 5x - 1 &= 0 \\ \Rightarrow x &= -1, \frac{1}{6} \end{aligned}$$

But when $x = -1$,

$$\tan^{-1} 2x = \tan^{-1}(-2) < 0$$

And $\tan^{-1} 3x = \tan^{-1}(-3) < 0$

This value will not satisfy the given equation

Hence, $x = \frac{1}{6}$

15 (c)

$$\sin^{-1} \frac{4}{5} + 2 \tan^{-1} \frac{1}{3} = \sin^{-1} \frac{4}{5} + \tan^{-1} \frac{2 \left(\frac{1}{3} \right)}{1 - \left(\frac{1}{3} \right)^2}$$

$$= \sin^{-1} \frac{4}{5} + \tan^{-1} \frac{3}{4} = \sin^{-1} \frac{4}{5} + \cos^{-1} \frac{4}{5} = \frac{\pi}{2}$$

$$\left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right]$$

16 (a)

Given equation is

$$\begin{aligned} 2 \cos^{-1} x + \sin^{-1} x &= \frac{11\pi}{6} \\ \Rightarrow \cos^{-1} x + (\cos^{-1} x + \sin^{-1} x) &= \frac{11\pi}{6} \end{aligned}$$

$$\Rightarrow \cos^{-1} x + \frac{\pi}{2} = \frac{11\pi}{6}$$

$$\Rightarrow \cos^{-1} x = \frac{4\pi}{3}$$

Which is not possible as $\cos^{-1} x \in [0, \pi]$.

17 (c)

$$\begin{aligned} & \cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\cos\frac{5\pi}{3}\right) \\ &= \cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left[\sin\left(\frac{\pi}{2} - \frac{5\pi}{3}\right)\right] \\ &= \frac{5\pi}{3} + \frac{\pi}{2} - \frac{5\pi}{3} = \frac{\pi}{2} \end{aligned}$$

Alternate

$$\text{Since, } \cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$$

$$\therefore \cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right) = \frac{\pi}{2}$$

18 (d)

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} = 30^\circ$$

19 (c)

We have,

$$\begin{aligned} & \sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x \\ & \Rightarrow \sin\{\sin^{-1} x + \sin^{-1}(1-x)\} = \sin(\cos^{-1} x) \\ & \Rightarrow x\sqrt{1-(1-x)^2} + \sqrt{1-x^2}(1-x) = \sqrt{1-x^2} \\ & \Rightarrow x\sqrt{1-(1-x)^2} = x\sqrt{1-x^2} \\ & \Rightarrow x = 0 \text{ or } 2x - x^2 = 1 - x^2 \Rightarrow x = 0 \text{ or } x = \frac{1}{2} \end{aligned}$$

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ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	D	C	C	B	D	D	C	D	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	A	B	A	C	A	C	D	C	C



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