



DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth

DATE :

SOLUTION

SUBJECT : MATHS

DPP NO. :2

Topic :-DETERMINANTS

1 (c)

$$\text{Let } A = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = \begin{vmatrix} 1 + \omega + \omega^2 & \omega & \omega^2 \\ 1 + \omega + \omega^2 & \omega^2 & 1 \\ 1 + \omega + \omega^2 & 1 & \omega \end{vmatrix}$$

$$[C_1 \rightarrow C_1 + C_2 + C_3]$$

$$= \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix} = 0 \quad [\because 1 + \omega + \omega^2 = 0]$$

2 (b)

Applying $C_1 \rightarrow C_1 + C_2$, we get

$$\begin{vmatrix} {}^{10}C_4 + {}^{10}C_5 & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 + {}^{11}C_7 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 + {}^{12}C_9 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} {}^{11}C_5 & {}^{10}C_5 & {}^{11}C_m \\ {}^{12}C_7 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{13}C_9 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$$

It means either two rows or two columns are identical.

$$\therefore {}^{11}C_5 = {}^{11}C_m, {}^{12}C_7 = {}^{12}C_{m+2}, {}^{13}C_9 = {}^{13}C_{m+4}$$

$$\Rightarrow m = 5$$

3 (b)

$$\text{Given, } \begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & 3 \\ 5 & -6 & x \end{vmatrix} = 29$$

$$\Rightarrow 1(0 + 18) - 1(2x - 15) = 29$$

$$\Rightarrow 2x = 4 \Rightarrow x = 2$$

4 (a)

Applying $C_1 \rightarrow C_1 + C_2$, we get

$$\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} = \begin{vmatrix} 1 & \cos^2 x & 1 \\ 1 & \sin^2 x & 1 \\ 2 & 12 & 2 \end{vmatrix} = 0$$

5 (b)

$$\text{Since, } |A| = -1, |B| = 3$$

$$\therefore |AB| = |A||B| = -3$$

$$\text{Now, } |3AB| = (3)^3(-3) = -81$$

7 (d)

Applying $C_3 \rightarrow C_3 - \alpha C_1 + C_2$ to the given determinant, we get

$$\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ 2 & 1 & -2\alpha + 1 \end{vmatrix} = (1 - 2\alpha)(ac - b^2)$$

So, if the determinant is zero, we must have

$$(1 - 2\alpha)(ac - b^2) = 0$$

$$\Rightarrow 1 - 2\alpha = 0$$

$$\text{or } (ac - b^2) = 0$$



$$\Rightarrow \alpha = \frac{1}{2} \text{ or } ac = b^2$$

Which means a, b, c are in GP.

8 (a)

We have,
$$\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

$$\Rightarrow (x+9) \begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0 \quad (R_1 \rightarrow R_1 + R_2 + R_3)$$

$$\Rightarrow (x+9)\{1(x^2 - 12) - 1(2x - 14) + 1(12 - 7x)\} = 0$$

$$\Rightarrow (x+9)(x^2 - 9x + 14) = 0$$

$$\Rightarrow (x+9)(x-2)(x-7) = 0$$

\therefore The other two roots are 2 and 7.

9 (a)

Let $A \equiv \begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} a+b+c-x & c & b \\ a+b+c-x & b-x & a \\ a+b+c-x & a & c-x \end{vmatrix}$$

$$= (a+b+c-x) \begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix}$$

$$\Rightarrow (a+b+c-x)[1\{(b-x)(c-x) - a^2\} - c(c-x-a) + b(a-b+x)] = 0$$

$$\Rightarrow (a+b+c-x)[bc - bx - cx + x^2 - a^2 - c^2 + xc + ac + ab - b^2 + bx] = 0$$

$$\Rightarrow (a+b+c-x)[x^2 - (a^2 + b^2 + c^2) + ab + bc + ca] = 0$$

$\therefore ab + bc + ca = 0$ (given)

$$\Rightarrow \text{either } x = a + b + c \text{ or } x = (a^2 + b^2 + c^2)^{1/2}$$

10 (b)

We have,

$$\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+1 & 1 & 1 \\ x+1 & x-1 & 1 \\ x+1 & 1 & x-1 \end{vmatrix} = 0 \quad [\text{Applying } C_1 \rightarrow C_1 + C_2 + C_3]$$

$$\Rightarrow (x+1) \begin{vmatrix} 1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$$

$$\Rightarrow (x+1) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-2 & 0 \\ 0 & 0 & x-2 \end{vmatrix} = 0 \quad \left[\begin{array}{l} \text{Applying } R_2 \rightarrow R_2 - R_1, \\ R_3 \rightarrow R_3 - R_1 \end{array} \right]$$

$$\Rightarrow (x+1)(x-2)^2 = 0$$

$$\Rightarrow x = -1, 2$$

11 (c)

$$\text{Let } A = \begin{vmatrix} 1 & 2 & 3 \\ 1^3 & 2^3 & 3^3 \\ 1^5 & 2^5 & 3^5 \end{vmatrix} = 1.2.3 \begin{vmatrix} 1 & 2 & 3 \\ 1^2 & 2^2 & 3^2 \\ 1^4 & 2^4 & 3^4 \end{vmatrix}$$

$$= 6 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 9 \\ 1 & 16 & 81 \end{vmatrix} = 6 \begin{vmatrix} 1 & 0 & 0 \\ 1 & 3 & 5 \\ 1 & 15 & 65 \end{vmatrix}$$

$$[C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_2]$$



$$= 6.3.5 \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 5 & 13 \end{vmatrix} = 90[1(13 - 5)] = 720 = 6!$$

12 (c)

$$\therefore |A^3| = |A|^3 = 125$$

$$\Rightarrow \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix} = 5$$

$$\Rightarrow \alpha^2 - 4 = 5 \Rightarrow \alpha = \pm 3$$

14 (b)

Given, angles of a triangle are A, B and C . We know that $A + B + C = \pi$, therefore

$$A + B = \pi - C$$

$$\Rightarrow \cos(A + B) = \cos(\pi - C) = -\cos C$$

$$\Rightarrow \cos A \cos B - \sin A \sin B = -\cos C$$

$$\Rightarrow \cos A \cos B + \cos C = \sin A \sin B \dots(i)$$

$$\text{Let } \Delta = \begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix}$$

$$= -(1 - \cos^2 A) + \cos C(\cos C + \cos A \cos B) + \cos B(\cos B + \cos A \cos C)$$

$$= -\sin^2 A + \cos C(\sin A \sin B) + \cos B(\sin A \sin C) \text{ [from Eq.(i)]}$$

$$= -\sin^2 A + \sin A(\sin B \cos C + \cos B \sin C)$$

$$= -\sin^2 A + \sin A \sin(B + C)$$

$$= -\sin^2 A + \sin^2 A = 0 \text{ [}\because \sin(B + C) = \sin(\pi - A) = \sin A\text{]}$$

15 (a)

We have,

$$\Delta = \begin{vmatrix} a^2 + x & ab & ac \\ ab & b^2 + x & bc \\ ac & bc & c^2 + x \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{abc} \begin{vmatrix} a^3 + ax & a^2b & a^2c \\ ab^2 & b^3 + bx & b^2c \\ ac^2 & bc^2 & c^3 + cx \end{vmatrix} \text{ [Applying } C_1(a), C_2(b), C_3(c)\text{]}$$

$$\Rightarrow \Delta = \begin{vmatrix} a^2 + x & a^2 & a^2 \\ b^2 & b^2 + x & b^2 \\ c^2 & c^2 & c^2 + x \end{vmatrix}$$

$$\Rightarrow \Delta = (a^2 + b^2 + c^2 + x) \begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2 + x & b^2 \\ c^2 & c^2 & c^2 + x \end{vmatrix}$$

[Applying $R_1 \rightarrow R_1 + R_2 + R_3$]

$$\Rightarrow \Delta = (a^2 + b^2 + c^2 + x)\{(b^2x + c^2x + x^2) - (b^2x) + (-c^2x)\}$$

$$\Rightarrow \Delta = x^2(a^2 + b^2 + c^2 + x)$$

$\Rightarrow x^2$ is a factor Δ

16 (a)

$$\text{Given that, } \begin{vmatrix} x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \\ x+a & x+b & x+c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -1 & -1 & x+3 \\ -1 & -1 & x+4 \\ a-b & b-c & x+c \end{vmatrix} = 0 \text{ (} C_1 \rightarrow C_1 - C_2, C_2 \rightarrow C_2 - C_3 \text{)}$$

$$\Rightarrow \begin{vmatrix} -1 & -1 & x+4 \\ a-b & b-c & x+c \\ a-b & b-c & x+c \end{vmatrix} = 0 \text{ (} R_1 \rightarrow R_1 - R_2 \text{)}$$

$$\Rightarrow (-1)(-b + c + a - b) = 0$$

$$\Rightarrow 2b - a - c = 0$$

$$\Rightarrow a + c = 2b$$

$\therefore a, b, c$ in AP.

17 (b)



Given, $A = \begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ x & x & 1 \end{vmatrix} \Rightarrow A = 1$

$\therefore A^3 - 4A^2 + 3A + I = (1)^3 - 4(1)^2 + 3(1) + I = I$

18 (a)

Let $\Delta = \begin{vmatrix} 1 & x & y \\ 2 & \sin x + 2x & \sin y + 3y \\ 3 & \cos x + 3x & \cos y + 3y \end{vmatrix}$

$= \begin{vmatrix} 1 & x & y \\ 0 & \sin x & \sin y \\ 0 & \cos x & \cos y \end{vmatrix} \begin{matrix} (R_2 \rightarrow R_2 - 2R_1) \\ (R_3 \rightarrow R_3 - 3R_1) \end{matrix}$

$= \sin x \cos y - \cos x \sin y = \sin(x - y)$

19 (d)

We have, $\Delta = \frac{1}{abc} \begin{vmatrix} a^3 + ax & a^2b & a^2c \\ ab^2 & b^3 + bx & b^2c \\ c^2a & c^2b & c^2 + xc \end{vmatrix}$

Taking a, b, c common in columns 1st, 2nd and 3rd, we get,

$\Delta = \begin{vmatrix} a^2 + x & a^2 & a^2 \\ b^2 & b^2 + x & b^2 \\ c^2 & c^2 & c^2 + x \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$

$= (a^2 + b^2 + c^2 + x) \begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2 + x & b^2 \\ c^2 & c^2 & c^2 + x \end{vmatrix}$

Applying $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$

$= (a^2 + b^2 + c^2 + x) \begin{vmatrix} 1 & 1 & 0 \\ b^2 & x & 0 \\ c^2 & 0 & x \end{vmatrix}$

$= x(x - b^2)(a^2 + b^2 + c^2 + x)$

Hence, option (d) is correct.

20 (a)

Given, $\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0$

$\Rightarrow (ab)^3 + (bc)^3 + (ca)^3 - 3a^2b^2c^2 = 0$

$\Rightarrow (ab + bc + ca)(a^2b^2 + b^2c^2 + c^2a^2 - ab^2c$

$-bc^2a - ca^2b) = 0$

$\Rightarrow ab + bc + ca = 0$

$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	B	A	B	A	D	A	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	B	B	A	A	B	A	D	A



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