# DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. : 2

## Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(b)**

Clearly, $f(x)$ is differentiable for all non-zero values of $x$. For $x \neq 0$, we have

$$f'(x) = \frac{x\, e^{-x^2}}{\sqrt{1-e^{-x^2}}}$$

Now,

$$(\text{LHD at } x=0) = \lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0} = \lim_{h\to 0}\frac{f(0-h)-f(0)}{x-0}$$

$$\Rightarrow (\text{LHD at } x=0) = \lim_{h\to 0}\frac{\sqrt{1-e^{-h^2}}}{-h} = \lim_{h\to 0} -\frac{\sqrt{1-e^{-h^2}}}{h}$$

$$\Rightarrow (\text{LHD at } x=0) = -\lim_{h\to 0}\sqrt{\frac{e^{h^2}-1}{h^2}} \times \frac{1}{\sqrt{e^{h^2}}} = -1$$

and, $(\text{RHD at } x=0) = \lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0} = \lim_{h\to 0}\frac{\sqrt{1-e^{-h^2}}-0}{h}$

$$\Rightarrow (\text{RHD at } x=0) = \lim_{h\to 0}\sqrt{\frac{e^{h^2}-1}{h^2}} \times \frac{1}{\sqrt{e^{h^2}}} = 1$$

So, $f(x)$ is not differentiable at $x=0$

Hence, the set of points of differentiability of $f(x)$ is $(-\infty, 0) \cup (0, \infty)$

2 **(c)**

Since $f(x)$ is continuous at $x=0$

$$\therefore f(0) = \lim_{x\to 0} x \sin\left(\frac{1}{x}\right) = 0$$

3 **(d)**

For $f(x)$ to be continuous everywhere, we must have,

$f(0) = \lim_{x\to 0} f(x)$

$$\Rightarrow f(0) = \lim_{x\to 0}\frac{2-(256-7x)^{1/8}}{(5x+32)^{1/5}-2} \quad \left[\text{Form}\frac{0}{0}\right]$$

$$\Rightarrow f(0) = \lim_{x\to 0}\frac{\frac{7}{8}(256-7x)^{-\frac{7}{8}}}{(5x+32)^{-4/5}} = \frac{7}{8}\times\frac{2^{-7}}{2^{-4}} = \frac{7}{64}$$

4 **(b)**

We have,

$$f(x) = |x|^3 = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x < 0 \end{cases}$$

$$\therefore (\text{LHD at } x=0) = \lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} -\frac{x^3}{x} = 0$$

and,

$$\therefore (\text{RHD at } x=0) = \lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{x^3}{x} = 0$$

Clearly, (LHD at $x=0$) = (RHD at $x=0$)

Hence, $f(x)$ is differentiable at $x = 0$ and its derivative at $x = 0$ is 0

5 **(a)**

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{4^x - 1}{x}\right)^3 \times \frac{\left(\frac{x}{a}\right)}{\sin\left(\frac{x}{a}\right)} \cdot \frac{ax^2}{\log\left(1 + \frac{1}{3}x^2\right)}$$

$$= (\log 4)^3 . 1 . a \lim_{x\to 0}\left(\frac{x^2}{\frac{1}{3}x^2 - \frac{1}{18}x^4 + \dots}\right)$$

$= 3a\,(\log 4)^3$

$\because \quad \lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \quad 3a\,(\log 4)^3 = 9(\log 4)^3$

$\Rightarrow \quad a = 3$

6 **(d)**

We have,

$f(x) = |[x]x|$ for $-1 < x \le 2$

$$\Rightarrow f(x) = \begin{cases} -x, & -1 < x < 0 \\ 0, & 0 \le x < 1 \\ x, & 1 \le x < 2 \\ 2x, & x = 2 \end{cases}$$

It is evident from the graph of this function that it is continuous but not differentiable at $x = 0$. Also, it is discontinuous at $x = 1$ and non-differentiable at $x = 2$

7 **(c)**

Given, $f(x) = [x^3 - 3]$

Let $g(x) = x^3 - x$ it is in increasing function

$\therefore \quad g(1) = 1 - 3 = -2$

and $g(2) = 8 - 3 = 5$

Here, $f(x)$ is discontinuous at six points

8 **(b)**

Given, $y = \cos^{-1}\cos(x - 1), \quad x > 0$

$\Rightarrow \quad y = x - 1, \qquad 0 \le x - 1 \le \pi$

$\therefore \quad y = x - 1, \qquad 1 \le x \le \pi + 1$

At $x = \frac{5\pi}{4} \in [1, \pi + 1]$

$$\Rightarrow \frac{dy}{dx} = 1 \quad \Rightarrow \quad \left(\frac{dy}{dx}\right)_{x=\frac{5\pi}{4}} = 1$$

9 **(d)**

We have,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(x) + f(h) - f(x)}{h} \qquad [\because f(x+y) = f(x) + f(y)]$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(h)}{h} = \lim_{h\to 0}\frac{h^2 g(h)}{h}$$

$$\Rightarrow f'(x) = 0 \times g(0) = 0 \qquad \left[\begin{array}{l} \because g \text{ is continuous} \\ \therefore \lim_{h\to 0} g(h) = g(0) \end{array}\right]$$

10 **(b)**

Using Heine's definition of continuity, it can be shown that $f(x)$ is everywhere discontinuous

11 **(b)**

For $x \neq -1$, we have

$f(x) = 1 - 2x + 3x^2 - 4x^3 + \cdots\infty$

$\Rightarrow f(x)=(1+x)^{-2}=\dfrac{1}{(1+x)^2}$

Thus, we have

$f(x)=\begin{cases}\dfrac{1}{(1+x)^2}, & x\neq -1\\ 1, & x=-1\end{cases}$

We have, $\lim_{x\to -1^-} f(x)\to\infty$ and $\lim_{x\to -1^-} f(x)\to\infty$

So, $f(x)$ is not continuous at $x=-1$

Consequently, it is not differentiable there at

12 **(b)**

At $x=a$,

LHL= $\lim_{x\to a^-} f(x)=\lim_{x\to a} 2a-x=a$

And RHL= $\lim_{x\to a^+} f(x)=\lim_{x\to a} 3x-2a=a$

And $f(a)=3(a)-2a=a$

$\therefore$ LHL=RHL= $f(a)$

Hence, it is continuous at $x=a$

Again, at $x=a$

LHD= $\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h}$

$=\lim_{h\to 0}\dfrac{2a-(a-h)-a}{-h}=-1$

and RHD= $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

$=\lim_{h\to 0}\dfrac{3(a+h)-2a-a}{h}=3$

$\therefore$ LHD≠RHD

Hence, it is not differentiable at $x=a$

13 **(b)**

We have,

$f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x)=\lim_{h\to 0}\dfrac{f(x)f(h)-f(x)}{h}$

$\Rightarrow f'(x)=f(x)\lim_{h\to 0}\dfrac{f(h)-1}{h}$

$\Rightarrow f'(x)=f(x)\lim_{h\to 0}\dfrac{1+(\sin 2h)g(h)-1}{h}$

$\Rightarrow f'(x)=f(x)\lim_{h\to 0}\dfrac{\sin 2h}{h}\times\lim_{h\to 0} g(h)=2f(x)g(0)$

14 **(c)**

If $-1\le x\le 1$, then $0\le x\sin\pi x\le 1/2$

$\therefore f(x)=[x\sin\pi x]=0$, for $-1\le x\le 1$

If $1<x<1+h$, where h is a small positive real number, then

$\pi<\pi x<\pi+\pi h\Rightarrow -1<\sin\pi x<0\Rightarrow -1<x\sin\pi x<0$

$\therefore f(x)=[x\sin\pi x]=-1$ in the right neighbourhood of $x=1$

Thus, $f(x)$ is constant and equal to zero in $[-1,1]$ and so $f(x)$ is differentiable and hence continuous on $(-1,1)$

At $x=1$, $f(x)$ is discontinuous because

$\Rightarrow \lim_{x\to 1^-} f(x)=0$ and $\lim_{x\to 1^+} f(x)=-1$

Hence, $f(x)$ is not differentiable at $x=1$

15 **(d)**

We have,

(LHD at $x = 0$) $= \left\{\frac{d}{dx}(1)\right\}_{x=0} = 0$

(RHD at $x = 0$) $= \left\{\frac{d}{dx}(1 + \sin x)\right\}_{x=0} = \cos 0 = 1$

Hence, $f'(x)$ at $x = 0$ does not exist

16 **(c)**

Here, $f'(x) = \begin{cases} 2bx + a, & x \geq -1 \\ 2a, & x < -1 \end{cases}$

Given, $f'(x)$ is continuous everywhere

$\therefore \lim_{x\to-1^+} f(x) = \lim_{x\to-1^-} f(x)$

$\Rightarrow -2b + a = -2a$

$\Rightarrow 3a = 2b$

$\Rightarrow a = 2, \quad b = 3$

or $a = -2, \ b = -3$

17 **(b)**

We have,

$$\lim_{x\to0}\frac{f(x) - f(0)}{x - 0} = \lim_{x\to0}\frac{\log\cos x}{\log(1 + x^2)}$$

$$\Rightarrow \lim_{x\to0}\frac{f(x) - f(0)}{x - 0} = \lim_{x\to0}\frac{\log(1 - 1 + \cos x)}{\log(1 + x^2)}\cdot\frac{1 - \cos x}{1 - \cos x}$$

$$\Rightarrow \lim_{x\to0}\frac{f(x) - f(0)}{x - 0} = \lim_{x\to0}\frac{\log\{1 - (1 - \cos x)\}}{1 - \cos x}\cdot\frac{1 - \cos x}{\log(1 + x^2)}$$

$$\Rightarrow \lim_{x\to0}\frac{f(x) - f(0)}{x - 0} = -\lim_{x\to0}\log\frac{[1 - (1 - \cos x)]}{-(1 - \cos x)}\times\frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}\times\frac{x^2}{\log(1 + x^2)}$$

$$\Rightarrow \lim_{x\to0}\frac{f(x) - f(0)}{x - 0} = -\frac{1}{2}$$

Hence, $f(x)$ is differentiable and hence continuous at $x = 0$

18 **(a)**

Since $f(x)$ is continuous at $x = 1$. Therefore,

$\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) \Rightarrow A - B = 3 \Rightarrow A = 3 + B$ ...(i)

If $f(x)$ is continuous at $x = 2$, then

$\lim_{x\to2^-} f(x) = \lim_{x\to2^+} f(x) \Rightarrow 6 = 4B - A$ ...(ii)

Solving (i) and (ii) we get $B = 3$

As $f(x)$ is not continuous at $x = 2$. Therefore, $B \neq 3$

Hence, $A = 3 + B$ and $B \neq 3$

19 **(a)**

We have,

$$f(x) = \begin{cases} x - 4, & x \geq 4 \\ -(x - 4), & 1 \leq x < 4 \\ (x^3/2) - x^2 + 3x + (1/2), & x < 1 \end{cases}$$

Clearly, $f(x)$ is continuous for all $x$ but it is not differentiable at $x = 1$ and $x = 4$

20 **(a)**

It is given that $f(x)$ is continuous at $x = 1$

$\therefore \lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = f(1)$

$\Rightarrow \lim_{x\to1^-} a[x + 1] + b[x - 1] = \lim_{x\to1^+} a[x + 1] + b[x - 1]$

$\Rightarrow a - b = 2a + 0 \times b$

$\Rightarrow a + b = 0$

| ANSWER-KEY | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **A.** | B | C | D | B | A | D | C | B | D | B |
| | | | | | | | | | | |
| Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| **A.** | B | B | B | C | D | C | B | A | A | A |
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