DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. :2

Topic :-INTEGRALS

1 **(b)**

We have,

$$I=\int_0^{\pi}\log(1+\cos x)\,dx$$

$$\Rightarrow I=\int_0^{\pi}\log\left(2\cos^2\frac{x}{2}\right)dx$$

$$\Rightarrow I=\int_0^{\pi}\left(\log 2+2\log\cos\frac{x}{2}\right)dx$$

$$\Rightarrow I=\pi\log 2+2\int_0^{\pi}\log\cos\frac{x}{2}\,dx$$

$$\Rightarrow I=\pi\log 2+2\times 2\int_0^{\pi/2}\log\cos t\,dt,\text{ where } t=\frac{x}{2}\text{ and } dx=2dt$$

$$\Rightarrow I=\pi\log 2+4\times-\frac{\pi}{2}\log 2=-\pi\log 2$$

2 **(a)**

Putting $\tan^{-1}x=t$, we have

$$I=\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$$

$$\Rightarrow I=\int e^t(\tan t+\sec^2 t)dt=e^t\tan t+C=x\,e^{\tan^{-1}x}+C$$

3 **(c)**

Given,

$$I_1=\int_a^{\pi-a}x\,f(\sin x)\,dx$$

and $I_2=\int_a^{\pi-a}f(\sin x)\,dx$

Now, $I_1=\int_a^{\pi-a}x\,f(\sin x)\,dx$

$$=\int_a^{\pi-a}(\pi-x)f[\sin(\pi-x)]\,dx$$

$$=\int_a^{\pi-a}(\pi-x)f(\sin x)\,dx$$

$$=\int_a^{\pi-a}\pi\,f(\sin x)dx-I_1$$

$$\Rightarrow 2I_1=\pi I_2\Rightarrow I_2=\frac{2}{\pi}I_1$$

4 **(b)**

$$\int\cos^{-1}\left(\frac{1}{x}\right)dx=\int\sec^{-1}x\cdot 1\,dx$$

$= \sec^{-1} x \int dx - \int \left[\frac{d}{dx}\sec^{-1} x \int dx\right] dx$

$= x\sec^{-1} x - \int \frac{1}{x\sqrt{x^2-1}} x\, dx$

$= x\sec^{-1} x - \int \frac{1}{\sqrt{x^2-1}}\, dx$

$= x\sec^{-1} x - \cosh^{-1} x + c$

5 **(d)**

Let $I = \int_0^1 \frac{dx}{x+\sqrt{1-x^2}}$

Put $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

$I = \int_0^{\pi/2} \frac{\cos\theta\, d\theta}{\sin\theta+\cos\theta}$...(i)

$$\Rightarrow\ I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-\theta\right) d\theta}{\sin\left(\frac{\pi}{2}-\theta\right)+\cos\left(\frac{\pi}{2}-\theta\right)}$$

$= \int_0^{\pi/2} \frac{\sin\theta}{\cos\theta+\sin\theta}\, d\theta$...(ii)

On adding Eqs.(i) and (ii), we get

$$2I = \int_0^{\pi/2} 1\, d\theta \ \Rightarrow I = \frac{\pi}{4}$$

6 **(b)**

Let

$$I = \int_0^{\sqrt{n}} [x^2] dx$$

$$\Rightarrow I = \sum_{r=1}^{n} \int_{\sqrt{r-1}}^{\sqrt{r}} [x^2]\, dx$$

$$\Rightarrow I = \sum_{r=1}^{n} \int_{\sqrt{r-1}}^{\sqrt{r}} (r-1)\, dx$$

$$\Rightarrow I = \sum_{r=1}^{n} (r-1)\left(\sqrt{r}-\sqrt{r-1}\right)$$

$\Rightarrow I = \left(\sqrt{2}-\sqrt{1}\right) + 2\left(\sqrt{3}-\sqrt{2}\right) + 3\left(\sqrt{4}-\sqrt{3}\right)$

$+ \cdots + (n-1)\left(\sqrt{n}-\sqrt{n-1}\right)$

$$\Rightarrow I = n\sqrt{n} - \left(\sqrt{1}+\sqrt{2}+\cdots+\sqrt{n}\right) = n\sqrt{n} - \sum_{r=1}^{n}\sqrt{r}$$

7 **(d)**

Let $I = \int \frac{1}{x}(\log_{ex} e) dx = \int \frac{1}{x(1+\log_e x)} dx$

Put $\log_e x = t \Rightarrow \frac{1}{x} dx = dt$

$$\therefore\ I = \int \frac{dt}{(1+t)} = \log_e(1+t) + c$$

$= \log_e(1+\log_e x) + c$

8 **(a)**

$$I_1 - I_2 = \int_0^{\pi/2} (\cos\theta - \sin 2\theta) f(\sin\theta + \cos^2\theta) d\theta$$

Put $\sin\theta + \cos^2\theta = t$

$\Rightarrow (\cos\theta - \sin 2\theta)d\theta = dt$

Then, $I_1 - I_2 = \int_1^1 f(t)dt = 0$

$\therefore\ I_1 = I_2$

9 **(b)**

$(1) I = \int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\, dx$

$$= \int_{-\pi/2}^{\pi/2} \sqrt{\cos x} \sin x\, dx$$

$$= 2\int_0^{\pi/2} \sqrt{\cos x} \sin x\, dx$$

$$= -\frac{4}{3}[\cos^{3/2} x]_0^{\pi/2} = \frac{4}{3}$$

$(2) I = \int_0^1 |x-1|dx + \int_1^4 |x-1|dx + \int_0^3 |x-3|dx + \int_3^4 |x-3|dx$

$$= \int_0^1 -(x-1)dx + \int_1^4 (x-1)dx + \int_0^3 -(x-3)dx + \int_3^4 (x-3)dx$$

$= 10$

10 **(a)**

Let $I = \int_{-\pi/4}^{\pi/4} \sin^{-4} x\, dx = \int_{-\pi/4}^{\pi/4} \operatorname{cosec}^{-4} x\, dx$

$$= \int_{-\pi/4}^{\pi/4} (1+\cot^2 x)\operatorname{cosec}^2 x\, dx$$

Put $\cot x = t$

$\Rightarrow\ -\operatorname{cosec}^2 x\, dx = dt$

$$\therefore\ I = -\int_{-1}^{1} (1+t^2)\, dt = -2\int_0^1 (1+t^2)\, dt$$

$$= -2\left[t + \frac{t^3}{3}\right]_0^1 = -2\left[1+\frac{1}{3}\right] = -\frac{8}{3}$$

11 **(d)**

We have,

$$\lim_{x\to\infty} \frac{\left(\int_0^x e^x\, dx\right)^2}{\int_0^x e^{2x^2}\, dx} = \lim_{x\to\infty} \frac{(e^x-1)^2}{\int_0^x e^{2x^2}\, dx}$$

$= \lim_{x\to\infty} \frac{2(e^x-1)e^x}{\int_0^x e^{2x^2}}$ [Using L'Hospital's rule]

$$= 2\lim_{x\to\infty} \frac{e^x - 1}{e^{2x^2 - x}}$$

$= 2\lim_{x\to\infty} \frac{e^x}{e^{2x^2-x}(4x-1)}$ [Using L'Hospital's rule]

$$= 2\lim_{x\to\infty} \frac{1}{e^{2x^2-2x}(4x-1)} = 0$$

12 **(a)**

Given, $\int_{\sin x}^{1} t^2\ f(t)dt = 1 - \sin x$

Now, $\frac{d}{dx}\int_{\sin x}^{1} t^2\ f(t)dt = \frac{d}{dx}(1-\sin x)$

$\Rightarrow [1^2 f(1)].(0) - (\sin^2 x).f(\sin x).\cos x = -\cos x$

[by Leibnitz formula]

$\Rightarrow$ Put $\sin x = 1/\sqrt{3}$

$$\therefore\ f\left(\frac{1}{\sqrt{3}}\right) = \left(\sqrt{3}\right)^2 = 3$$

13 **(b)**

We have,

$f(a-x)+f(a+x)=0$

$\Rightarrow f(2a-x)+f(x)=0$ [On replacing $x$by $x-a$]

$\Rightarrow f(2a-x)=-f(x)$

$\because \int_0^{2a} f(x)dx=\int_0^a \{f(x)+f(2a-x)\}\,dx$

$\therefore \int_0^{2a} f(x)\,dx=\int_0^a \{f(x)-f(x)\}\,dx=0$

14 **(a)**

$\int \frac{dx}{\sin(x-a)\sin(x-b)}$

$=\frac{1}{\sin(a-b)}\int \frac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)}\,dx$

$=\frac{1}{\sin(a-b)}$

$\int \frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\,dx$

$=\frac{1}{\sin(a-b)}\left[\int \cot(x-a)dx-\int \cot(x-b)dx\right]$

$=\frac{1}{\sin(a-b)}[\log\sin(x-a)-\log\sin(x-b)]+c$

$=\frac{1}{\sin(a-b)}\log\left|\frac{\sin(x-a)}{\sin\,(x-b)}\right|+c$

15 **(b)**

Putting $2+x=t^2$, we get

$I=\int \sqrt{\frac{5-x}{2+x}}dx=2\int \sqrt{7-t^2}\,\,dt$

$\Rightarrow I=t\sqrt{7-t^2}+7\sin^{-1}\frac{t}{\sqrt{7}}+C$

$\Rightarrow I=\sqrt{x+2}\sqrt{5-x}+7\sin^{-1}\frac{\sqrt{x+2}}{7}+C$

16 **(a)**

We have,

$I=\int \frac{2x^2+3}{(x^2-1)(x^2+4)}dx=\int \frac{1}{x^2-1}dx+\int \frac{1}{x^2+4}dx$

$\Rightarrow I=\frac{1}{2}\log\left(\frac{x-1}{x+1}\right)+\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+C$

$\Rightarrow I=-\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)+\frac{1}{2}\tan^{-1}x+C$

$\therefore a=-1/2$and$b=1/2$

17 **(c)**

$\int e^x\frac{(x-1)}{x^2}\,dx$

$=\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)\,dx$

$= \frac{e^x}{x} + c$

18 **(d)**

We have,

$$I = \int_0^{3\alpha} \operatorname{cosec}(x-\alpha)\operatorname{cosec}(x-2\alpha)dx$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\int_0^{3\alpha} \frac{\sin\alpha}{\sin(x-\alpha)\sin(x-2\alpha)}dx$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\int_0^{3\alpha} \frac{\sin\{(x-\alpha)-(x-2\alpha)\}}{\sin(x-\alpha)\sin(x-2\alpha)}dx$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\int_0^{3\alpha} \cot(x-2\alpha)-\cot(x-\alpha)\,dx$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\left[\log\frac{\sin(x-2\alpha)}{\sin(x-\alpha)}\right]_0^{3\alpha}$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\left[\log\frac{\sin\alpha}{\sin 2\alpha} - \log\frac{\sin 2\alpha}{\sin\alpha}\right]$$

$$\Rightarrow I = \frac{1}{\sin\alpha}\left[\log\left(\frac{\sin\alpha}{2\sin\alpha\cos\alpha}\right)\right] = \frac{2}{\sin\alpha}\log\left(\frac{1}{2}\sec\alpha\right)$$

$$\Rightarrow I = 2\operatorname{cosec}\alpha\log\left(\frac{1}{2}\sec\alpha\right)$$

19 **(a)**

$$\int_0^3 \frac{3x+1}{x^2+9}\,dx = \frac{3}{2}\int_0^3 \frac{2x}{x^2+9}\,dx + \int_0^3 \frac{1}{x^2+9}\,dx$$

$$= \frac{3}{2}[\log(x^2+9)]_0^3 + \frac{1}{3}\left[\tan^{-1}\frac{x}{3}\right]_0^3$$

$$= \frac{3}{2}[\log 18 - \log 9] + \frac{1}{3}[\tan^{-1}(1) - \tan^{-1}(0)]$$

$$= \frac{3}{2}[\log 2] + \frac{\pi}{12}$$

$$= \log(2\sqrt{2}) + \frac{\pi}{12}$$

20 **(a)**

Let $I = \int \frac{dx}{\sqrt{(1-x)(x-2)}}$

$$= \int \frac{dx}{\sqrt{-x^2+3x-2)}} = \int \frac{dx}{\sqrt{\frac{1}{4} - \left(x-\frac{3}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{\left(x-\frac{3}{2}\right)}{\frac{1}{2}}\right) + c = \sin^{-1}(2x-3) + c$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	C	B	D	B	D	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	B	A	B	A	C	D	A	A