DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. :2

Topic :-DIFFERENTIAL EQUATIONS

1 **(c)**

Given, $\frac{dy}{dx} - \frac{2}{x} y = x^2 e^x$

$\therefore$ IF $= e^{-\int \frac{2}{x} dx} = e^{-\log x^2} = \frac{1}{x^2}$

$\therefore$ Complete solution is $\frac{y}{x^2} = \int \frac{x^2 e^x}{x^2} dx + c$

$\Rightarrow \frac{y}{x^2} = e^x + c$

$\Rightarrow y = x^2(e^x + c)$

When $y = 0, x = 1$, then $c = -e$

$\therefore y = x^2(e^x - e)$

2 **(b)**

Given, $\frac{dy}{dx} + \frac{2x}{1+x^2} . y = \frac{4x^2}{1+x^2}$

$\therefore$ IF $= e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2) = (1+x^2)}$

$\therefore$ Complete solution is

$y.(1 + x^2) = \int (1 + x^2) . \frac{4x^2}{1+x^2} dx$

$\Rightarrow y(1 + x^2) = \frac{4x^3}{3} + c_1$

$\Rightarrow 3y(1 + x^2) = 4x^3 + c$

3 **(a)**

Given, $k = PQ$ =length of normal

$\Rightarrow k = y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$\Rightarrow \frac{k^2}{y^2} = 1 + \left(\frac{dy}{dx}\right)^2$

$\therefore y \frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$

4 **(a)**

We have,

$y_1 y_3 = 3 y_2^2 \Rightarrow \frac{y_3}{y_2} = 3\frac{y_2}{y_1}$

Integrating both sides, we get

$\log y_2 = 3 \log y_1 + \log c_1$

$\Rightarrow y_2 = c_1 y_1^3 \Rightarrow \frac{y_2}{y_1^3} = c_1 \Rightarrow \frac{d\, y_1}{y_1^3} = c_1$

Integrating both sides w.r.t. x, we get

$-\frac{1}{2y_1^2} = c_1 x + c_2$

$\Rightarrow y_1^2 = \frac{1}{(-2\, c_1)x + (-2\, c_2)}$

$\Rightarrow y_1^2 = \frac{1}{ax+b}$, where $a = -2c_1, b = -2\, c_2$

$\Rightarrow y_1 = \frac{1}{\sqrt{ax+b}}$

Integrating both sides w.r.t. x, we get

$y = \frac{2}{a}\sqrt{ax+b} + c_3$

$\Rightarrow \frac{ay - c_3}{2} = \sqrt{ax+b}$

$\Rightarrow ax + b = \left(\frac{ax - c_3}{2}\right)^2$

$\Rightarrow x = \frac{a}{4}y^2 - \frac{c^3}{2}y + \frac{1}{a}\left(\frac{c_3^2}{4} - b\right) \Rightarrow x = A_1y^2 + A_2y + A_3,$

where $= A_1 = \frac{a}{4}, A_2 = -\frac{c_3}{2}$ and $A_3 = \frac{1}{a}\left(\frac{c_3^2}{4} - b\right)$

5 **(a)**

Here, $x = A\cos 4t + B\sin 4t$

On differentiating w. r. t. t, we get

$\frac{dx}{dt} = -4A\sin 4t + 4B\cos 4t$

Again, on differentiating w. r. t. t, we get

$\frac{d^2x}{dt^2} = -16A\cos 4t - 16B\sin 4t$

$= -16(A\cos 4t + B\sin 4t)$

$\Rightarrow \frac{d^2x}{dt^2} = -16x$

6 **(a)**

We have,

$y = c_1 + c_2e^x + c_3e^{-2x+c_4}$

$\Rightarrow y = c_1 + c_2e^x + c_3e^{-2x} \cdot e^{c_4}$

$\Rightarrow y = c_1 + c_2e^x + c_3'e^{-2x}$, where $c_3' = c_3e^{c_4}$

It is an equation containing three arbitrary constants. So, the associated differential equation is of order 3

7 **(b)**

Equation of parabolas family can be taken as

$x = ay^2 + by + c$

Differentiating $w.r.t.$, y we get

$\frac{dx}{dy} = 2ay + b$

$\Rightarrow \frac{d^2x}{dy^2} = 2a \quad \Rightarrow \frac{d^3x}{dy^3} = 0$

8 **(a)**

Given $\frac{1-y}{y^2}\,dy + \frac{1+x}{x^2}\,dx = 0$

$\Rightarrow \int\left(\frac{1}{y^2} - \frac{1}{y}\right)dy + \int\left(\frac{1}{x^2} + \frac{1}{x}\right)\,dx = 0$

$\Rightarrow \log\left(\frac{x}{y}\right) = \frac{1}{x} + \frac{1}{y} + c$

9 **(b)**

Given, $\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}$

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\,\frac{dv}{dx}$

$\therefore v + x\frac{dv}{dx} = \frac{\sqrt{x^2+v^2x^2}+vx}{x}$

$\Rightarrow \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$

$\Rightarrow \quad \log\left(v+\sqrt{1+v^2}\right)=\log x+\log c$

$\Rightarrow \quad \log\left(\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right)=\log cx$

$\Rightarrow \quad y+\sqrt{x^2+y^2}=cx^2$

10 **(a)**

Given, $\frac{dy}{dx}=\frac{x\log x^2+x}{\sin y+y\cos y}$

$\Rightarrow \quad (\sin y+y\cos y)dy=(x\log x^2+x)dx$

$\left(\frac{d}{dy}y\sin y\right)dy=\left(\frac{d}{dx}x^2\log x\right)dx$

$\Rightarrow \quad y\sin y=x^2\log x+c$

11 **(d)**

Given, $x(1-x^2)dy+(2x^2y-y-ax^3)dx=0$

$\Rightarrow \quad \frac{dy}{dx}+\frac{(2x^2-1)}{x(1-x)}y=\frac{ax^3}{(1-x^2)}$

Here, $P=\frac{2x^2-1}{x(1-x^2)}$

12 **(c)**

We have,

$\frac{dy}{dx}+\frac{y}{x}=x^2 \Rightarrow x\frac{dy}{dx}+y=x^3 \Rightarrow \frac{d}{dx}(xy)=x^3$

Integrating, we get

$xy=\frac{x^4}{4}+C \Rightarrow y=\frac{x^3}{4}+C\,x^{-1}$

13 **(c)**

Let $\quad x^2+y^2-2gx=0 \quad$ ....(i)

$\Rightarrow \quad 2x+2y\frac{dy}{dx}-2g=0$

$\Rightarrow \quad 2g=\left(2x+2y\frac{dy}{dx}\right)$

On putting the value of 2g in eq. (i), we get

$x^2+y^2-\left(2x+2y\frac{dy}{dx}\right)x=0$

$\Rightarrow \quad y^2=x^2+2xy\frac{dy}{dx}$

14 **(d)**

Given differential equation can be written as

$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0$

$\Rightarrow \quad (m^2-3m+2)y=0$

$\Rightarrow \quad (m-1)(m-2)y=0$

$\Rightarrow \quad m=1,2$

$\therefore$ Solution is $y=c_1e^x+c_2e^{2x}$

$y'=c_1e^x+2c_2e^{2x}$

From given condition

$y(0)=1$

$\Rightarrow c_1+c_2=1$.....(i)

And y'(0)=0

$\Rightarrow c_1+c_2=1\ldots\ldots(ii)$

On solving Eqs. (i) and (ii)we get

$-c_2=1$

$\Rightarrow \quad c_2=-1$

And $c_1 = 2$

$\therefore y = 2e^x - e^{2x}$

$\therefore$ $at\ x = \log_e 2$

$y = 2e^{log 2} - e^{2\log 2}$

$= 2 \times 2 - 2^2 = 0$

15 **(b)**

The equation of straight line touching the given circle is

$$x \cos\theta + y \sin\theta = a \quad \text{...(i)}$$

On differentiating w. r. t. x, regarding θ as a constant

$\Rightarrow \quad \cos\theta + \frac{dy}{dx}\sin\theta = 0 \quad$ ...(ii)

From eqs. (i) and (ii), we get

$$\cos\theta = \frac{a\frac{dy}{dx}}{x\frac{dy}{dx} - y} \text{ and } \sin\theta = -\frac{a}{x\frac{dy}{dx} - y}$$

$\therefore \quad \cos^2\theta + \sin^2\theta = 1$

$\therefore \quad \frac{a^2\left(\frac{dy}{dx}\right)^2 + a^2}{\left(x\frac{dy}{dx} - y\right)^2} = 1$

$\Rightarrow \quad \left(y - x\frac{dy}{dx}\right)^2 = a^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$

16 **(a)**

The given differential equation can be rewritten as

$\Rightarrow \quad \left(\frac{1}{y^2} - \frac{1}{y}\right)dy = -\left(\frac{1}{x^2} + \frac{1}{x}\right)dx$

$\Rightarrow \quad -\frac{1}{y} - \log y = -\left(-\frac{1}{x} + \log x\right) + c \quad$ [integrating]

$\Rightarrow \quad \log\left(\frac{x}{y}\right) = \frac{1}{x} + \frac{1}{y} + c$

17 **(a)**

We have, $(xy - x^2) = y^2$

$\Rightarrow \quad y^2\frac{dx}{dy} = xy - x^2$

$\Rightarrow \quad \frac{1}{x^2}\frac{dx}{dy} - \frac{1}{x}.\frac{1}{y} = -\frac{1}{y^2}$

Put $\frac{1}{x} = v \quad \Rightarrow -\frac{1}{x^2}\frac{dx}{dy} = \frac{dv}{dy}$

$\therefore \quad \frac{dv}{dy} + \frac{v}{y} = \frac{1}{y^2}$, which is linear

$\therefore \quad IF = e^{\int\frac{1}{y}dy} = e^{\log y} = y$

$\therefore$ The solution is $vy = \int\frac{1}{y^2}\,y\,dy + c$

$\Rightarrow \quad \frac{y}{x} = \log y + c$

$\Rightarrow \quad y = x(\log y + c)$

This passes through the point (-1,1)

$\therefore \quad 1 = -1(\log 1 + c)$

ie., $\quad c = -1$

thus, the equation of the curve is

$$y = x(\log y - 1)$$

18 **(d)**

Given, $y = 2e^{2x} - e^{-x}$

$\Rightarrow \quad y_1 = 4e^{2x} + e^{-x}$

$\Rightarrow \quad y_2 = 8e^{2x} - e^{-x}$
$\Rightarrow \quad y_2 = 4e^{2x} + e^{-x} + 4e^{2x} - 2e^{-x}$
$\Rightarrow \quad y_2 = y_1 + 2(2e^{2x} - e^{-x})$
$\Rightarrow \quad y_2 = y_1 + 2y$
$\Rightarrow \quad y_2 = y_1 + 2y$
$\Rightarrow \quad y_2 - y_1 - 2y = 0$

19 **(c)**

Given equation is $\frac{dy}{dx} - y = 1 \Rightarrow \frac{dy}{1+y} = dx$

On integrating both sides, we get

$$\int \frac{1}{1+y} dy = \int dx$$

$\Rightarrow \log(1+y) = x + c$
$\Rightarrow 1 + y = e^x \cdot e^c \quad$ ...(i)
At $x = 0, y = -1$
Then $1 - 1 = e^0 \cdot e^c \Rightarrow e^c = 0$
On putting the value of e^c in Eq. (i).
Therefore, solution becomes
$1 + y = e^x \times 0 \Rightarrow y(x) = -1$

20 **(d)**

Let family of circles be

$$(x-\alpha)^2 + (y-2)^2 = 5^2$$

$\Rightarrow \quad x^2 + \alpha^2 - 2\alpha x + y^2 - 21 - 4y = 0 \quad$ ...(i)

$\Rightarrow \quad 2x - 2\alpha + 2y\frac{dy}{dx} - 4\frac{dy}{dx} = 0$

$\Rightarrow \quad \alpha = x + \frac{dy}{dx}(y-2)$

On putting the value of α in Eq. (i), we get

$$\left(x - x - \frac{dy}{dx}(y-2)\right)^2 + (y-2)^2 = 5^2$$

$\Rightarrow \quad \left(\frac{dy}{dx}\right)^2 (y-2)^2 = 25 - (y-2)^2$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	A	A	A	A	B	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	C	C	D	B	A	A	D	C	D