

DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTION

SUBJECT : MATHS
DPP NO. :2

Topic :-VECTOR ALGEBRA

1 (d)

$$\begin{aligned} & (\vec{a} - \vec{b}) \cdot (\vec{b} + \vec{c}) \times (\vec{c} + \vec{a}) \\ &= (\vec{a} - \vec{b}) \cdot [\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{c} + \vec{c} \times \vec{a}] \\ &= \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (\vec{c} \times \vec{a}) - \vec{b} \cdot (\vec{b} \times \vec{c}) - \vec{b} \cdot (\vec{b} \times \vec{a}) - \vec{b} \cdot (\vec{c} \times \vec{a}) \\ &= \vec{a} \cdot (\vec{b} \times \vec{c}) - \vec{a} \cdot (\vec{b} \times \vec{c}) \\ &= [\vec{a} \vec{b} \vec{c}] - [\vec{a} \vec{b} \vec{c}] = 0 \end{aligned}$$

2 (b)

$\therefore \vec{a}, \vec{b}$ and \vec{c} are coplanar vectors, so $2\vec{a} - \vec{b}, 2\vec{b} - \vec{c}$ and $2\vec{c} - \vec{a}$ are also coplanar. Thus

$$[2\vec{a} - \vec{b}, 2\vec{b} - \vec{c}, 2\vec{c} - \vec{a}] = 0$$

3 (b)

Clearly, angle between \vec{a} and $\vec{b} = \frac{\pi}{2}$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0$$

$$\therefore |\vec{a} + \vec{b}|^2 = a^2 + b^2 + 2\vec{a} \cdot \vec{b}$$

$$= 1 + 1 + 0 = 2$$

$$\Rightarrow |\vec{a} + \vec{b}| = \sqrt{2}$$

5 (d)

$$\text{Given, } (\vec{a} \times \vec{b}) \times \vec{c} = -\frac{1}{4} |\vec{b}| |\vec{c}| \vec{a}$$

$$\Rightarrow (\vec{c} \cdot \vec{a}) \vec{b} - (\vec{c} \cdot \vec{b}) \vec{a} = -\frac{1}{4} |\vec{b}| |\vec{c}| \vec{a}$$

On comparing both sides, we get

$$(\vec{c} \cdot \vec{a}) \vec{b} = 0$$

$$|\vec{c}| |\vec{a}| \cos \theta = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

6 (c)

$$\text{Now, } (\hat{i} + \hat{j} + \hat{k}) \times (\hat{i} + \hat{j}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= \hat{i}(-1) + \hat{j}(1) + \hat{k}(0) = -\hat{i} + \hat{j}$$

$$\text{and } |(\hat{i} + \hat{j} + \hat{k}) \times (\hat{i} + \hat{j})| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Vector perpendicular to both of the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + \hat{j}$

$$= \frac{(\hat{i} + \hat{j} + \hat{k}) \times (\hat{i} + \hat{j})}{|(\hat{i} + \hat{j} + \hat{k}) \times (\hat{i} + \hat{j})|}$$

$$= \frac{-\hat{i} + \hat{j}}{\sqrt{2}} = \frac{-1}{\sqrt{2}} (\hat{i} - \hat{j})$$

$$= c(\hat{i} - \hat{j}), c \text{ is a scalar.}$$

7 (b)

It is given that $(\vec{a} + \vec{b}) \parallel \vec{c}$ and $(\vec{c} + \vec{a}) \parallel \vec{b}$
 $\therefore (\vec{a} + \vec{b}) \times \vec{c} = 0$ and $(\vec{c} + \vec{a}) \times \vec{b} = 0$
 $\Rightarrow \vec{a} \times \vec{c} + \vec{b} \times \vec{c} = 0$ and $\vec{c} \times \vec{b} + \vec{a} \times \vec{b} = 0$
 $\Rightarrow \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

Hence, $\vec{a}, \vec{b}, \vec{c}$ form the sides of a triangle

8 (a)

\therefore Displacement, $\vec{AB} = (3 - 2)\hat{i} + (1 + 1)\hat{j} + (2 - 1)\hat{k}$
 $= \hat{i} + 2\hat{j} + \hat{k}$

and force, $\vec{F} = \frac{\sqrt{6}(\hat{i} + 2\hat{j} + \hat{k})}{\sqrt{6}}$

$= (\hat{i} + 2\hat{j} + \hat{k})$

\therefore Work done $= \vec{F} \cdot \vec{AB} = (1 + 2\hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} + \hat{k}) = 6$

9 (c)

let $\vec{a} = l\hat{i} + m\hat{j} + n\hat{k}$ makes an angle $\frac{\pi}{4}$ with z-axis

Also, $l^2 + m^2 + n^2 = 1$

Here, $n = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$, $l^2 + m^2 = \frac{1}{2}$ (i)

$\therefore \vec{a} = l\hat{i} + m\hat{j} + \frac{\hat{k}}{\sqrt{2}}$

$\Rightarrow \vec{a} + \hat{i} + \hat{j} = (l + 1)\hat{i} + (m + 1)\hat{j} + \frac{\hat{k}}{\sqrt{2}}$

$\Rightarrow |\vec{a} + \hat{i} + \hat{j}| = \sqrt{(l + 1)^2 + (m + 1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$

$\Rightarrow 1 = l^2 + m^2 + 2 + 2l + 2m + \frac{1}{2}$

$\Rightarrow l + m = -1$ (From Eq. (i))

$\Rightarrow l^2 + m^2 + 2lm = 1$

$\Rightarrow 2lm = \frac{1}{2}$

$\Rightarrow l = m = -\frac{1}{2}$

($\because l = m = \frac{1}{2}$, is not satisfied the given equation)

$\therefore \vec{a} = -\frac{\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{\sqrt{2}}$

10 (b)

Given, $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 (\sin^2\theta + \cos^2\theta) = 144$

$\Rightarrow 16|\vec{b}|^2 = 144$

$\Rightarrow |\vec{b}| = 3$

11 (c)

Since, $m\vec{a}$ is a unit vector, if and only, if

$|m\vec{a}| = 1 \Rightarrow |m||\vec{a}| = 1 \Rightarrow m|\vec{a}| = 1$

$\Rightarrow m = \frac{1}{|\vec{a}|}$

12 (b)

Resultant force \vec{F} is given by

$$\vec{F} = (2\hat{i} - 5\hat{j} + 6\hat{k}) - (-\hat{i} + 2\hat{j} - \hat{k}) = \hat{i} - 3\hat{j} + 5\hat{k}$$

Let \vec{d} be the displacement vector. Then,

$$\vec{d} = A\vec{B}$$

$$\Rightarrow \vec{d} = (6\hat{i} + \hat{j} - 3\hat{k}) - (4\hat{i} - 3\hat{j} - 2\hat{k}) = 2\hat{i} + 4\hat{j} - \hat{k}$$

$\therefore W = \text{Work done}$

$$\Rightarrow W = \vec{F} \cdot \vec{d}$$

$$\Rightarrow W = (\hat{i} - 3\hat{j} + 5\hat{k}) \cdot (2\hat{i} + 4\hat{j} - \hat{k})$$

$$\Rightarrow W = 2 - 12 - 5 = -15 \text{ units}$$

13 (d)

Since, P, Q, R are collinear. Therefore,

$$\vec{PQ} = m \vec{QR} \text{ for same scalar } m$$

$$\Rightarrow -2\hat{j} = m[(a-1)\hat{i} + (\hat{b} + 1)\hat{j} + c\hat{k}] \text{ for some non-zero scalar } m$$

$$\Rightarrow (a-1)m = 0, (b+1)m = -2, cm = 0$$

$$\Rightarrow a = 1, c = 0, b \in R$$

14 (b)

The direction cosines of a vector making equal angles with the coordinate axes are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Therefore, the unit vector along the vector making equal angles with the coordinate axes is

$$\vec{b} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

$$\therefore \text{Projection of } \vec{a} \text{ on } \vec{b} = \vec{a} \cdot \vec{b}$$

$$= (4\hat{i} - 3\hat{j} + 2\hat{k}) \cdot \left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right) = \frac{4 - 3 + 2}{\sqrt{3}} = \sqrt{3}$$

15 (a)

$$[2\hat{i} \ 3\hat{j} \ -5\hat{k}]$$

$$= -30 [\hat{i} \ \hat{j} \ \hat{k}]$$

$$= -30 (\because [\hat{i} \ \hat{j} \ \hat{k}] = 1)$$

16 (b)

We have,

$$(\vec{a} \times \vec{b}) \times (\vec{a} \times \vec{c}) \cdot \vec{d}$$

$$= \{((\vec{a} \times \vec{b}) \cdot \vec{c})\vec{a} - ((\vec{a} \times \vec{b}) \cdot \vec{a})\vec{c}\} \cdot \vec{d}$$

$$= \{[\vec{a} \ \vec{b} \ \vec{c}]\vec{a} - 0\} \cdot \vec{d} = [\vec{a} \ \vec{b} \ \vec{c}](\vec{a} \cdot \vec{d})$$

17 (d)

$$\text{Resultant force } \vec{F} = (2\hat{i} - 5\hat{j} + 6\hat{k}) + (-\hat{i} + 2\hat{j} - \hat{k})$$

$$= \hat{i} - 3\hat{j} + 5\hat{k}$$

$$\text{and displacement, } \vec{d} = (6\hat{i} + \hat{j} - 3\hat{k}) - (4\hat{i} - 3\hat{j} - 2\hat{k})$$

$$= 2\hat{i} + 4\hat{j} - \hat{k}$$

$$\therefore \text{work done } W = \vec{F} \cdot \vec{d}$$

$$= (\hat{i} - 3\hat{j} + 5\hat{k}) \cdot (2\hat{i} + 4\hat{j} - \hat{k})$$

$$= -15$$

$$= 15 \text{ units [neglecting -ve sign]}$$

18 (a)

The resultant force is given by

$$\vec{F} = 6 \frac{(\hat{i} - 2\hat{j} + 2\hat{k})}{\sqrt{1+4+4}} + 7 \frac{(2\hat{i} - 3\hat{j} - 6\hat{k})}{\sqrt{4+9+36}} = 4\hat{i} - 7\hat{j} - 2\hat{k}$$

$$\vec{d} = \text{Displacement} = \vec{PQ}$$

$$\vec{d} = (5\hat{i} - \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} - 3\hat{k}) = 3\hat{i} + 4\hat{k}$$

$$\therefore \text{Work done} = \vec{F} \cdot \vec{d} = 12 + 0 - 8 = 4 \text{ units}$$

19 (c)

$$\begin{aligned}
 &\text{We know, } [\vec{b} \times \vec{c} \ \vec{c} \times \vec{a} \ \vec{a} \times \vec{b}] \\
 &= (\vec{b} \times \vec{c}) \cdot [(\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b})] \\
 &= (\vec{b} \times \vec{c}) \cdot [((\vec{c} \times \vec{a}) \cdot \vec{b}) \vec{a} - ((\vec{c} \times \vec{a}) \cdot \vec{a}) \vec{b}] \\
 &= (\vec{b} \times \vec{c}) \cdot ([\vec{c} \vec{a} \vec{b}] \vec{a} - [\vec{c} \vec{a} \vec{a}] \vec{b}) \\
 &= (\vec{b} \times \vec{c}) \cdot \vec{a} [\vec{a} \vec{b} \vec{c}] - 0 \\
 &= [\vec{a} \vec{b} \vec{c}] [\vec{a} \vec{b} \vec{c}] \\
 &= [\vec{a} \vec{b} \vec{c}]^2
 \end{aligned}$$

20 (d)

$$\begin{aligned}
 &\therefore \vec{QP} \text{ is parallel to } \vec{AB} \text{ and } \vec{DC}. \\
 &\therefore \vec{AB} + \vec{DC} = \vec{QP} + \vec{QP} = 2\vec{QP}
 \end{aligned}$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	D	B	B	B	D	C	B	A	C	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	D	B	A	B	D	A	C	D