DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. :2

Topic :-THREE DIMENSIONAL GEOMETRY

1 **(b)**

Given equation of lines are

$\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1}=k$ [say] ..... (i)

and $\frac{x+3}{-36}=\frac{y-3}{2}=\frac{z-6}{4}$ (ii)

Any point on the line (i) is $P(3k+5,-k+7,k-2)$

This point is satisfied the Eq. (ii),

$\therefore \frac{3k+5+3}{-36}=\frac{-k+7-3}{2}=\frac{k-2-6}{4}$

$\Rightarrow \frac{3k+8}{-36}=\frac{-k+4}{2}=\frac{k-8}{4}$

$\Rightarrow 3k+8=18k-72 \Rightarrow k=\frac{16}{3}$

$\therefore P\left(16+5,-\frac{16}{3}+7,\frac{16}{3}-2\right)$

$ie, P\left(21,\frac{5}{3},\frac{10}{3}\right)$

2 **(a)**

We have, $\vec{A}B=-2\hat{\imath}-3\hat{\jmath}-6\hat{k}$

So, vector equation of the plane is

$\{\vec{r}-(\hat{\imath}-2\hat{\jmath}-4\hat{k})\}\cdot\vec{A}B=0$

$\Rightarrow \vec{r}\cdot(-2\hat{\imath}-3\hat{\jmath}-6\hat{k})=(\hat{\imath}-2\hat{\jmath}-4\hat{k})\cdot(-2\hat{\imath}-3\hat{\jmath}-6\hat{k})$

$\Rightarrow -2x-3y-6z=-2+6+24 \Rightarrow 2x+3y+6x+28=0$

3 **(c)**

Let point is (α,β,γ)

$\therefore (\alpha-\alpha)^2+\beta^2+\gamma^2=\alpha^2+(\beta-b)^2+\gamma^2$

$=\alpha^2+\beta^2+(\gamma-c)^2$

$=\alpha^2+\beta^2+\gamma^2$

We get, $\alpha=\frac{a}{2}, \beta=\frac{b}{2}$ and $\gamma=\frac{c}{2}$

$\therefore$ Required point is $\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$

4 **(b)**

Let equation of plane is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$, then $A(\alpha,0,0), B(0,\beta,0)$ and $C(0,0,\gamma)$ are the points on coordinate axes

Since, the centroid of a triangle is (1, 2, 4)

Now, $\frac{\alpha}{3}=1$

$\therefore \alpha=3, \frac{\beta}{3}=2 \Rightarrow \beta=6$

And $\frac{\gamma}{3}=4 \Rightarrow \gamma=12$

$\therefore$ Equation of plane is

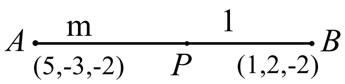
$\frac{x}{3} + \frac{y}{6} + \frac{z}{12} = 1$
$\Rightarrow\ 4x + 2y + z = 12$

5 **(c)**

$\because$ Vertices of ΔABC are $A(-1, 3, 2), B(2, 3, 5)$ and $C(3, 5, -2)$
$\Rightarrow AB = \sqrt{9 + 0 + 9} = \sqrt{18}$
$CA = \sqrt{16 + 4 + 16} = 6$
And $BC = \sqrt{1 + 4 + 49} = \sqrt{54}$
$\because AB^2 + CA^2 = BC^2$
$\Delta\ ABC$ is right angled triangle at A
$\therefore\ \angle A = 90°$

7 **(a)**

Let the point $P(x, y, z)$ divides the line joining the points A and B in the ratio m: 1.



Since, point P is on XOZ-plane
$\therefore y$ coordinate $= 0$
$\Longrightarrow\ \frac{2m - 3}{m + 1} = 0\ \Longrightarrow m = \frac{3}{2}$
Now, $\quad x = \frac{3 + 2 \times 5}{3 + 2} = \frac{13}{5}$
and $z = \frac{3 \times (-2) + 2 \times (-2)}{5} = -2$
$\therefore$ Required points is $\left(\frac{13}{5}, 0, -2\right)$

8 **(d)**

Let the equation of plane is $-\frac{x}{6} + \frac{y}{3} + \frac{z}{4} = 1$
$\therefore$ The perpendicular distance from origin to the above plane
$= \frac{|0 + 0 + 0 - 1|}{\sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2}}$
$= \frac{1}{\sqrt{\frac{4 + 16 + 9}{144}}}$
$= \frac{12}{\sqrt{29}}$

9 **(b)**

Equation of plane is $a(x - 1) + b((y + 1) + cz = 0$
$(\because$ plane is passing through$(1,2,-1))$
Above plane also passing through $(0, 2, -1)$
$\therefore\ -a + 3b - c = 0$
Also $2a - b + 3c = 0$
$\Longrightarrow \frac{a}{8} = \frac{b}{1} = \frac{c}{-5}$
Hence, equation of plane is
$8x + y - 5z - 7 = 0$

10 **(c)**

$\because$ Mid point of line joining (2, 3, 4) and (6, 7, 8) is (4, 5, 6). This point satisfied the equation
$x + y + z - 15 = 0$
$\therefore\ x + y + z - 15 = 0$ is required equation of plane

11 **(c)**

The distance between given points

$= \sqrt{(2-1)^2 + (2-4)^2 + (3-5)^2}$

$= \sqrt{1+4+4} = 3$

12 **(b)**

Equation of plane through (1, 2, 3) is

$a(x-1) + b(y-2) + c(z-3) = 0$ ....(i)

$\because$ It passes through $(-1, 4, 2)$ and $(3, 1, 1)$

$\therefore -2a + 2b - c = 0$ and $2a - b - 2c = 0$

$\Rightarrow \frac{a}{-5} = \frac{b}{-6} = \frac{c}{-2}$

$\therefore$ Equation of plane is

$-5x - 6y - 2z + 5 + 12 + 6 = 0$

$\Rightarrow 5x + 6y + 2z - 23 = 0$

Alternate

Equation plane is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-3 \\ -2 & 2 & -1 \\ 2 & -1 & -2 \end{vmatrix} = 0$$

$\Rightarrow (x-1)(-4-1) - (y-2)(4+2) + (z-3)(2-4) = 0$

$\Rightarrow -5x + 5 - 6y + 12 - 2z + 6 = 0$

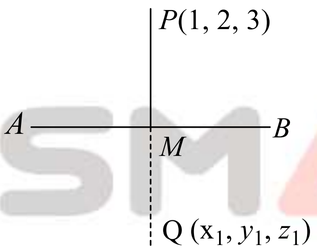
$\Rightarrow 5x + 6y + 2z - 23 = 0$

13 **(a)**

Given planes are parallel to each other but only $x + y + 3z - 6 = 0$ is equidistant from $x + 2y + 3z - 5 = 0$ and $x + 2y + 3z - 7 = 0$ having distance $\frac{1}{\sqrt{14}}$

14 **(c)**

Equation of given line is $\frac{x}{2} = \frac{y-1}{3} = \frac{z-1}{3} = k$ (say)



Any point on the line is $M(2k, 3k+1, 3k+1)$

Direction ratio of PM are $(2k-1, 3k-1, 3k-2)$ since, the line PMis perpendicular to AB

$\therefore 2(2k-1) + 3(3k-1) + 3(3k-2) = 0$

$\Rightarrow 22k - 11 = 0$

$\Rightarrow k = \frac{1}{2}$

$\therefore$ Point M is $\left(1, \frac{5}{2}, \frac{5}{2}\right)$

Let the image of P about the line AB is Q, where M is the mid point of PQ

$\therefore \frac{x_1+1}{2} = 1, \frac{y_1+2}{2} = \frac{5}{2}, \frac{z_1+3}{2} = \frac{5}{2}$

$\Rightarrow x_1 = 1, y_1 = 3, z_1 = 2$

15 **(b)**

The equation of straight line passing through origin and direction cosine (l, m, n) is

$\frac{x}{l} = \frac{y}{m} = \frac{z}{n} = r$ (say)

Coordinates of any point P are (lr, mr, nr)

Here, $l = \frac{-1}{\sqrt{1^2+2^2+2^2}} = \frac{-1}{3}, m = \frac{2}{3}, n = \frac{-2}{3}$

and $r = 3$ (given)

$\therefore$ Coordinates of P are $(-1, 2, -2)$

16 **(b)**

Since, the given sphere touching the three coordinates planes. So, it is clear that centre is (a, a, a) and radius is a

$\therefore$ The equation of sphere at the centre (a, a, a) and radius a is

$(x-a)^2 + (y-a)^2 + (z-a)^2 = a^2$

$\Rightarrow x^2 + y^2 + z^2 - 2ax - 2ay - 2az + 3a^2 = a^2$

$\therefore x^2 + y^2 + z^2 - 2a(x+y+z) + 2a^2 = 0$ is the required equation of sphere

17 **(b)**

Angle between the plane and line is given by

$$\sin\theta = \frac{aa' + bb' + cc'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}$$

$$\therefore \sin\theta = \frac{2\times\frac{3}{4} + 3\times\frac{2}{4} - 4\times\frac{3}{4}}{\sqrt{2^2+3^2+4^2}\sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{2}{4}\right)^2 + \left(\frac{-3}{4}\right)^2}}$$

$$= \frac{\frac{6}{4} + \frac{6}{4} - \frac{12}{4}}{\sqrt{4+9+16}\sqrt{\frac{9}{16} + \frac{4}{16} + \frac{9}{16}}} = 0$$

$\therefore \sin\theta = \sin 0°$

$\Rightarrow \theta = 0°$

18 **(b)**

Given that equation of planes are,

$4x + 4y - 5z = 12$ ...(i)

And $8x + 12y - 13z = 32$ ...(ii)

Let direction ratios of the line are (l, m, n)

$\therefore$ Eqs. (i) and (ii) becomes

$4l + 4m - 5n = 0$ ...(iii)

And $8l + 12m - 13n = 0$ ...(iv)

$\Rightarrow \frac{l}{8} = \frac{m}{12} = \frac{n}{16} \Rightarrow \frac{l}{2} = \frac{m}{3} = \frac{n}{4}$

Now, we take intersection point with $z = 0$ given by

$4x + 4y = 12$ ...(v)

and $8x + 12y = 32$ ...(vi)

On solving Eqs. (v) and (vi), we get (1, 2, 0)

$\therefore$ Required line is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z}{4}$

19 **(b)**

DR's of given line are $(3, -5, 2)$

DR's of normal to the plane $= (1, 3, -\alpha)$

$\therefore$ Line is perpendicular to the normal

$\Longrightarrow 3(1) - 5(3) + 2(-\alpha) = 0$

$\Longrightarrow 3 - 15 - 2\alpha = 0$

$\Longrightarrow 2\alpha = -12$

$\Longrightarrow \alpha = -6$

Also point $(2, 1, -2)$ lies on the plane

$2 + 3 + 6(-2) + \beta = 0$

$\Longrightarrow \beta = 7$

$\therefore (\alpha, \beta) = (-6, 7)$

20 **(b)**

We know

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$

where α, β, γ and δ are the angles with diagonals of cube.

$\therefore 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma + 1 - \sin^2\delta = \frac{4}{3}$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = \frac{8}{3}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	C	B	C	C	A	D	B	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	B	A	C	B	B	B	B	B	B