DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIONS

SUBJECT : MATHS
DPP NO. :3

Topic :-RELATIONS AND FUNCTIONS

1 **(d)**
We observe that
Period of $\sin\frac{\pi x}{2}$ is $\frac{2\pi}{\pi/2} = 4$, Period of $\cos\frac{\pi x}{3}$ is $\frac{2\pi}{\pi/3} = 6$,
and,
Period of $\tan\frac{\pi x}{4}$ is $\frac{\pi}{\pi/4} = 4$
$\therefore$ Period of $f(x) =$ LCM of $(4, 6, 4) = 12$
2 **(c)**
We have,
$f(x) = \lim_{x\to\infty} \frac{x^n + x^{-n}}{x^n + x^{-n}}$
$\Rightarrow f(x) = \lim_{x\to\infty} \frac{x^{2n} - 1}{x^{2n} + 1} = \frac{0-1}{0+1} = -1, \text{if} -1 < x < 1$
If $|x| > 1$, then $x^{2n} \to \infty$ as $n \to \infty$
$\therefore f(x) = \lim_{x\to\infty} \frac{1 - \frac{1}{x^{2n}}}{1 + \frac{1}{x^{2n}}} = \frac{1-0}{1+1}, = 1, \text{if } |x| > 1$
If $|x| = 1$, then $x^{2n} = 1$
$\therefore f(x) = \lim_{x\to\infty} \frac{x^{2n} - 1}{x^{2n} + 1} = \frac{1-1}{1+1} = 0$
Thus, we have
$$f(x) = \begin{cases} -1, & \text{if } |x| < 1 \\ 0, & \text{if } |x| = 1 \\ 1, & \text{if } |x| > 1 \end{cases}$$
3 **(c)**
$R = \{(1,3), (4,2), (2,4), (2,3), (3,1)\}$ is a relation on
$A = \{1, 2, 3, 4\}$, then
(a) since, $(2,4) \in R$ and $(2,3) \in R$, so R is not a function.
(b) since, $(1,3) \in R$ and $(3,1) \in R$ but $(1,1) \notin R$.
So, R is not transitive.
(c) since, $(2,3) \in R$ but $(3,2) \notin R$, so R is not symmetric.
(d) since, $(4,4) \notin R$, so R is not reflexive.
4 **(a)**
We have,
$f(x) = {}^{16-x}C_{2x-1} + {}^{20-3x}P_{4x-5}$
Clearly, $f(x)$ is defined, if
$16 - x \geq 2x - 1 > 0, 20 - 3x \geq 4x - 5 > 0$ and $x \in Z$
$\Rightarrow x \in \{1,2,3,4,5\}, x \in \{2,3\}$ and $x \in Z$
$\Rightarrow x \in \{2,3\}$
$\therefore$ Domain $(f) = \{2,3\}$
5 **(d)**
Given, $f(x) = e^{2ix}$ and $f: R \to C$. Function $f(x)$ is not one-one, because after some values of $x(ie, \pi)$ it will

give the same values.
Also, $f(x)$ is not onto, because it has minimum and maximum values $-1-i$ and $1+i$ respectively.

6 **(a)**

For $f(x)$ to be defined,
$x-4\geq 0$ and $6-x\geq 0 \Rightarrow x\geq 4$ and $x\leq 6$
Therefore, the domain is [4, 6].

7 **(d)**

We have,
$hogof\ (x)=\cos^{-1}(|\sin x|)$
and, $fogoh\ (x)=\sin^2\left(\sqrt{\cos^{-1}x}\right)$
Clearly, $hogof\ (x)\neq fogoh(c)$
Thus, option (a) is not correct
Now,
$gofoh\ (x)=|\sin(\cos^{-1}x)|=\left|\sin\left(\sin^{-1}\sqrt{1-x^2}\right)\right|=\sqrt{1-x^2}$
and, $fohog\ (x)=\sin^2(\cos^{-1}\sqrt{x})=1-\cos^2(\cos^{-1}\sqrt{x})$
$\Rightarrow fohog\ (x)=1-\left\{\cos\left(\cos^{-1}\sqrt{x}\right)\right\}^2=1-x$
$\therefore gofoh\ (x)\neq fohog(x)$
Thus, option (b) is correct
Also,
$hogof(x)=\cos^{-1}(|\sin x|)$ and, $fohog\ (x)=1-x$
$\therefore hogof(x)\neq fohog(x)$
Thus, option (c) is not correct
Hence, option (d) is correct

8 **(a)**

We have,
$$f(x)=\frac{2^x+2^{-x}}{2}$$
$$\therefore f(x+y)f(x-y)=\frac{2^{x+y}+2^{-x-y}}{2}\times\frac{2^{x-y}+2^{-x+y}}{2}$$
$$\Rightarrow f(x+y)f(x-y)=\frac{2^{2x}+2^{-2y}+2^{2y}+2^{-2x}}{4}$$
$$\Rightarrow f(x+y)f(x-y)=\frac{1}{2}\left(\frac{2^{2x}+2^{-2x}}{2}+\frac{2^{2y}+2^{-2y}}{2}\right)$$
$$\Rightarrow f(x+y)\,f\,(x-y)=\frac{1}{2}\{f(2x)+f\,(2y)\}$$

9 **(b)**

$R=\{(a,b):a,b\in N,a-b=3\}$
$=\{[(n+3),n]:n\in N\}$
$=\{(4,1),(5,2),(6,3),\dots.\}$

10 **(a)**

Clearly, $f(x)=\sin^{-1}\left\{\log_3\left(\frac{x}{3}\right)\right\}$ exists if
$$-1\leq\log_3\left(\frac{x}{3}\right)\leq 1\Leftrightarrow 3^{-1}\leq\frac{x}{3}\leq 3^1\Leftrightarrow 1\leq x\leq 9$$
Hence, domain of $f(x)$ is [1, 9]

11 **(c)**

For $f(x)$ to be defined, we must have
$\frac{\sqrt{4-x^2}}{1-x}>0, 4-x^2>0$ and $1-x\neq 0$
$\Rightarrow 1-x>0, 4-x^2>0$ and $1-x\neq 0$
$\Rightarrow x<1, x\in(-2,2)$ and $x\neq 1\Rightarrow x\in(-2,1)$

$\therefore$ Domain $(f) = (-2, 1)$
Now, for $x \in (-2, 1)$, we have

$$-\infty < \log\left(\frac{\sqrt{4-x^2}}{1-x}\right) < \infty$$

$$\Rightarrow -1 \leq \sin\left\{\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right\} \leq 1 \Rightarrow -1 \leq f(x) \leq 1$$

Hence, Range $(f) = [-1, 1]$

12 **(a)**

Given, $f(x) = \frac{ax+b}{cx+d}$ and $fof(x) = x$

$$\Rightarrow \quad f\left(\frac{ax+b}{cx+d}\right) = x$$

$$\Rightarrow \frac{a\left(\frac{ax+b}{cx+d}\right)+b}{c\left(\frac{ax+b}{cx+d}\right)+d} = x$$

$$\Rightarrow \quad \frac{x(a^2+bc)+ab+bd}{x(ac+cd)+bc+d^2} = x$$

$$\Rightarrow \quad d = -a$$

13 **(c)**

If $f: C \to C$ given by $f(x) = \frac{ax+b}{cx+d}$ is a constant function, then
$f(x) =$ Constant $(= \lambda$, say) for all $x \in C$

$$\Rightarrow \frac{ax+b}{cx+d} = \lambda \text{ for all } x \in C$$

$\Rightarrow (a - \lambda c)\, x + (b - \lambda d) = 0$ for all $x \in C$
$\Rightarrow a - \lambda c = 0$ and $b - \lambda d = 0 \Rightarrow \frac{a}{c} = \frac{b}{d} \Rightarrow ad = bc$

14 **(d)**

Periods of $\sin \lambda\, x + \cos \lambda\, x$ and $|\sin x| + |\cos x|$ are $\frac{2\pi}{\lambda}$ and $\frac{\pi}{2}$ respectively

$$\therefore \frac{\pi}{2} = \frac{2\pi}{\lambda} \Rightarrow \lambda = 4$$

15 **(b)**

We have, $f(x) = \sqrt{\log_{16} x^2}$
Clearly, $f(x)$ exists, if
$\log_{16} x^2 \geq 0 \Rightarrow x^2 \geq 1 \Leftrightarrow |x| \geq 1$

16 **(b)**

Since, $f(x)$ is an even function, therefore $f'(x)$ is an odd function
ie, $\quad f'(-e) = -f'(e)$
$\therefore \ f'(e) + f'(-e) = 0$

17 **(c)**

We have,

$$f(x) = \log\left(\frac{1+x}{1-x}\right)$$

$$\therefore f\left(\frac{2x}{1+x^2}\right) = \log\left\{\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right\} = \log\left(\frac{x+1}{1-x}\right)^2$$

$$\Rightarrow f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+x}{1-x}\right) = 2\, f\,(x)$$

18 **(c)**

$f(x) = \cos^2 x + \sin^4 x = 1 - \cos^2 x + \cos^4 x$

$\Rightarrow f(x) = \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4}$ for all x

Also, $f(x) = \cos^2 x + \sin^4 x \leq \cos^2 x + \sin^2 x = 1$

$\therefore$ Range $(f) = [3/4, 1]$

Hence, $f(R) = [3/4,1]$

19 **(d)**

For domain of given function

$$-1 \leq \log_2 \left\{\frac{x}{12}\right\} \leq 1$$

$\Rightarrow\ 2^{-1} \leq \frac{x}{12} \leq 2$

$\Rightarrow\ 6 \leq x \leq 24$

$\Rightarrow\ x \in [6, 24]$

20 **(d)**

Given, $f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$

Here, 4^{-x^2} is defined for $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$, $\cos^{-1}\left(\frac{x}{2} - 1\right)$ is defined,

If $-1 \leq \frac{x}{2} - 1 \leq 1 \Rightarrow 0 \leq x \leq 4$

And $\log(\cos x)$ is defined, if $\cos x > 0$

$\Rightarrow -\frac{\pi}{2} < x < \frac{\pi}{2}$

Hence, $f(x)$ is defined for $x \in \left[0, \frac{\pi}{2}\right]$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	C	A	D	A	D	C	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	C	D	B	B	C	C	D	D