DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. :3

Topic :-MATRICES

2 **(a)**

Given equations are $x - cy - bz = 0$

$cx - y + az = 0$ and $bx + ay - z = 0$

For non-zero solution

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$\Rightarrow 1(1 - a^2) + c(-c - ab) - b(ac + b) = 0$

$\Rightarrow \; a^2 + b^2 + c^2 + 2abc = 1$

3 **(b)**

We have,

Det $(I_n) = 1(\neq 0) \Rightarrow$ rank $(I_n) = n$

4 **(a)**

The given matrix A is singular, if

$$|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \; 8(7\lambda - 16) + 6(-6\lambda + 8) + 2(24 - 14) = 0$

$\Rightarrow \; 56\lambda - 128 - 36\lambda + 48 + 20 = 0$

$\Rightarrow \; 20\lambda = 60$

$\Rightarrow \; \lambda = 3$

5 **(c)**

Let $B = I + A + A^2 + A^3 \dots \infty$

$\Rightarrow AB = A + A^2 + A^3 + \cdots \infty$

$\Rightarrow B - AB = I$

$\Rightarrow B(I - A) = I$

$\Rightarrow B = (I - A)^{-1}$

$$\Rightarrow B = \begin{bmatrix} 0 & -2 \\ -3 & -3 \end{bmatrix}^{-1} = -\frac{1}{6}\begin{bmatrix} -3 & 2 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/3 \\ -1/2 & 0 \end{bmatrix}$$

6 **(a)**

Since A is non-singular matrix

$\therefore |A| \neq 0 \Rightarrow \text{rank}(A) = n$

7 **(b)**

$$A^2 = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix}$$

Now, $f(A) = A^2 - 3A + 7$

$$= \begin{bmatrix} -7 & -12 \\ 24 & 17 \end{bmatrix} - 3\begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix}$$

$$\therefore f(A) + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} -3 & -6 \\ 12 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

8 **(a)**

The given system of equations will have a unique solution, if

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0 \Rightarrow k \neq 0$

9 **(a)**

Given, $2x + y - z = 7$ ...(i)

$x - 3y + 2z = 1$ ...(ii)

and $x + 4y - 3z = 5$ ...(iii)

From Eqs.(i) and (ii), we get

$5x - y = 15$ ...(iv)

From Eqs. (i) and (iii)

$5x - y = 16$ ...(v)

Eqs. (iv) and (v) shows that they are parallel and solution does not exist.

10 **(d)**

We have,

$X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \Rightarrow X^2 = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

Clearly for $n = 2$, the matrices in options (a), (b), (c) do not tally with $\begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

12 **(a)**

We have,

$A = [a_{ij}]$ $\therefore |k\,A| = k^n|A|$

13 **(b)**

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

14 **(d)**

Given that, $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} \alpha^2 + 0 & 0 + 0 \\ \alpha + 1 & 0 + 1 \end{bmatrix}$

$= \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$

Also, $B = A^2$ (given)

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$

Clearly this is not satisfied by any real value of α

15 **(b)**

We have,

$A\,(\text{adj}\,A) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$\Rightarrow |A|\,I = 4\,I$ $[\because A(\text{adj}\,A) = |A|\,I]$

$\Rightarrow |A| = 4$

$\Rightarrow |\text{adj}\,A| = |A|^2$ $[|\text{adj}\,A| = |A|^{n-1}]$

16 **(a)**

Given, $A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$

$A^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^3 & 0 \\ 0 & \omega^3 \end{bmatrix}$

Similarly, $A^{50}=\begin{bmatrix}\omega^{50} & 0\\ 0 & \omega^{50}\end{bmatrix}$

$=\begin{bmatrix}(\omega^2)^{16}\omega^2 & 0\\ 0 & (\omega^3)^{16}\omega^2\end{bmatrix}$

$=\begin{bmatrix}\omega^2 & 0\\ 0 & \omega^2\end{bmatrix}$

$=\omega^2$A

17 **(a)**

We have,

$AB=I_3$

$\Rightarrow\begin{bmatrix}1 & 2 & x\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & -2 & y\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$\Rightarrow\begin{bmatrix}1 & 0 & x+y\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$\Rightarrow x+y=0$

18 **(a)**

Let $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$\therefore$ adj(A) $=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

19 **(a)**

$\because |A|=0-1(1-9)+2(1-6)$

$=8-10=-2\neq 0$

adj $(A)=\begin{bmatrix}-1 & 1 & -1\\ 8 & -6 & 2\\ -5 & 3 & -1\end{bmatrix}$

$\therefore\ A^{-1}=\frac{1}{-2}\begin{bmatrix}-1 & 1 & -1\\ 8 & -6 & 2\\ -5 & 3 & -1\end{bmatrix}=\begin{bmatrix}1/2 & -1/2 & 1/2\\ -4 & 3 & -1\\ 5/2 & -3/2 & 1/2\end{bmatrix}$

20 **(a)**

$|f(\theta)|=1(\cos^2\theta+\sin^2\theta)=1$

Now, adj$\{f(\theta)\}=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

$\therefore\ \{f(\theta)\}^{-1}=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}=f(-\theta)$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	A	B	A	C	A	B	A	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	B	D	B	A	A	A	A	A