DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. :3

Topic :-DETERMINANTS

1 **(c)**

Given, $f(x)=\begin{vmatrix}1 & 2(x-1) & 3(x-1)(x-2)\\ x-1 & (x-1)(x-2) & (x-1)(x-2)(x-3)\\ x & x(x-1) & x(x-1)(x-2)\end{vmatrix}$

$=(x-1)(x-1)(x-2)\begin{vmatrix}1 & 2 & 3\\ x-1 & x-2 & x-3\\ x & x & x\end{vmatrix}$

Applying $C_1\to C_1-C_2, C_2\to C_2-C_3$

$=(x-1)^2(x-2)\begin{vmatrix}-1 & -1 & 3\\ 1 & 1 & x-3\\ 0 & 0 & x\end{vmatrix}$

$=(x-1)^2(x-2)x(-1+1)$ =0

$\Rightarrow f(x)=0$

$\therefore f(49)=0$

2 **(b)**

Given that, $\begin{vmatrix}1+ax & 1+bx & 1+cx\\ 1+a_1x & 1+b_1x & 1+c_1x\\ 1+a_2x & 1+b_2x & 1+c_2x\end{vmatrix}$

$=A_0+A_1x+A_2x^2+A_3x^3$

On putting $x=0$ on both sides, we get

$\begin{vmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{vmatrix}=A_0$

$\Rightarrow A_0=0$

4 **(d)**

We have,

$\begin{vmatrix}\cos\alpha & \sin\alpha & 0\\ \cos\beta & \sin\beta & 0\\ \cos\gamma & \sin\gamma & 0\end{vmatrix}\begin{vmatrix}\cos\alpha & \sin\alpha & 0\\ \cos\beta & \sin\beta & 0\\ \cos\gamma & \sin\gamma & 0\end{vmatrix}$

$=\begin{vmatrix}1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha)\\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta)\\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1\end{vmatrix}$

$\therefore \begin{vmatrix}1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha)\\ \cos(\alpha+\beta) & 1 & \cos(\gamma-\beta)\\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1\end{vmatrix}=0$

5 **(a)**

Given, $\begin{vmatrix}x & 3 & 7\\ 2 & x & 2\\ 7 & 6 & x\end{vmatrix}$=0

$\Rightarrow x(x^2-12)-3(2x-14)+7(12-7x)=0$

$\Rightarrow x^3-67x+126=0$

$\Rightarrow (x+9)(x^2-9x+14)=0$

$\Rightarrow (x+9)(x-2)(x-7)=0$

$\Rightarrow x = -9, 2, 7$

Hence, the other two roots are 2, 7

6 **(c)**

From the sine rule, we have

$\Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k(\text{say}),$

$\Rightarrow \sin A = ak, \sin B = bk$ and $\sin C = ck$

$\therefore \begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos(B-C) \\ c\sin A & \cos(B-C) & 1 \end{vmatrix}$

$= \begin{vmatrix} a^2 & abk & ack \\ abk & 1 & \cos(B-C) \\ ack & \cos(B-C) & 1 \end{vmatrix}$

$= a^2 \begin{vmatrix} 1 & \sin B & \sin C \\ \sin B & 1 & \cos(B-C) \\ \sin C & \cos(B-C) & 1 \end{vmatrix}$

$= a^2 \begin{vmatrix} 1 & \sin(A+C) & \sin(A+B) \\ \sin(A+C) & 1 & \cos(B-C) \\ \sin(A+B) & \cos(B-C) & 1 \end{vmatrix}$

$= a^2 \begin{vmatrix} \sin A & \cos A & 0 \\ \cos C & \sin C & 0 \\ \cos B & \sin B & 0 \end{vmatrix} \begin{vmatrix} \sin A & \cos A & 0 \\ \cos C & \sin C & 0 \\ \cos B & \sin B & 0 \end{vmatrix} = a^2 \times 0 = 0$

7 **(b)**

Given, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Applying $C_2 \to C_2 - C$ and $C_3 \to C_3 - C_1$

$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = xy$

Hence, D is divisible by both x and y.

8 **(a)**

Taking x common from $R_2$and $x(x-1)$common from R_3,we get

$f(x) = x^2(x-1) \begin{vmatrix} 1 & x & (x+1) \\ 2 & (x-1) & (x+1) \\ 3 & (x-2) & (x+1) \end{vmatrix}$

$\Rightarrow f(x) = x^2(x-1)(x+1) \begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 1 \\ 3 & x-2 & 1 \end{vmatrix}$

$= x^2(x^2-1) \begin{vmatrix} 1 & x & 1 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{vmatrix} \begin{bmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{bmatrix}$

$\Rightarrow f(x) = x^2(x^2-1)(-2+2) = 0$

$\Rightarrow \quad f(x) = 0$for all x

$\therefore \quad f(11) = 0$

9 **(b)**

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$,we get

$\begin{vmatrix} 1 & 4 & 20 \\ 0 & -6 & -15 \\ 0 & 2x-4 & 5x^2-20 \end{vmatrix}$=0

$\Rightarrow 1[-6(5x^2-20) + 15(2x-4)] = 0$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow \quad x=-1,2$

10 **(b)**

We have,

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix}=0 \quad \text{Applying } C_1 \to C_1+C_2+C_3$$

$$\Rightarrow (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix}=0$$

$$\Rightarrow (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 0 & 3x-11 & 0 \\ 0 & 0 & 3x-11 \end{vmatrix}=0 \quad \text{Applying } R_2 \to R_2-R_1,\ R_3 \to R_3-R_1$$

$\Rightarrow (3x-2)(3x-11)^2=0$

$\Rightarrow x=2/3, 11/3$

11 **(a)**

We have,

$$\begin{vmatrix} \alpha & x & x & x \\ x & \beta & x & x \\ x & x & \gamma & x \\ x & x & x & \delta \end{vmatrix}$$

$$=\begin{vmatrix} \alpha & x-\alpha & x-\alpha & x-\alpha \\ x & -(x-\beta) & 0 & 0 \\ x & 0 & -(x-\gamma) & 0 \\ x & 0 & 0 & -(x-\delta) \end{vmatrix}$$

$$=\alpha\begin{vmatrix} -(x-\beta) & 0 & 0 \\ 0 & -(x-\gamma) & 0 \\ 0 & 0 & -(x-\delta) \end{vmatrix}-x\begin{vmatrix} x-\alpha & x-\alpha & x-\alpha \\ 0 & -(x-\gamma) & 0 \\ 0 & 0 & -(x-\delta) \end{vmatrix}$$

$$+x\begin{vmatrix} x-\alpha & x-\alpha & x-\alpha \\ -(x-\beta) & 0 & 0 \\ 0 & 0 & -(x-\delta) \end{vmatrix}-x\begin{vmatrix} x-\alpha & x-\alpha & x-\alpha \\ -(x-\beta) & 0 & 0 \\ 0 & -(x-\gamma) & 0 \end{vmatrix}$$

$=-\alpha(x-\beta)(x-\gamma)(x-\delta)-(x-\alpha)(x-\gamma)(x-\delta)$

$-x(x-\alpha)(x-\beta)(x-\delta)-x(x-\alpha)(x-\beta)(x-\gamma)$

$=-\alpha(x-\beta)(x-\gamma)(x-\delta)+x(x-\beta)(x-\gamma)(x-\delta)$

$-x(x-\beta)(x-\gamma)(x-\delta)-x(x-\alpha)(x-\gamma)(x-\delta)$

$-x(x-\alpha)(x-\beta)(x-\delta)-x(x-\alpha)(x-\beta)(x-\gamma)$

$=(x-\beta)(x-\gamma)(x-\delta)(x-\alpha)-x[(x-\alpha)(x-\beta)(x-\gamma)+(x-\beta)(x-\gamma)(x-\delta)$

$+(x-\gamma)(x-\delta)(x-\alpha)+(x-\alpha)(x-\beta)(x-\delta)]$

$=f(x)-xf'(x)$, where, $f(x)=(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$

12 **(b)**

Given $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix}=0$

$\Rightarrow c^2-ab-a(c-a)+b(b-c)=0$

$\Rightarrow a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow \frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]=0$

$\Rightarrow a=b=c$

So, ΔABC is equilateral triangle.

$\therefore \angle A=60°, \angle B=60°, \angle C=60°$

$\sin^2 A+\sin^2 B+\sin^2 C=\sin^2 60°+\sin^2 60°+\sin^2 60°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

$= 3 \times \frac{3}{4} = \frac{9}{4}$

14 **(a)**

Given that, $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$= \begin{vmatrix} 1+\omega^n+\omega^{2n} & \omega^n & \omega^{2n} \\ 1+\omega^n+\omega^{2n} & 1 & \omega^n \\ 1+\omega^n+\omega^{2n} & \omega^{2n} & 1 \end{vmatrix}$

$= \begin{vmatrix} 0 & \omega^n & \omega^{2n} \\ 0 & 1 & \omega^n \\ 0 & \omega^{2n} & 1 \end{vmatrix}$

($\because$ If n multiple of 3, then $1 + \omega^n + \omega^{2n} = 0$)

$= 0$

15 **(b)**

$\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{bmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{bmatrix} = 0$

$\Rightarrow \begin{vmatrix} x+9 & x+9 & x+9 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 9+x & x+9 & 9+x \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix}$

$= \begin{bmatrix} 9+x & 9+x & 9+x \\ 5 & x & 4 \\ x & 4 & 5 \end{bmatrix} = 0$

$\Rightarrow (x+9)\begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & x-3 \\ 6 & x-6 & -3 \end{vmatrix} = (9+x)\begin{vmatrix} 1 & 0 & 0 \\ x & 7-x & 2-x \\ 7 & -5 & x-7 \end{vmatrix}$

$= (9+x)\begin{vmatrix} 1 & 0 & 0 \\ 5 & x-5 & -1 \\ x & 4-x & 5-x \end{vmatrix}$

$= 0$

$\Rightarrow x + 9 = 0 \Rightarrow x = -9$

16 **(b)**

The given system of equations will have a unique solution, if

$\begin{vmatrix} k & 2 & -1 \\ 0 & k-1 & -2 \\ 0 & 0 & k+2 \end{vmatrix} \neq 0 \Rightarrow k(k-1)(k+2) \neq 0 \Rightarrow k \neq 0,1,-2$

18 **(d)**

$\begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$

$\Rightarrow \begin{vmatrix} a & b-y & c-z \\ -x & y & 0 \\ 0 & -y & z \end{vmatrix} = 0$

$\Rightarrow a(yz) + x(bz - yz + cy - yz) = 0$

$\Rightarrow ayz + bzx + cyx = 2xyz$

$\Rightarrow \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$

19 **(a)**

Given,

$$\begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$$ is symmetric determinant.

∴ Its value is

$1 + 2\cos(\alpha-\beta)\cos\alpha\cos\beta$

$-\cos^2\alpha - \cos^2\beta - \cos^2(\alpha-\beta)$

$= 1 - \cos^2\alpha - \cos^2\beta - \cos(\alpha-\beta)$

$[\cos(\alpha-\beta) - 2\cos\alpha\cos\beta]$

$= 1 - \cos^2\alpha - \cos^2\beta - \cos(\alpha-\beta)$

$[\cos(\alpha-\beta) - \cos(\alpha+\beta) - \cos(\alpha-\beta)]$

$= 1 - \cos^2\alpha - \cos^2\beta + \cos(\alpha-\beta)\cos(\alpha+\beta)$

$= 1 - \cos^2\alpha - \cos^2\beta + \cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta$

$= 1 - \cos^2\alpha - \cos^2\beta(1-\cos^2\alpha) - \sin^2\alpha\sin^2\beta$

$= (1-\cos^2\alpha)(1-\cos^2\beta) - \sin^2\alpha\sin^2\beta$

$= \sin^2\alpha\sin^2\beta - \sin^2\alpha\sin^2\beta = 0$

20 **(d)**

We have,

$$\Delta = \begin{vmatrix} 2\sin A\cos A & \sin C & \sin B \\ \sin C & 2\sin B\cos B & \sin A \\ \sin B & \sin A & 2\sin C\cos C \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 2ka\cos A & kc & kb \\ kc & 2kb\cos B & ka \\ kb & ka & 2kc\cos C \end{vmatrix} \text{ [Using: Sine rule]}$$

$$\Rightarrow \Delta = k^3\begin{vmatrix} 2a\cos A & c & b \\ c & 2b\cos B & a \\ b & a & 2c\cos C \end{vmatrix}$$

$$\Rightarrow \Delta = k^3\begin{vmatrix} a\cos A + a\cos A & a\cos B + b\cos A & c\cos A + a\cos C \\ a\cos B + b\cos A & b\cos B + b\cos B & b\cos C + c\cos B \\ c\cos A + a\cos C & b\cos C + c\cos B & c\cos C + c\cos C \end{vmatrix}$$

$$\Rightarrow \Delta = k^3\begin{vmatrix} \cos A & a & 0 \\ \cos B & b & 0 \\ \cos C & c & 0 \end{vmatrix}\begin{vmatrix} a & \cos A & 0 \\ b & \cos B & 0 \\ c & \cos C & 0 \end{vmatrix}$$

$\Rightarrow \Delta = k^3 \times 0 \times 0 = 0$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	C	D	A	C	B	A	B	B

Q.	11	12	13	14	15	16	17	18	19	20
A.	A	B	A	A	B	B	B	D	A	D