DPP
DAILY PRACTICE PROBLEMS

Class : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. : 3

Topic :- CONTINUITY AND DIFFERENTIABILITY

1 **(c)**

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \lambda[x] = 0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 5^{1/x} = 0$

And $f(0) = \lambda[0] = 0$

$\therefore$ f is continuous only whatever λ may be

2 **(b)**

We have,

$y(x) = f(e^x)\, e^{f(x)}$

$\Rightarrow y'(x) = f'(e^x) \cdot e^x \cdot e^{f(x)} + f(e^x)\, e^{f(x)}\, f'(x)$

$\Rightarrow y'(0) = f'(1)e^{f(0)} + f(1)e^{f(0)} f'(0)$

$\Rightarrow y'(0) = 2 \qquad [\because f(0) = f(1) = 0, f'(1) = 2]$

3 **(b)**

Since $f(x)$ is differentiable at $x = 1$. Therefore,

$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\Rightarrow \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{a(1-h)^2 - b - 1}{-h} = \lim_{h\to 0} \frac{\frac{1}{|1+h|} - 1}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{(a-b-1) - 2\,ah + ah^2}{-h} = \lim_{h\to 0} \frac{-h}{h(1+h)}$$

$$\Rightarrow \lim_{h\to 0} \frac{-(a-b-1) - 2\,ah - ah^2}{h} = -1$$

$\Rightarrow -(a-b-1) = 0$ and so $\lim_{h\to 0} \frac{2ah - ah^2}{h} = -1$

$\Rightarrow a - b - 1 = 0$ and $2a = -1 \Rightarrow a = -\frac{1}{2}, b = -\frac{3}{2}$

4 **(c)**

We have,

$f(x) = \frac{\sin 4\pi[x]}{1+[x]^2} = 0$ for all x $[\because 4\pi[x]$is a multiple of $\pi]$

$\Rightarrow f'(x) = 0$ for all x

5 **(d)**

We have,

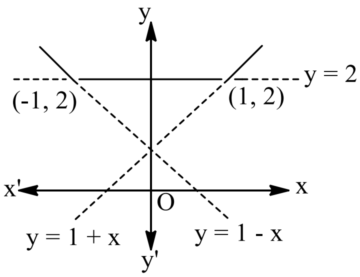
$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \sin\frac{1}{x}$$

$\Rightarrow \lim_{x\to 0} f(x) =$ An oscillating number which oscillates between -1 and 1

Hence, $\lim_{x\to 0} f(x)$ does not exist

Consequently, $f(x)$ cannot be continuous at $x = 0$ for any value of k

6 **(c)**



It is clear from the graph that $f(x)$ is continuous everywhere and also differentiable everywhere except $\{-1, 1\}$ due to sharp edge

7 **(d)**

We have,

$\log\left(\frac{x}{y}\right) = \log x - \log y$ and $\log(e) = 1$

$\therefore f(x) = \log x$

Clearly, $f(x)$ is unbounded because $f(x) \to -\infty$ as $x \to 0$ and $f(x) \to +\infty$ as $x \to \infty$

We have,

$$f\left(\frac{1}{x}\right) = \log\left(\frac{1}{x}\right) = -\log x$$

As $x \to 0, f\left(\frac{1}{x}\right) \to \infty$

Also,

$$\lim_{x\to 0} xf(x) = \lim_{x\to 0} x\log x = \lim_{x\to 0}\frac{\log x}{1/x}$$

$$\Rightarrow \lim_{x\to 0} xf(x) = \lim_{x\to 0}\frac{1/x}{-1/x^2} = -\lim_{x\to 0} x = 0$$

9 **(c)**

Since $g(x)$ is the inverse of $f(x)$. Therefore,

$fog(x) = x$, for all x

$\Rightarrow \frac{d}{dx}\{fog\,(x)\} = 1$, for all x

$\Rightarrow f'(g(x))\, g'(x) = 1$, for all x

$\Rightarrow \frac{1}{1+\{g(x)\}^3} \times g'(x) = 1$ for all x $\quad \left[\because f'(x) = \frac{1}{1+x^3}\right]$

$\Rightarrow g'(x) = 1 + \{g(x)\}^3$, for all x

10 **(d)**

We have,

$f(x) = |x^2 - 4x + 3|$

$$\Rightarrow f(x) = \begin{cases} x^2 - 4x + 3, \text{if } x^2 - 4x + 3 \geq 0 \\ -(x^2 - 4x + 3), \quad \text{if } x^2 - 4x + 3 < 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} x^2 - 4x + 3, \text{if } x \leq 1 \text{ or } x \geq 3 \\ -x^2 + 4x - 3, \quad \text{if } 1 < x < 3 \end{cases}$$

Clearly, $f(x)$ is everywhere continuous

Now,

$$(\text{LHD at } x = 1) = \left(\frac{d}{dx}(x^2 - 4x + 3)\right)_{\text{at } x=1}$$

$\Rightarrow (\text{LHD at } x = 1) = (2x - 4)_{\text{at } x=1} = -2$

and,

$$(\text{RHD at } x = 1) = \left(\frac{d}{dx}(-x^2 + 4x - 3)\right)_{\text{at } x=1}$$

$\Rightarrow (\text{RHD at } x = 1) = (-2x + 4)_{\text{at } x=1} = 2$

Clearly, $(\text{LHD at } x = 1) \neq (\text{RHD at } x = 1)$

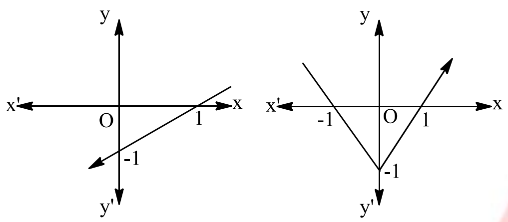
So, $f(x)$ is not differentiable at $x = 1$
Similarly, it can be checked that $f(x)$ is not differentiable at $x = 3$ also
ALITER We have,
$f(x) = |x^2 - 4x + 3| = |x - 1|\,|x - 3|$
Since, $|x - 1|$ and $|x - 3|$ are not differentiable at 1 and 3 respectively
Therefore, $f(x)$ is not differentiable at $x = 1$ and $x = 3$

11 **(c)**

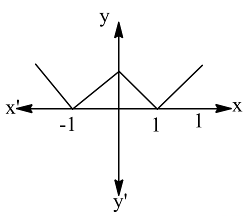
The point of discontinuity of $f(x)$ are those points where $\tan x$ is infinite.
ie, $\tan x = \tan \infty$
$\Rightarrow \quad x = (2n+1)\frac{\pi}{2}, \qquad n \in I$

12 **(a)**

Using graphical transformation



(i) y = x - 1 (ii) y = |x| - 1



(iii) y = ||x| - 1|

As, we know the function is not differentiable at6 sharp edges and in figure (iii) $y = \big||x| - 1\big|$ we have 3 sharp edges at $x = -1, 0, 1$
$\therefore f(x)$ is not differentiable at $\{0, \pm 1\}$

13 **(c)**

$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} 2(0 - h) = 0$
And $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} 2(0 + h) + 1 = 1$
$\because \quad \lim_{x \to 0^-} f(x) \neq \lim_{x \to 1^+} f(x)$
$\therefore \quad f(x)$ is discontinuous at $x = 0$

14 **(b)**

Draw a rough sketch of $y = f(x)$ and observe its properties

15 **(c)**

$$\lim_{x \to \pi} \frac{(1 + \cos x) - \sin x}{(1 + \cos x) + \sin x}$$
$$= \lim_{x \to \pi} \frac{2\cos^2 x/2 - 2(\sin x/2) \cos \ x/2}{2\cos^2 x/2 + 2(\sin x/2) \cos \ x/2}$$
$$= \lim_{x \to \pi} \tan\left(\frac{\pi}{4} - \frac{\pi}{2}\right) = -1$$
Since, $f(x)$ is continuous at $x = \pi$
$\therefore \quad f(\pi) = \lim_{x \to \pi} f(x) = -1$

16 **(d)**

$$f'(1^-) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h}$$
$$= \lim_{h\to 0} \frac{(1-h-1).\sin\left(\frac{1}{1-h-1}\right)-0}{-h}$$
$$= -\lim_{h\to 0} \sin\frac{1}{h}$$
And $f'(1^+) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$
$$= \lim_{h\to 0} \frac{(1+h-1)\sin\left(\frac{1}{1+h-1}\right)-0}{h}$$
$$= \lim_{h\to 0} \sin\frac{1}{h}$$
$\therefore \quad f'(1^-) \neq f'(1^+)$
f is not differentiable at $x=1$
Again, now
$$f'(0^+) = \lim_{h\to 0} \frac{(0+h-1)\sin\left(\frac{1}{0+h-1}\right)-\sin 1}{h}$$
$$= \lim_{h\to 0} \frac{\left[-\left\{(h-1)\cos\left(\frac{1}{h-1}\right)\times\left(\frac{1}{(h-1)^2}\right)\right\}+\sin\left(\frac{1}{h-1}\right)\right]}{1}$$
[using L 'Hospital's rule]
$= \cos 1 - \sin 1$
And $f'(0^-) = \lim_{h\to 0} \frac{(0-h-1)\sin\left(\frac{1}{0-h-1}\right)-\sin 1}{-h}$
$$= \lim_{h\to 0} \frac{(-h-1)\cos\left(\frac{1}{-h-1}\right)\left(\frac{1}{(-h-1)^2}\right)-\sin\left(\frac{1}{-h-1}\right)}{-1}$$
[using L 'Hospital's rule]
$= \cos 1 - \sin 1$
$\Rightarrow \quad f'(0^-) = f'(0^+)$
$\therefore \quad f$ is differentiable at $x=0$

17 **(c)**

As $f(x)$ is continuous at $x = \frac{\pi}{2}$
$\therefore \lim_{x\to \frac{\pi}{2}^-} f(x) = \lim_{x\to \frac{\pi}{2}^+} f(x)$
$\Rightarrow m\frac{\pi}{2}+1 = \sin\frac{\pi}{2}+n \Rightarrow m\frac{\pi}{2}+1 = 1+n \Rightarrow n = \frac{m\pi}{2}$

18 **(d)**

Since, $\frac{f(6)-f(1)}{6-1} \geq 2 \quad \left[\because \quad f'(x) = \frac{y_2-y_1}{x_2-x_1}\right]$
$\Rightarrow \quad f(6)-f(1) \geq 10$
$\Rightarrow f(6)+2 \geq 10$
$\Rightarrow \quad f(6) \geq 8$

19 **(b)**

We have,
$\lim_{x\to a^-} f(x)\ g(x) = \lim_{x\to a^-} f(x) . \lim_{x\to a^-} g(x) = m\times l = ml$
and,
$\lim_{x\to a^+} f(x)\ g(x) = \lim_{x\to a^+} f(x) \lim_{x\to a^+} g(x) = lm$
$\therefore \lim_{x\to a^-} f(x)\ g(x) = \lim_{x\to a^+} f(x)\ g(x) = lm$
Hence, $\lim_{x\to a} f(x)\ g(x)$ exists and is equal to lm

20 **(c)**

We have,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h\to 0}\frac{f(x)f(h)-f(x)}{h}$$

$$\Rightarrow f'(x) = f(x)\lim_{h\to 0}\frac{f(h)-1}{h} \qquad [\because f(x+y)=f(x)f(y)]$$

$$\Rightarrow f'(x) = f(x)\left\{\lim_{h\to 0}\frac{1+h\,g(h)-1}{h}\right\} \qquad [\because f(x)=1+x\,g(x)]$$

$$\Rightarrow f'(x) = f(x)\lim_{h\to 0} g(h) = f(x)\cdot 1 = f(x)$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	B	C	D	C	D	B	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	A	C	B	C	D	C	D	B	C