

DPP

DAILY PRACTICE PROBLEMS

Class : XIth

Date :

Solutio

Subject : MATHS

DPP No. : 3

Topic :- Applications of Derivatives

1

(c)

From mean value theorem $f'(c) = \frac{f(b)-f(a)}{b-a}$

Given, $a = 0 \Rightarrow f(a) = 0$

and $b = \frac{1}{2} \Rightarrow f(b) = \frac{3}{8}$

Now, $f'(x) = (x-1)(x-2) + x(x-2) + x(x-1)$

$\therefore f'(c) = (c-1)(c-2) + c(c-2) + c(c-1)$

$= c^2 - 3c + 2 + c^2 - 2c + c^2 - c$

$\Rightarrow f'(c) = 3c^2 - 6c + 2$

By definition of mean value theorem

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$\Rightarrow 3c^2 - 6c + 2 = \frac{\left(\frac{3}{8}\right) - 0}{\left(\frac{1}{2}\right) - 0} = \frac{3}{4}$

$\Rightarrow 3c^2 - 6c + \frac{5}{4} = 0$

This is a quadratic equation in c

$\therefore c = \frac{6 \pm \sqrt{36 - 15}}{2 \times 3} = \frac{6 \pm \sqrt{21}}{6} = 1 \pm \frac{\sqrt{21}}{6}$

Since, ' c ' lies between $\left[0, \frac{1}{2}\right]$

$\therefore c = 1 - \frac{\sqrt{21}}{6}$ (neglecting $c = 1 + \frac{\sqrt{21}}{6}$)

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(a)

$\therefore f(x) = \frac{1}{4x^2 + 2x + 1}$

On differentiating w.r.t. x , we get

$f'(x) = \frac{-(8x+2)}{(4x^2+2x+1)^2} \dots (i)$

For maxima or minima put $f'(x) = 0$

$\Rightarrow 8x + 2 = 0 \Rightarrow x = -\frac{1}{4}$

Again differentiating w.r.t. x of Eq. (i), we get

$f''(x) = -\frac{[(4x^2+2x+1)^2(8)-(8x-2)2 \times (4x^2+2x+1)(8x+2)]}{(4x^2+2x+1)^4}$

At $x = -\frac{1}{4}$, $f''\left(-\frac{1}{4}\right) = -ve$



$$f(x) \text{ is maximum at } x = -\frac{1}{4}$$

$$\therefore \text{ maximum value of } f(x)$$

$$f\left(-\frac{1}{4}\right)_{\max} = \frac{1}{4 \times \frac{1}{16} - 2 \times \frac{1}{4} + 1}$$

$$= \frac{1}{\frac{1}{4} - \frac{2}{4} + 1}$$

$$= \frac{4}{1 - 2 + 4} = \frac{4}{3}$$

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(b)

$$\text{Let } f(x) = 2x + 3y$$

$$\therefore f(x) = 2x + \frac{18}{x} \quad [\because xy = 6, \text{ given}]$$

$$\Rightarrow f'(x) = 2 - \frac{18}{x^2}$$

Put $f'(x) = 0$ for maxima or minima

$$\Rightarrow 0 = 2 - \frac{18}{x^2} \Rightarrow x = \pm 3$$

$$\text{And } f''(x) = \frac{36}{x^3} \Rightarrow f''(3) = \frac{36}{3^3} > 0$$

\therefore At $x = 3$, $f(x)$ is minimum.

The minimum value is $f(3) = 12$

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(c)

On differentiating given curve w. r. t. x , we get

$$4y^3 \frac{dy}{dx} = 3ax^2$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(a,a)} = \frac{3a^3}{4a^3} = \frac{3}{4}$$

\therefore Equation of normal at point (a, a) is

$$y - a = -\frac{4}{3}(x - a)$$

$$\Rightarrow 4x + 3y = 7a$$

7

(b)

$$\text{Let } f(x) = x^3 - px + q$$

$$\text{Then } f'(x) = 3x^2 - p$$

$$\text{Put } f'(x) = 0 \Rightarrow x = \sqrt{\frac{p}{3}}, -\sqrt{\frac{p}{3}}$$

$$\text{Now, } f''(x) = 6x$$

$$\therefore \text{ At } x = \sqrt{\frac{p}{3}}, f''(x) = 6\sqrt{\frac{p}{3}} > 0, \text{ minima}$$

$$\text{And at } x = -\sqrt{\frac{p}{3}}, f''(x) < 0, \text{ maxima}$$

8

(a)

$$\text{For } x = p, y = ap^2 + bp + c$$

$$\text{And for } x = q, y = aq^2 + bq + c$$

$$\text{Slope} = \frac{aq^2 + bq + c - ap^2 - bp - c}{q - p}$$

$$= a(q + p) + b$$

$$\frac{dy}{dx} = 2ax + b = a(q + p) + b$$



(according to the equation)

$$\therefore x = \frac{q+p}{2}$$

9

(c)

Clearly, $f(x)$ is continuous and differentiable on the intervals $[0, 3]$ and $(0, 3)$ respectively for all $n \in \mathbb{N}$

Also, $f(0) = f(3) = 0$

It is given that Rolle's theorem for the function $f(x)$ defined on $[0, 3]$ is applicable with $c = \frac{3}{4}$

$$\therefore f'(c) = 0$$

$$\Rightarrow 2(c-3)^n + 2nc(c-3)^{n-1} = 0$$

$$[\because f(x) = 2x(x-3)^n \therefore f'(x) = 2(x-3)^n + 2nx(x-3)^{n-1}]$$

$$\Rightarrow 2(c-3)^{n-1}(c-3+nc) = 0$$

$$\Rightarrow \frac{3}{4} - 3 + \frac{3}{4}n = 0 \Rightarrow n = 3 \quad \left[\because c = \frac{3}{4} \right]$$

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(a)

Given,

$$y - x = 1$$

$$\Rightarrow y = x + 1$$

$$\frac{dy}{dx} = 1$$

And $y^2 = x$

$$\Rightarrow 2y \frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}$$

$$\therefore 1 = \frac{1}{2y}$$

$$\Rightarrow 2y = 1$$

$$\Rightarrow y = \frac{1}{2}$$

\therefore point on the curve is $\left(\frac{1}{4}, \frac{1}{2}\right)$

\therefore required shortest distance

$$= \left| \frac{\frac{1}{4} - \frac{1}{2} + 1}{\sqrt{2}} \right| = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$$

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(d)

$$\text{Since, } s^3 \propto v \Rightarrow \frac{ds}{dt} = ks^3 \dots\dots(i)$$

$$\Rightarrow \frac{d^2s}{dt^2} = 3ks^2 \frac{ds}{dt}$$

$$\Rightarrow \frac{d^2s}{dt^2} = 3k^2s^5 \quad [\text{from Eq. (i)}]$$

Hence, acceleration of particle is proportional to s^5 .

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(a)

$$\text{Given, } s = t^3 + 2t^2 + t$$

$$\Rightarrow v = \frac{ds}{dt} = 3t^2 + 4t + 1$$

Speed of the particle after 1 s



$$v_{(t=1)} = \left(\frac{ds}{dt}\right)_{(t=1)}$$

$$= 3 \times 1^2 + 4 \times 1 + 1 = 3 + 5 = 8 \text{ cm/s}$$

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(b)

Given, $y = a^x \Rightarrow \frac{dy}{dx} = a^x \log a$

Now, length of subtangent at any point (x_1, y_1)

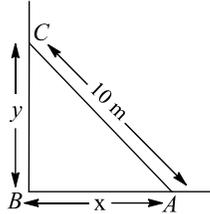
$$= \frac{y}{dy/dx} = \frac{a^{x_1}}{a^{x_1} \log a} = \frac{1}{\log a}$$

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(c)

Let $AB = xm$, $BC = ym$ and $AC = 10m$

$$\therefore x^2 + y^2 = 100 \dots(i)$$



On differentiating w.r.t t, m we get

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$\Rightarrow 2x(3) - 2y(4) = 0$$

$$\Rightarrow x = \frac{4y}{3}$$

On putting this value in Eq. (i), we get

$$\frac{16}{9}y^2 + y^2 = 100$$

$$\Rightarrow y^2 = \frac{100 \times 9}{25} = 36 \Rightarrow y = 6m$$

17

(c)

Since $f(x)$, which is of degree 3, has relative minimum/maximum at $x = -1$ and $x = \frac{1}{3}$. Therefore

$x = -1, x = \frac{1}{3}$ are roots of $f'(x) = 0$. Thus, $x + 1$ and $3x - 1$ are factors of $f'(x)$

Consequently, we have

$$f'(x) = \lambda(x + 1)(3x - 1) = \lambda(3x^2 + 2x - 1)$$

$$\Rightarrow f(x) = \lambda(x^3 + x^2 - x) + c$$

Now, $f(-2) = 0$ [Given]

$$\Rightarrow c = 2\lambda \dots(i)$$

We have,

$$\int_{-1}^1 f(x) dx = \frac{14}{3}$$

$$\Rightarrow \int_{-1}^1 [\lambda(x^3 + x^2 - x) + c] dx = \frac{14}{3}$$

$$\Rightarrow \lambda \int_{-1}^1 x^2 dx + \int_{-1}^1 c dx = \frac{14}{3} \Rightarrow \frac{2\lambda}{3} + 2c = \frac{14}{3} \Rightarrow \lambda + 3c = 7 \dots(ii)$$

Solving (i) and (ii), we get $\lambda = 1, c = 2$

Hence, $f(x) = x^3 + x^2 - x + 2$

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(d)

$$f(x) = \cos x + \frac{1}{2} \cos 2x - \frac{1}{3} \cos 3x$$



$$\Rightarrow f'(x) = -\sin x - \sin 2x + \sin 3x$$

Put $f'(x) = 0$

$$\Rightarrow 2 \sin \frac{3x}{2} \cos \frac{x}{2} = 2 \sin \frac{3x}{2} \cos \frac{3x}{2}$$

$$\Rightarrow \sin \frac{3x}{2} = 0, \cos \frac{3x}{2} = \cos \frac{x}{2}$$

$$\Rightarrow x = \frac{2n\pi}{3}, \frac{3x}{2} = 2n\pi \pm \frac{x}{2}$$

$$\Rightarrow \text{At } x = 0, \frac{2\pi}{3},$$

At $x = 0$

$$f(x) = 1 + \frac{1}{2} - \frac{1}{3} = \frac{7}{6}$$

At $x = \frac{2\pi}{3},$

$$f(x) = -\frac{1}{2} - \frac{1}{4} - \frac{1}{3} = -\frac{13}{12}$$

$$\therefore \text{difference} = \frac{7}{6} + \frac{13}{12} = \frac{27}{12} = \frac{9}{4}$$

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(a)

Let $f(x) = e^{x-1} + x - 2$. Then,

$$f(1) = e^0 + 1 - 2 = 0$$

$\Rightarrow x = 1$ is a real root of the equation $f(x) = 0$

Let $x = \alpha$ be other real root of $f(x) = 0$ such that $\alpha > 1$

Consider the interval $[1, \alpha]$

Clearly, $f(1) = f(\alpha) = 0$

So, by Rolle's theorem $f'(x) = 0$ has a root in $(1, \alpha)$

But, $f'(x) = e^{x-1} + 1 > 1$ for all x

$\therefore f'(x) \neq 0$ for any $x \in (1, \alpha)$

This is a contradiction

Hence, $f(x) = 0$ has no real root other than 1

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(d)

We have,

$$f(x) = \sin^6 x + \cos^6 x$$

$$\Rightarrow f(x) = (\sin^2 x + \cos^2 x)^3 - 3(\sin^2 x + \cos^2 x)(\sin^2 x \cos^2 x)$$

$$\Rightarrow f(x) = 1 - 3(\sin^2 x \cos^2 x) = 1 - \frac{3}{4} \sin^2 2x$$

$$\therefore f(x) \leq 1, \text{ for all } x \quad \left[\because -\frac{3}{4} \sin^2 2x < 0 \right]$$

$$\text{and, } f(x) > 1 - \frac{3}{4} = \frac{1}{4} \text{ for all } x \quad \left[\because -\frac{3}{4} \sin^2 2x \geq -\frac{3}{4} \right]$$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	C	A	C	B	C	A	B	A	C	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	D	B	B	C	C	D	A	D

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