DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. :3

Topic :-INTEGRALS

1 **(d)**

Let $g_n(x) = 1 + x^2 + x^4 + \ldots + x^{2n} = \frac{x^{2n+2}-1}{x^2-1}$

$\therefore h_n(x) = g_n'(x) = \dfrac{2x(nx^{2n+2} - (n+1)x^{2n} + 1)}{(x^2-1)^2}$

Now, $f(x) = \lim_{n\to\infty} h_n(x) = \frac{2x}{(x^2-1)^2}$

As $0 < x < 1$

Thus, $\int f(x)dx = \int \frac{2x}{(x^2-1)^2} dx$

$= -\dfrac{1}{x^2-1} = \dfrac{1}{1-x^2}$

2 **(c)**

$\int \dfrac{dx}{\sin x - \cos x + \sqrt{2}}$

$= \int \dfrac{dx}{\sin x \dfrac{\sqrt{2}}{\sqrt{2}} - \cos x \dfrac{\sqrt{2}}{\sqrt{2}} + \sqrt{2}}$

$= \int \dfrac{dx}{\sqrt{2}(\sin x \sin\frac{\pi}{4} - \cos x \cos\frac{\pi}{4} + 1)}$

$= \dfrac{1}{\sqrt{2}} \int \dfrac{dx}{1 - \cos\left(x + \frac{\pi}{4}\right)}$

$= \dfrac{1}{\sqrt{2}} \int \dfrac{dx}{1 - \cos 2\left(\frac{x}{2} + \frac{\pi}{8}\right)}$

$= \dfrac{1}{\sqrt{2}} \int \dfrac{dx}{2\sin^2\left(\frac{x}{2} + \frac{\pi}{8}\right)}$

$= \dfrac{1}{2\sqrt{2}} \int \text{cosec}^2\left(\frac{x}{2} + \frac{\pi}{8}\right) dx$

$= -\frac{1}{2\sqrt{2}} \cdot \frac{-\cot\left(\frac{x}{2}+\frac{\pi}{8}\right)}{\frac{1}{2}} + c = \frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$

3 **(a)**

Let $I = \int_0^{\pi/2} \frac{\cos x}{1+\sin x}\, dx$

$= [\log(1 + \sin x)]_0^{\pi/2}$

$= \log 2 - \log 1 = \log 2$

4 **(b)**

Let $I = \int_0^1 \frac{2\sin^{-1}\frac{x}{2}}{x} dx$

Put $\sin^{-1}\frac{x}{2} = t \Rightarrow \frac{x}{2} = \sin t$

$\Rightarrow x = 2\sin I$

$\Rightarrow dx = 2\cos t\, dt$

$\therefore I = \int_0^{\frac{\pi}{6}} \frac{2t}{2\sin t}.2\cos t\, dt$

$= \int_0^{\frac{\pi}{6}} \frac{2t}{\tan t} dt$

$= \int_0^{\frac{\pi}{6}} \frac{2x}{\tan x} dx$

5 **(a)**

Given ,$\int_2^e \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right) dx = a + \frac{b}{\log 2}$

Put $\log x = z \Rightarrow x = e^z \Rightarrow dx = e^z dz$

$\therefore \int_2^e \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right) dx = \int_{\log 2}^1 \left(\frac{1}{z} - \frac{1}{z^2}\right) e^z\ dz$

$= \int_{\log 2}^1 e^2 \left(\frac{1}{z} + d\left(\frac{1}{z}\right)\right) dz$

$= \left[e^2.\frac{1}{z}\right]_{\log 2}^1 = e - \frac{2}{\log 2}$

$\therefore\ a = e$ and $b = -2$

7 **(c)**

Let $I = \int_0^1 \frac{x\, dx}{[x+\sqrt{1-x^2}]\sqrt{1-x^2}}$

Put $x = \sin\theta \Rightarrow dx = \cos\theta d\theta$

$\therefore\quad I = \int_0^{\pi/2} \frac{\sin\theta\cos\theta d\theta}{(\sin\theta + \cos\theta).\cos\theta}$

$\Rightarrow I = \int_0^{\pi/2} \frac{\sin\theta}{\sin\theta + \cos\theta} d\theta \ldots \text{(i)}$

$\Rightarrow I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)} d\theta$

$\Rightarrow I = \int_0^{\pi/2} \frac{\cos\theta}{\sin\theta + \cos\theta} d\theta \ldots\text{(ii)}$

On addingEqs. (i) and (ii), we get

$2I = \int_0^{\pi/2} \left(\frac{\sin\theta + \cos\theta}{\sin\theta + \cos\theta}\right) d\theta$

$= \int_0^{\pi/2} 1 d\theta = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

8 **(d)**

Let $I = \int_0^1 \cot^{-1}(1 - x + x^2) dx$

$= \int_0^1 \tan^{-1}\left(\frac{1}{x^2 - x + 1}\right) dx$

$= \int_0^1 \tan^{-1}\left(\frac{x - (x-1)}{1 + x(x-1)}\right) dx$

$= \int_0^1 \tan^{-1} x\, dx - \int_0^1 \tan^{-1}(x-1) dx$

$= \int_0^1 \tan^{-1} x\, dx - \int_0^1 \tan^{-1}(1 - x - 1) dx$

$= 2\int_0^1 \tan^{-1} x\, dx = 2\left[x\tan^{-1}x - \int \frac{x}{1+x^2}dx\right]_0^1$

$= 2\left[x\tan^{-1}x - \frac{1}{2}\log(1+x^2)\right]_0^1$

$= 2\left[\left(1\tan^{-1}1 - \frac{1}{2}\log 2\right) - \left(0 - \frac{1}{2}\log 1\right)\right]$

$= \frac{\pi}{2} - \log 2$

9 **(a)**

Let $I = \int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}}$

Put $x = \tan^2\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$

$dx = 2\tan\theta\sec^2\theta\, d\theta$

$\therefore I = \int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \frac{2\tan\theta\sec^2\theta}{(\tan^2\theta - 3)\sqrt{\tan^2\theta + 1}}d\theta$

$= \int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \frac{2\tan\theta\sec^2\theta}{(\sec^2\theta - 1 - 3)\sec\theta}d\theta$

$= \int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \frac{2\tan\theta\sec^2\theta}{(\sec^2\theta - 4)\sec\theta}d\theta$

$= \int_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \frac{2\tan\theta\sec\theta}{(\sec^2\theta - 4)}d\theta$

$= \left[\frac{1}{2}\log\frac{(\sec\theta - 2)}{(\sec\theta + 2)}\right]_{\tan^{-1}\sqrt{8}}^{\tan^{-1}\sqrt{15}} \left[\because \int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)\right]$

$= \frac{1}{2}\left[\log\left(\frac{\sec\tan^{-1}\sqrt{15} - 2}{\sec\tan^{-1}\sqrt{15} + 2}\right) - \log\left(\frac{\sec\tan^{-1}\sqrt{8} - 2}{\sec\tan^{-1}\sqrt{8} + 2}\right)\right]$

$= \frac{1}{2}\left[\log\left(\frac{\sec\sec^{-1}4 - 2}{\sec\sec^{-1}4 + 2}\right) - \log\left(\frac{\sec\sec^{-1}3 - 2}{\sec\sec^{-1}3 + 2}\right)\right]$

$= \frac{1}{2}\left[\log\frac{2}{6} - \log\frac{1}{5} = \frac{1}{2}\log\frac{5}{3}\right]$

10 **(c)**

We have,

$\Rightarrow I = \int \{1 + 2\tan x(\tan x + \sec x)\}^{1/2} dx$

$\Rightarrow I = \int \{1 + \tan^2 x + \tan^2 x + 2\tan x\sec x\}^{1/2}\, dx$

$\Rightarrow I = \int (\sec^2 x + \tan^2 x + 2\tan x\sec x)^{1/2} dx$

$\Rightarrow I = \int (\tan x + \sec x) dx$

$\Rightarrow I = \log\sec x + \log(\sec x + \tan x) + C$

$\Rightarrow I = \log\sec x(\sec x + \tan x) + C$

11 **(a)**

We have,

$I_1 = \int_0^\infty \frac{1}{1+x^4}dx$ and $I_2 = \int_0^\infty \frac{x^2}{1+x^4}dx$

Putting $x=\frac{1}{t}$in I_1, we get

$$I_1=\int_{\infty}^{0}\frac{t^4}{1+t^4}\times-\frac{1}{t^2}dt=\int_{0}^{\infty}\frac{t^2}{t+t^4}dt=I_2\Rightarrow\frac{I_1}{I_2}=1$$

12 **(c)**

Let $I=\int\frac{\sin x}{3+4\cos^2 x}dx$

Put $\cos x=t\ \Rightarrow -\sin x\,dx=dt$

$$\therefore I=\int\frac{-dt}{3+4t^2}=\frac{-1}{4}\int\frac{dt}{t^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow I=-\frac{1}{4\cdot\frac{\sqrt{3}}{2}}\cdot\tan^{-1}\frac{t}{\left(\frac{\sqrt{3}}{2}\right)}+c$$

$$=-\frac{1}{2\sqrt{3}}\tan^{-1}\frac{2t}{\sqrt{3}}+c$$

$$\Rightarrow I=\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right)+c$$

13 **(c)**

Let $f(x)=e^{\cos^2 x.}\cos^3(2n+1)x$

Then, $f(\pi-x)=e^{\cos^2(\pi-x).}\cos^3[(2n+1)\pi-(2n+1)x]$

$=-e^{\cos^2 x}.\cos^3(2n+1)x$

$\Rightarrow f(\pi-x)=-f(x)$

$\therefore f(x)$is an odd function.

Hence, $\int_0^{\pi}e^{\cos^2 x}\ .\cos^3(2n+1)x\,dx=0$

14 **(b)**

Given ,$\frac{d}{dx}\{f(x)\}=\frac{1}{1+x^2}$

On integrating both sides, we get

$f(x)=\tan^{-1}x$

$$\therefore \frac{d}{dx}f(x^3)=\frac{d}{dx}(\tan^{-1}x^3)$$

$$=\frac{1}{1+(x^3)^2}.3x^2$$

$$=\frac{3x^2}{1+x^6}$$

15 **(d)**

Let $I=\int_0^{\pi}[\cot x]dx$ ... (i)

$$\Rightarrow I=\int_0^{\pi}[\cot(\pi-x)]dx=\int_0^{\pi}[-\cot x]dx \quad \text{... (ii)}$$

On addingEqs. (i) and (ii),

$$2I=\int_0^{\pi}[\cot x]dx+\int_0^{\pi}[-\cot x]dx$$

$$=\int_0^{\pi}(-1)dx$$

$[\because [x]+[-x]=-,\text{if}x\notin z\ and[x]+[-x]=0,\text{if}x\in z]$

$=[-x]_0^{\pi}=-\pi$

$\therefore\ I = -\frac{\pi}{2}$

16 **(c)**

Let $I = \int_{-3}^{2}(|x+1| + |x+2| + |x-1|\quad)dx$

Again, let $f(x) = |x+1| + |x+2| + |x-1|$

$$= \begin{cases} -(x+1)-(x+2)-(x-1), & -3 < x \le -2 \\ -(x+1)+x+2-(x-1), & -2 < x \le -1 \\ 1+x+x+2-(x-1) & -1 < x \le 0 \\ 1+x+x+2-(x-1) & 0 \le x < 1 \\ 1+x+x+2+x-1 & 1 \le x < 2 \end{cases}$$

$$= \begin{cases} -3x-2, & -3 < x \le -2 \\ -x+2, & -2 < x \le -1 \\ x+4, & -1 \le x < 1 \\ 3x+2, & 1 \le x < 2 \end{cases}$$

$$\therefore I = \int_{-3}^{-2}(-3-2)dx + \int_{-2}^{-1}(-x+2)dx$$

$$+ \int_{-1}^{1}(x+4)dx + \int_{1}^{2}(3x+2)dx$$

$$= \left[-\frac{3x^2}{2} - 2x\right]_{-3}^{-2} + \left[-\frac{x^2}{2} + 2x\right]_{-2}^{-1}$$

$$+ \left[\frac{x^2}{2} + 4x\right]_{-1}^{1} + \left[\frac{3x^2}{2} + 2x\right]_{1}^{2}$$

$$= \left[-6+4-\left(-\frac{27}{2}+6\right)\right] + \left[-\frac{1}{2}-2-(-2-4)\right]$$

$$+ \left[\frac{1}{2}+4-\left(\frac{1}{2}-4\right)\right]\left[6+4-\left(\frac{3}{2}+2\right)\right]$$

$$= \frac{11}{2} + \frac{7}{2} + 8 + \frac{13}{2}$$

$$= \frac{31}{2} + 8 = \frac{47}{2}$$

Alternate

Let $I = \int_{-3}^{2}\{|x+1| + |x+2| + |x-1|\}dx$

$$= \int_{-3}^{-1}|x+1|dx + \int_{-1}^{2}|x+1|dx + \int_{-3}^{-2}|x+2|dx$$

$$+ \int_{-2}^{2}|x+2|dx + \int_{-3}^{1}|x-1|dx$$

$$+ \int_{1}^{2}|x-1|dx$$

$$= -\int_{-3}^{-1}(x+1)dx + \int_{-1}^{2}(x+1)dx - \int_{-3}^{-2}(x+2)dx$$

$$+ \int_{-2}^{2}(x+2)dx - \int_{-3}^{1}(x-1)dx + \int_{1}^{2}(x-1)dx$$

$$= -\left(\frac{x^2}{2}+x\right)_{-3}^{-1} + \left(\frac{x^2}{2}+x\right)_{-1}^{2} - \left(\frac{x^2}{2}+2x\right)_{-3}^{-2}$$

$$+\left(\frac{x^2}{2}+2x\right)_{-2}^{2}-\left(\frac{x^2}{2}-x\right)_{-3}^{1}-\left(\frac{x^2}{2}-x\right)_{1}^{2}$$
$$=\frac{47}{2}$$

17 **(b)**

$$\int_0^3 |x^3+x^2+3x|dx=\int_0^3 (x^3+x^2+3x)dx$$
$$=\left[\frac{x^4}{4}+\frac{x^3}{3}+\frac{3x^2}{2}\right]_0^3$$
$$=\frac{81}{4}+\frac{27}{3}+\frac{27}{2}=\frac{171}{4}$$

18 **(b)**

$$\int \frac{dx}{\sin x\cos x}=\int \frac{\cos x}{\sin x\cos^2 x}dx$$
$$=\int \frac{1}{\tan x}\sec^2 x\,dx=\log|\tan x|+c$$

19 **(b)**

Let $I=\int_0^{2n\pi}\left\{|\sin x|-\frac{1}{2}|\sin x|\right\}dx$

$$=\int_0^{2n\pi}\frac{1}{2}|\sin x|dx$$
$$=\frac{1}{2}\left[\int_0^{2\pi}|\sin x|dx+\int_{2\pi}^{4\pi}|\sin x|\,dx+\ldots\right.$$
$$\left.+\int_{2(n-1)\pi}^{2n\pi}|\sin x\}dx\quad\right]$$

Now, $I_1=\int_0^{2\pi}|\sin x|dx$

$$=\int_0^{\pi}\sin x\,dx-\int_{\pi}^{2\pi}\sin x\,dx$$
$$=[-\cos x]_0^{\pi}+[\cos x\quad]_{\pi}^{2\pi}=4$$
$$\therefore\ I=\frac{1}{2}[4+4+4+\ldots+n\text{ times}]$$
$$=\frac{1}{2}(4n)=2n$$
$$I=\int_0^{2n\pi}\frac{1}{2}|\sin x|\,dx$$

Since $|\sin x|$ is periodic with period π.

$$\therefore\ I=2n\int_0^{\pi}\frac{|\sin x|dx}{2}$$
$$I=n\int_0^{\pi}\sin x\,dx$$
$$=[-\cos x\quad]_0^{\pi}$$
$$\Rightarrow I=2n$$

Alternate

$$I=\int_0^{2n\pi}\frac{1}{2}|\sin x|dx$$

Since, $|\sin x|$is periodic with period π.

$\therefore I + 2n\int_0^{\pi}\frac{|\sin x|}{2}dx = n\int_0^{\pi}\sin x\,dx = n[-\cos x\quad]_0^{\pi}$

$\Rightarrow\ \ I = 2n$

20 **(d)**

Given, $\int_a^b x^3\,dx = 0$ and $\int_a^b x^2\,dx = \frac{2}{3}$

If we take $a = -1$and $b = 1$, then it will satisfy the given integration.

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	D	C	A	B	A	A	C	D	A	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	C	B	D	C	B	B	B	D