DPP
DAILY PRACTICE PROBLEMS

Class : XIth
Date :

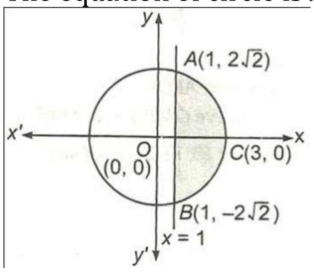
Solutio

Subject : Maths
DPP No. : 3

Topic :-Applications of integrales

1 **(b)**

The equation of circle is $x^2+y^2=9$



∴ Area of the smaller segment cut off from the circle $x^2+y^2=9$ by $x=1$, given by

∴ Required area, $A=2\int_1^3\sqrt{9-x^2}\,dx$

$=2\cdot\frac{1}{2}\left[x\sqrt{9-x^2}+9\sin^{-1}\frac{x}{3}\right]_1^3$

$=\left[\frac{9\pi}{2}-\sqrt{8}-9\sin^{-1}\left(\frac{1}{3}\right)\right]$

$=\left[9\left(\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{3}\right)\right)-\sqrt{8}\right]$

$=\left[9\cos^{-1}\left(\frac{1}{3}\right)-\sqrt{8}\right]$

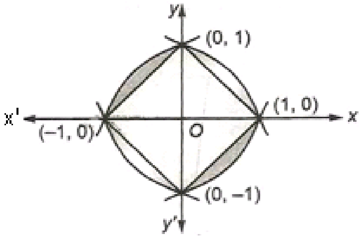
$=[9\sec^{-1}(3)-\sqrt{8}]$ sq unit

2 **(b)**

Since, $|x|+|y|=1$

$\Rightarrow\begin{cases}x+y=1, x>0, y>0\\ x-y=1, x>0, y<0\\ -x+y=1, x<0, y>0\\ -x-y=1, x<0, y<0\end{cases}$

and $1-y^2=|x|$



$\Rightarrow\begin{cases}1-y^2=x, x\ge 0\\ 1-y^2=-x, x<0\end{cases}$

∴ Required area $=\left|2\int_0^1\sqrt{(1-x)}\,dx\right|+\left|2\int_{-1}^0\sqrt{(x+1)}\,dx\right|-4\left(\frac{1}{2}\cdot 1\cdot 1\right)$

$= \frac{2}{3}$ sq unit

5 **(c)**

Required area

$= 2\int_{-1}^{3}\sqrt{x^2-1}\,dx$

$= 2\left[\frac{x\sqrt{x^2-1}}{2} - \frac{1}{2}\log|x+\sqrt{x^2-1}\right]_{-1}^{3}$

$= \left(x\sqrt{x^2-1} - \log|x+\sqrt{x^2-1}|\right)_{-1}^{3}$

$= 6\sqrt{2} - \log|3+2\sqrt{2}|$

6 **(c)**

Requred area $= \int_1^2 \log x \; dx$

$= [x\log x - x]_1^2 = 2\log 2 - 1$

$= \log 4 - \log e = \log\left(\frac{4}{e}\right)$

8 **(d)**

$\int_1^b f(x)dx = \sqrt{(b^2+1)} - \sqrt{2}$

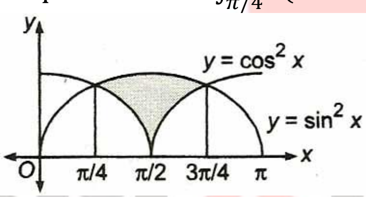
On differentiating both sides w.r.t. b, we get

$f(b) = \frac{b}{\sqrt{(b^2+1)}}$

Hence, $f(x) = \frac{x}{\sqrt{(x^2+1)}}$

9 **(c)**

Required area $= \int_{\pi/4}^{3\pi/4}(\sin^2 x - \cos^2 x)dx$



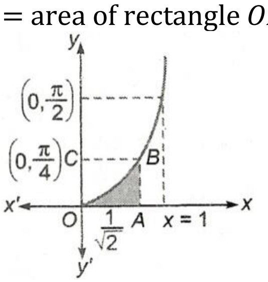
$= -\int_{\pi/4}^{3\pi/4}\cos(2x)dx = -\left[\frac{\sin 2x}{2}\right]_{\pi/4}^{3\pi/4}$

$= -\frac{1}{2}\left(\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right) = 1$ sq unit

10 **(d)**

Required area

= area of rectangle $OABC$ − area of curve $OBCO$



$= \frac{\pi}{4\sqrt{2}} - \int_0^{\frac{\pi}{4}}\sin y\,dy$

$= \frac{\pi}{4\sqrt{2}} + [\cos y]_0^{\frac{\pi}{4}}$

$= \left[\frac{\pi}{4\sqrt{2}} + \left(\frac{1}{\sqrt{2}} - 1\right)\right]$ sq unit

13 **(d)**

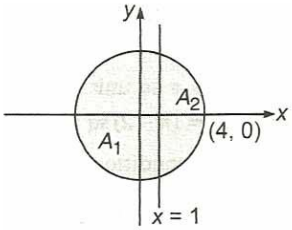
Required area $= \int_1^{1.7} [x] dx$

$= \int_1^{1.7} dx = 1.7 - 1 = 0.7 = \frac{7}{10}$

14 **(d)**

Equation of circle is $x^2 + y^2 = 16$

$\therefore$ Total area of circle $= A_1 + A_2 = 16\pi$ ...(i)



$\frac{A_1}{A_2} = \frac{16\pi}{A_2} - 1$ [on dividing Eq. (i) by A_2]

and $A_2 = 2\int_1^4 \sqrt{16 - x^2} dx$

$A_2 = 2\left\{\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right\}_1^4$

$= 2\left\{4\pi - \frac{\sqrt{15}}{2} - 8\sin^{-1}\left(\frac{1}{4}\right)\right\}$

$= 8\pi - \sqrt{15} - 16\sin^{-1}\left(\frac{1}{4}\right)$

$\therefore \frac{A_1}{A_2} = \frac{16\pi}{8\pi - \sqrt{15} - 16\sin^{-1}\left(\frac{1}{4}\right)} - 1$

15 **(b)**

Given equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Here, $a = 5, b = 4$

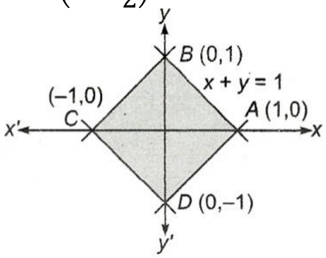
We know that the area of an ellipse $\frac{x^4}{a^2} + \frac{y^2}{b^2} = 1$ is

$\pi ab = \pi(5)(4) = 20\pi$ sq unit

17 **(d)**

Area $= 4\int_0^1 (1 - x) dx = 4\left[x - \frac{x^2}{2}\right]_0^1$

$= 4\left(1 - \frac{1}{2}\right) = 2$ sq units



Alternate

From figure $ABCD$ is square, whose diagonals AC and BD are of length 2 unit.

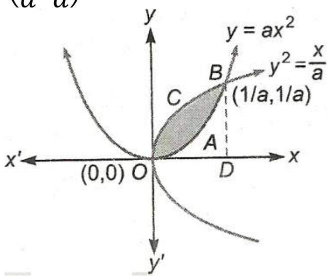
Hence, Required area $= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 2 \times 2$

$= 2$ sq units

19 **(a)**

The points of intersection of given curves are (0,0) and $\left(\frac{1}{a}, \frac{1}{a}\right)$



$\therefore$ Required area $OABCO =$ area of $OCBDO -$ area of $OABDO$

$\Rightarrow \int_0^{1/a} \left(\sqrt{\frac{x}{a}} - ax^2\right) dx = 1$ [given]

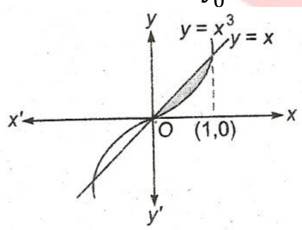
$\Rightarrow \left(\frac{1}{\sqrt{a}} \cdot \frac{x^{3/2}}{3/2} - \frac{ax^3}{3}\right)_0^{1/a} = 1$

$\Rightarrow \frac{2}{3a^2} - \frac{1}{3a^2} = 1$

$\Rightarrow a^2 = \frac{1}{3} \Rightarrow a = \frac{1}{\sqrt{3}}$ [as $a > 0$]

20 **(a)**

Required area $= 2\int_0^1 (x - x^3) dx$



$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 2\left[\frac{1}{2} - \frac{1}{4}\right] = \frac{1}{2}$ sq unit

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	B	B	B	A	C	C	C	D	C	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	A	C	D	D	B	C	D	B	A	A