DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. :3

Topic :-DIFFERENTIAL EQUATIONS

1 **(a)**

It is a linear differential equation of the form of $\frac{dy}{dx} + Py = Q$.

$\Rightarrow P = \sec^2 x, Q = \tan x \sec^2 x$

$\therefore \text{IF} = e^{\int P\,dx} = e^{\int \sec^2 x\,dx} = e^{\tan x}$

Solution is $ye^{\tan x} = \int \tan x e^{\tan x} \sec^2 x\,dx + c$

$\Rightarrow ye^{\tan x} = \tan x\, e^{\tan x} - e^{\tan x} + c$

$\Rightarrow y = \tan x - 1 + ce^{-\tan x}$

2 **(b)**

We have,

$$y\frac{dy}{dx} + x = a \Rightarrow y\,dy + x\,dx = a\,dx$$

Integrating, we get

$\frac{y^2}{2} + \frac{x^2}{2} = ax + C \Rightarrow x^2 + y^2 - 2\,ax + 2\,C = 0,$

which represents a set of circles having centre on x-axis

3 **(d)**

$\because$ Equation of normal at (x, y) is

$$Y - y = \frac{dx}{dy}(X - x)$$

Put, $y = 0$

Then, $X = x + y\,\frac{dy}{dx}$

Given, $y^2 = 2x\,X$

$$\Rightarrow y^2 = 2x\left(x + y\,\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - 2x^2}{2xy} = \frac{\left(\frac{y}{x}\right)^2 - 2}{2\left(\frac{y}{x}\right)}$$

Put $y = vx$, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Then, $v + x\frac{dv}{dx} = \frac{v^2-2}{2v}$

$$\Rightarrow x\frac{dv}{dx} = -\frac{(2+v^2)}{2v}$$

$$\Rightarrow \frac{2v\,dv}{(2+v^2)} + \frac{dv}{x} = 0$$

On integrating both sides, we get

In $(2 + v^2) +$ In$|x| =$ In c

$\Rightarrow$ In $\left(|x|(2+v^2)\right) =$ In c

$$\Rightarrow |x|\left(2 + \frac{y^2}{x^2}\right) = c$$

∵ It passes through (2, 1), then

$$2\left(2+\frac{1}{4}\right)=c$$

$$\Rightarrow c=\frac{9}{2}$$

Then, $|x|\left(2+\frac{y^2}{x^2}\right)=\frac{9}{2}$

$$\Rightarrow 2x^2+y^2=\frac{9}{2}|x|$$

$$\Rightarrow 4x^2+2y^2=9|x|$$

4 **(a)**

We have,

$$y\,dx-x\,dy-3x^2y^2\,e^{x^3}dx=0$$

$$\Rightarrow y\,dx-x\,dy=3x^2y^2e^{x^3}dx$$

$$\Rightarrow \frac{y\,dx-x\,dy}{y^2}=3x^2e^{x^3}dx$$

$$\Rightarrow d\left(\frac{x}{y}\right)=d\left(e^{x^3}\right)\Rightarrow \frac{x}{y}=e^{x^3}+C$$

5 **(c)**

Given, $\frac{dy}{dx}=\frac{ax+h}{by+k}$

$$\Rightarrow \int(by+k)\,dy=-\int(ax+h)\,dx$$

$$\Rightarrow \frac{by^2}{2}+ky=\frac{ax^2}{2}+hx+c$$

Thus, above equation represents a parabola, if

$a=0$ and $b\neq o$

Or $b=0$ and $a\neq 0$

6 **(b)**

The equations of the ellipses centred at the origin are given by $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a,b are arbitrary constants

Differentiating both sides w.r.t. to x, we get

$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$$

$\Rightarrow \frac{x}{a^2}+\frac{y\,y_1}{b^2}=0$ ...(i)

Differentiating (i) w.r.t. x, we get

$\frac{1}{a^2}+\frac{y_1^2}{b^2}+\frac{y\,y_2}{b^2}=0$ ...(ii)

Multiplying (ii) by x and subtracting it from (i), we get

$$\frac{1}{b^2}\{y\,y_1-x\,y_1^2-x\,y\,y_2\}=0\Rightarrow xy\,y_2+xy_1^2-y\,y_1=0$$

7 **(b)**

Given equation is $y=ax^{n+1}+bx^{-n}$

On differentiating with respect to x, we get

$$\frac{dy}{dx}=a(n+1)x^n-bnx^{-n-1}$$

Again, on differentiating , we get

$$\frac{d^2y}{dx^2}=an(n+1)x^{n-1}+bn(n+1)x^{-n-2}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2}=an(n+1)x^{n+1}+bn(n+1)x^{-n}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2}=n(n+1)(ax^{n+1}+bx^{-n})$$

$\Rightarrow \dfrac{x^2 d^2 y}{dx^2} = n(n+1)y$

8 **(b)**

Given, $y = a\cos(x+b)$

$\Rightarrow \dfrac{dy}{dx} = -a\sin(x+b)$

$\Rightarrow \dfrac{d^2y}{dx^2} = -a\cos(x+b) = -y$

$\Rightarrow \dfrac{d^2y}{dx^2} + y = 0$

9 **(a)**

Here, $\frac{dy}{dt} - \left(\frac{1}{1+t}\right)y = \frac{1}{(1+t)}$ and $y(0) = -1$

Which represents linear differential equation of first order.

$\text{IF} = e^{\int -\left(\frac{t}{1+t}\right)dt} = e^{-t+\log|1+t|} = e^{-t}(1+t)$

$\therefore$ Required solution is

$y(\text{IF}) = [\int Q(\text{IF})dt\,] + c$

$\Rightarrow ye^{-t}(1+t) = \int \frac{1}{1+t} \cdot e^{-t}(1+t)dt + c = \int e^{-t}dt + c$

$\Rightarrow ye^{-t}(1+t) = -e^{-t} + c$

Since, $y(0) = -1 \Rightarrow -1 \cdot e^0(1+0) = -e^0 + c$

$\Rightarrow c = 0$

$\therefore y = -\frac{1}{(1+t)}$ and $y(1) = -\frac{1}{2}$

10 **(b)**

Given, $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1 - \sqrt{x}$

$\therefore \quad \text{IF} = e^{\int \frac{1}{(1-x)\sqrt{x}}dx}$

Put $\sqrt{x} = t$

$\Rightarrow \quad \frac{1}{2\sqrt{x}}\,dx = dt$

$\therefore \quad \text{IF} = e^{\int \frac{2}{1-t^2}dt}$

$= e^{\frac{2}{2}\log\left(\frac{1+t}{1-t}\right)} = \frac{1+t}{1-t} = \frac{1+\sqrt{x}}{1-\sqrt{x}}$

11 **(b)**

Given, $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

Put $y = vx$

$\Rightarrow \quad \frac{dy}{dx} = v + x\,\frac{dv}{dx}$

$\therefore \quad v + x\,\frac{dv}{dx} = \frac{x^2+v^2x^2}{2xvx}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{1-v^2}{2v}$

$\Rightarrow \quad \frac{2v}{1-v^2}dv = \frac{dx}{x}$

$\Rightarrow \quad -\log(1-v^2) = \log x + \log c$

$\Rightarrow \quad \log(1-v^2)^{-1} = \log xc$

$\Rightarrow \quad \left(\frac{x^2-y^2}{x^2}\right)^{-1} = xc$

$\Rightarrow \quad \frac{x^2}{x^2-y^2} = xc$

$\Rightarrow \quad x = c(x^2-y^2)$

12 **(c)**

$\because y = u^n$

$\therefore \dfrac{dy}{dx} = nu^{n-1}\dfrac{du}{dx}$

On substituting the values of y and $\frac{dy}{dx}$ in the given equation, then

$2x^4 \cdot u^n \cdot nu^{n-1}\dfrac{du}{dx} + u^{4n} = 4x^6$

$\Rightarrow \dfrac{du}{dx} = \dfrac{4x^6 - u^{4n}}{2nx^4u^{2n-1}}$

Since, it is homogeneous. Then, the degree of $4x^6 - u^{4n}$ and $2nx^4u^{2n-1}$ must be same.

$\therefore 4n = 6$ and $4 + 2n - 1 = 6$

Then, we get $n = \frac{3}{2}$

13 **(b)**

Given equation is $y = ax\cos\left(\frac{1}{x} + b\right)$ .....(i)

On differentiating Eq. (i), we get

$y_1 = a\left[\cos\left(\dfrac{1}{x} + b\right) - x\sin\left(\dfrac{1}{x} + b\right)\left(\dfrac{-1}{x^2}\right)\right]$

$\Rightarrow y_1 = a\left[\cos\left(\frac{1}{x} + b\right) + \frac{1}{x}\sin\left(\frac{1}{x} + b\right)\right]$ .....(ii)

Again, on differentiating Eq. (ii), we get

$y_2 = a\left[-\sin\left(\dfrac{1}{x} + b\right)\left(-\dfrac{1}{x^2}\right) + \dfrac{1}{x}\cos\left(\dfrac{1}{x} + b\right)\left(-\dfrac{1}{x^2}\right) - \dfrac{1}{x^2}\sin\left(\dfrac{1}{x} + b\right)\right]$

$\Rightarrow y_2 = \dfrac{-a}{x^3}\cos\left(\dfrac{1}{x} + b\right) = \dfrac{-ax}{x^4}\cos\left(\dfrac{1}{x} + b\right) = \dfrac{-y}{x^4}$

$\Rightarrow x^4y_2 + y = 0$

14 **(b)**

Given, $dy = x\log x \; dx$

$\Rightarrow \quad y = \frac{x^2}{2}\log x - \int \frac{x}{2}dx$ [integrating]

$\Rightarrow \quad y = \frac{x^2}{2}\log x - \frac{x^2}{4} + c$

15 **(c)**

Given equation is $y = \sec(\tan^{-1}x)$

On differentiating w. r. t. x, we get

$\dfrac{dy}{dx} = \sec(\tan^{-1}x)\tan(\tan^{-1}x) \cdot \dfrac{1}{1+x^2} = \dfrac{xy}{1+x^2}$ $[\because \tan(\tan^{-1}x) = x]$

$\Rightarrow (1+x^2)\dfrac{dy}{dx} = xy$

16 **(a)**

$\dfrac{dy}{dx}\tan y = \sin(x+y) + \sin(x-y)$

$\Rightarrow \dfrac{dy}{dx}\tan y = 2\sin x\cos y$

$\Rightarrow \displaystyle\int \tan y \sec y \; dy = 2\int \sin x \; dx$

$\Rightarrow \sec y + 2\cos x = c$

17 **(b)**

Equation of family of parabolas with focus at $(0, 0)$ and x-axis as axis is

$y^2 = 4a(x+a)$ ....(i)

On differentiating Eq. (i), we get

$2yy_1 = 4a$, putting the value of a in Eq. (i)

$\Rightarrow y^2 = 2yy_1\left(x + \dfrac{yy_1}{2}\right)$

$\Rightarrow y = 2xy_1 + yy_1^2$

$\Rightarrow y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} = y$

18 **(b)**

We have,

$\frac{dy}{dx} + y = \frac{1+y}{x} \Rightarrow \frac{dy}{dx} + \left(1 - \frac{1}{x}\right)y = \frac{1}{x}$

$\therefore \text{I. F.} = e^{\int\left(1-\frac{1}{x}\right)dx} = e^{x-\log x} = \frac{1}{x}e^x$

19 **(c)**

Given, $y^2 = 4a(x-b)$

$\Rightarrow \quad 2y\frac{dy}{dx} = 4a$

$\Rightarrow \quad 2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 0$

$\Rightarrow \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

20 **(a)**

The given equation can be rewritten as, $\frac{d^2y}{dx^2} = -\sin x$.

On integrating the given equation

$\int \frac{d^2y}{dx^2}dx = \int -\sin x\, dx + c$

$\Rightarrow \frac{dy}{dx} = -(-\cos x) + c = \cos x + c$

Again, on integrating, we get

$\int \frac{dy}{dx}dx = \int \cos x\, dx + \int c\, dx + d$

$y = \sin x + cx + d$

ANSWER-KEY

Q.	1	2	3	4	5	6	7	8	9	10
A.	A	B	D	A	C	B	B	B	A	B
Q.	11	12	13	14	15	16	17	18	19	20

A.	B	C	B	B	C	A	B	B	C	A