

## DPP

DAILY PRACTICE PROBLEMS

CLASS : XII<sup>th</sup>  
DATE :

**SOLUTION**

SUBJECT : MATHS  
DPP NO. :3

### Topic :-VECTOR ALGEBRA

1 (a)

Taking A as the origin, let the position vectors of B and C be  $\vec{b}$  and  $\vec{c}$  respectively

$$\therefore \vec{BE} + \vec{AF} = \left(\frac{\vec{c}}{2} - \vec{b}\right) + \left(\frac{\vec{b} + \vec{c}}{2} - \vec{0}\right) = \vec{c} - \frac{\vec{b}}{2} = \vec{DC}$$

2 (a)

Since,  $\vec{a}, \vec{b}, \vec{c}$  are mutually perpendicular unit vectors.

$$\Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$\text{and } \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0 \quad \dots(i)$$

$$\text{Now, } |\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c})$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

$$= 1 + 1 + 1 + 0 = 3 \quad [\text{from Eq. (i)}]$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$$

3 (c)

Any vector lying in the plane of  $\vec{a}$  and  $\vec{b}$  is of the form  $x\vec{a} + y\vec{b}$

It is given that  $\vec{c}$  is parallel to the plane of  $\vec{a}$  and  $\vec{b}$

$$\therefore \vec{c} = \lambda(x\vec{a} + y\vec{b}) \text{ for some scalar } \lambda$$

$$\Rightarrow d\hat{i} + \hat{j} + (2d - 1)\hat{k} = \lambda\{x(\hat{i} - 2\hat{j} + 3\hat{k}) + y(3\hat{i} + 3\hat{j} - \hat{k})\}$$

$$\Rightarrow d\hat{i} + \hat{j} + (2d - 1)\hat{k} = \lambda\{(x + 3y)\hat{i} + (-2x + 3y)\hat{j} + (3x - y)\hat{k}\}$$

$$\Rightarrow \lambda(x + 3y) = d, \lambda(-2x + 3y) = 1 \text{ and } \lambda(3x - y) = (2d - 1)$$

[ $\because \hat{i}, \hat{j}, \hat{k}$  are non-coplanar]

Solving  $\lambda(x + 3y) = d$  and  $3x - y = 2d - 1$ , we get

$$x = \frac{7d-3}{10\lambda} \text{ and } y = \frac{d+1}{10\lambda}$$

Substituting these values in  $\lambda(x + 3y) = d$ , we get  $11d = -1$

ALTER clearly,  $\vec{c}$  is perpendicular to  $\vec{a} \times \vec{b}$

$$\therefore \vec{c} \cdot (\vec{a} \times \vec{b}) = 0$$

$$\Rightarrow [\vec{c}\vec{a}\vec{b}] = 0 \Rightarrow \begin{vmatrix} d & 1 & 2d-1 \\ 1 & -2 & 3 \\ 3 & 3 & -1 \end{vmatrix} = 0 \Rightarrow 11d = -1$$

4 (c)

$\because \vec{p}, \vec{q}, \vec{r}$  are reciprocal vectors  $\vec{a}, \vec{b}, \vec{c}$  respectively.

$$\therefore \vec{p} \cdot \vec{a} = 1, \vec{p} \cdot \vec{b} = 0, \vec{p} \cdot \vec{c} \text{ etc.}$$

$$\therefore (l\vec{a} + m\vec{b} + n\vec{c}) \cdot (l\vec{p} + m\vec{q} + n\vec{r}) = l^2 + m^2 + n^2$$

5 (b)

$$\text{Given expression} = 2(1 + 1 + 1) - 2\sum(\vec{a} \cdot \vec{b})$$

$$= 6 - 2\sum(\vec{a} \cdot \vec{b}) \quad \dots(i)$$

$$\text{But } (\vec{a} + \vec{b} + \vec{c})^2 \geq 0$$

$$\therefore (1 + 1 + 1) + 2 \sum \vec{a} \cdot \vec{b} \geq 0$$

$$\therefore 3 \geq -2 \sum \vec{a} \cdot \vec{b} \quad \dots \text{(ii)}$$

From relations (i) and (ii), we get

$$\text{Given expression} \leq 6 + 3 = 9$$

6 (a)

$$\text{Let } \vec{OA} = \hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{OB} = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

$$\therefore \vec{AB} = 2\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\therefore \text{work done, } W = \vec{F} \cdot \vec{AB}$$

$$= (2\hat{i} - 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} + 2\hat{j} + 2\hat{k})$$

$$= 4 - 6 + 4 = 2$$

7 (d)

$$\vec{AC} = (a\hat{i} - 3\hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = (a - 2)\hat{i} - 2\hat{j}$$

$$\text{and } \vec{BC} = (a\hat{i} - 3\hat{j} + \hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k}) = (a - 1)\hat{i} + 6\hat{k}$$

Since, the  $\Delta ABC$  is right angled at  $C$ , then

$$\vec{AC} \cdot \vec{BC} = 0$$

$$\Rightarrow \{(a - 2)\hat{i} - 2\hat{j}\} \cdot \{(a - 1)\hat{i} + 6\hat{k}\} = 0$$

$$\Rightarrow (a - 2)(a - 1) = 0 \Rightarrow a = 1 \text{ and } 2$$

8 (a)

We have,

$$(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$$

$$\Leftrightarrow -\vec{c} \times (\vec{a} \times \vec{b}) = \vec{a} \times (\vec{b} \times \vec{c})$$

$$\Leftrightarrow -\{(\vec{c} \cdot \vec{b})\vec{a} - (\vec{c} \cdot \vec{a})\vec{b}\} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

$$\Leftrightarrow (\vec{a} \cdot \vec{b})\vec{c} - (\vec{c} \cdot \vec{b})\vec{a} = 0$$

$$\Leftrightarrow (\vec{b} \cdot \vec{a})\vec{c} - (\vec{b} \cdot \vec{c})\vec{a} = 0$$

$$\Leftrightarrow \vec{b} \times (\vec{c} \times \vec{a}) = 0$$

9 (b)

Clearly,

$$(\vec{a} + \vec{b}) \times \{\vec{c} - (\vec{a} + \vec{b})\}$$

$$= (\vec{a} + \vec{b}) \times \vec{c} - (\vec{a} + \vec{b}) \times (\vec{a} + \vec{b}) = (\vec{a} + \vec{b}) \times \vec{c}$$

10 (a)

$$\vec{PQ} = (2\hat{i} - \hat{j} + 3\hat{k}) - (\hat{i} - \hat{j} + 2\hat{k})$$

$$= \hat{i} + \hat{k}$$

$$\text{and } \vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$$

$$\therefore \text{Moment} = |\vec{PQ} \times \vec{F}|$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 3 & 2 & -4 \end{vmatrix}$$

$$= -2\hat{i} + 7\hat{j} + 2\hat{k}$$

$$\therefore \text{Magnitude of moment} = \sqrt{4 + 49 + 4} = \sqrt{57}$$

11 (b)

$$\text{Since, } |\vec{a} + \vec{b}| = \sqrt{3}$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 3$$

$$\Rightarrow \vec{a} \cdot \vec{b} = \frac{1}{2} \dots \dots \text{(i)}$$

$$\therefore [|\vec{a}| = |\vec{b}| = 1, \text{ given}]$$

$$\therefore (3\vec{a} - 4\vec{b}) \cdot (2\vec{a} + 5\vec{b}) = 6 + 7\vec{a} \cdot \vec{b} - 20$$

$$= 6 + \frac{7}{2} - 20$$

$$= -\frac{21}{2} \quad [\text{from Eq. (i)}]$$

12 (c)

We have,

$$\hat{a} \times (\hat{b} \times \hat{c}) = \frac{1}{2} \hat{b}$$

$$\Rightarrow (\hat{a} \cdot \hat{c}) \hat{b} - (\hat{a} \cdot \hat{b}) \hat{c} = \frac{1}{2} \hat{b}$$

$$\Rightarrow \left\{ (\hat{a} \cdot \hat{c}) - \frac{1}{2} \right\} \hat{b} - (\hat{a} \cdot \hat{b}) \hat{c} = 0$$

$$\Rightarrow \hat{a} \cdot \hat{c} - \frac{1}{2} = 0 \text{ and } \hat{a} \cdot \hat{b} = 0 \quad \left[ \begin{array}{l} \because \hat{b}, \hat{c} \\ \text{are non-collinear vectors} \end{array} \right]$$

$$\Rightarrow \cos \theta = \frac{1}{2}, \text{ where } \theta \text{ is the angle between } \hat{a} \text{ and } \hat{c}$$

$$\Rightarrow \theta = \pi/3$$

14 (b)

The given line is parallel to the vector  $\vec{n} = \hat{i} - \hat{j} + 2\hat{k}$ . The required plane passing through the point  $(2, 3, 1)$  i.e.  $2\hat{i} + 3\hat{j} + \hat{k}$  and is perpendicular to the vector

$$\vec{n} = \hat{i} - \hat{j} + 2\hat{k}$$

$\therefore$  Its equation is

$$[(\vec{r} - (2\hat{i} + 3\hat{j} + \hat{k})) \cdot (\hat{i} - \hat{j} + 2\hat{k})] = 0$$

$$\Rightarrow \vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 1$$

15 (c)

$$(\vec{a} - \vec{b}) \cdot (\vec{b} \times \vec{c} - \vec{b} \times \vec{a} + \vec{c} \times \vec{a})$$

$$= \vec{a} \cdot (\vec{b} \times \vec{c}) - \vec{b} \cdot (\vec{c} \times \vec{a})$$

$$= [\vec{a} \vec{b} \vec{c}] - [\vec{b} \vec{c} \vec{a}] = 0$$

16 (a)

We have,

$$|\hat{n}_1 + \hat{n}_2|^2 = |\hat{n}_1|^2 + |\hat{n}_2|^2 + 2\hat{n}_1 \cdot \hat{n}_2$$

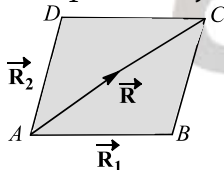
$$\Rightarrow |\hat{n}_1 + \hat{n}_2|^2 = |\hat{n}_1|^2 + |\hat{n}_2|^2 + 2|\hat{n}_1| |\hat{n}_2| \cos \theta$$

$$\Rightarrow |\hat{n}_1 + \hat{n}_2|^2 = 1 + 1 + 2 \cos \theta = 4 \cos^2 \frac{\theta}{2}$$

$$\therefore \cos \frac{\theta}{2} = \frac{1}{2} |\hat{n}_1 + \hat{n}_2|$$

17 (d)

$$\text{Let } \vec{R}_1 = 2\hat{i} + 4\hat{j} - 5\hat{k}$$



$$\text{and } \vec{R}_2 = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\therefore \vec{R} \text{ (along } \vec{AC}) = \vec{R}_1 + \vec{R}_2$$

$$= 3\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\therefore \vec{a} \text{ (unit vector along } AC) = \frac{\vec{R}}{|\vec{R}|}$$

$$= \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{9 + 36 + 4}}$$

$$= \frac{1}{7}(3\hat{i} + 6\hat{j} - 2\hat{k})$$

18 (a)

Let  $P(60\hat{i} + 3\hat{j})$ ,  $Q(40\hat{i} - 8\hat{j})$  and  $R(a\hat{i} - 52\hat{j})$  be the collinear points. Then  $\overrightarrow{PQ} = \lambda\overrightarrow{QR}$  for some scalar  $\lambda$

$$\Rightarrow (-20\hat{i} - 11\hat{j}) = \lambda[(a - 40)\hat{i} - 44\hat{j}]$$

$$\Rightarrow \lambda(a - 40) = -20, -44\lambda = -11$$

$$\Rightarrow \lambda(a - 40) = -20, \lambda = \frac{1}{4}$$

$$\therefore a - 40 = -20 \times 4 \Rightarrow a = -40$$

19 (a)

We have,

$$\vec{a} + \vec{b} + \vec{c} = \alpha\vec{d} \text{ and } \vec{b} + \vec{c} + \vec{d} = \beta\vec{a}$$

$$\Rightarrow \vec{a} + \vec{b} + \vec{c} + \vec{d} = (\alpha + 1)\vec{d} \text{ and } \vec{a} + \vec{b} + \vec{c} + \vec{d} = (\beta + 1)\vec{a}$$

$$\Rightarrow (\alpha + 1)\vec{d} = (\beta + 1)\vec{a}$$

If  $\alpha \neq -1$ , then

$$(\alpha + 1)\vec{d} = (\beta + 1)\vec{a} \Rightarrow \vec{d} = \frac{\beta + 1}{\alpha + 1}\vec{a}$$

$$\therefore \vec{a} + \vec{b} + \vec{c} = \alpha\vec{d}$$

$$\Rightarrow \vec{a} + \vec{b} + \vec{c} = \alpha \left( \frac{\beta + 1}{\alpha + 1} \right) \vec{a}$$

$$\Rightarrow \left\{ 1 - \frac{\alpha(\beta + 1)}{\alpha + 1} \right\} \vec{a} + \vec{b} + \vec{c} = 0$$

$$\Rightarrow \vec{a}, \vec{b}, \vec{c} \text{ are coplanar}$$

It is a contradiction to the given condition

$$\therefore \alpha = -1 \Rightarrow \vec{a} + \vec{b} + \vec{c} = 0$$

20 (c)

Let the unit vector  $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$  is perpendicular to  $\hat{i} - \hat{j}$ , then we get

$$\frac{(\hat{i} + \hat{j}) \cdot (\hat{i} - \hat{j})}{\sqrt{2}} = \frac{1 - 1}{\sqrt{2}} = 0$$

$$\therefore \frac{\hat{i} + \hat{j}}{\sqrt{2}} \text{ is the unit vector}$$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	C	C	B	A	D	A	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	C	C	B	C	A	D	A	A	C



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