DPP
DAILY PRACTICE PROBLEMS

CLASS : XIIth
DATE :

SOLUTIO

SUBJECT : MATHS
DPP NO. :3

Topic :-THREE DIMENSIONAL GEOMETRY

1 **(c)**

Given equation of line is

$$\frac{3-x}{1}=\frac{y-2}{5}=\frac{2z-3}{1}$$

$$\Rightarrow \frac{x-3}{-1}=\frac{y-2}{5}=\frac{z-\frac{3}{2}}{\frac{1}{2}}$$

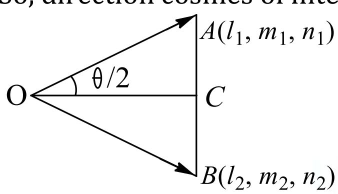
$\therefore$ Direction ratios of line are $-1, 5, \frac{1}{2}$

2 **(b)**

$\because \ \overrightarrow{\mathbf{OC}}=\left(\frac{l_1+l_2}{2}, \frac{m_1+m_2}{2}, \frac{n_1+n_2}{2}\right)$

And $|\overrightarrow{\mathbf{OC}}|=\cos\frac{\theta}{2}$

So, direction cosines of internal angle bisector are



$$\frac{l_1+l_2}{2\cos\frac{\theta}{2}}, \frac{m_1+m_2}{2\cos\frac{\theta}{2}}, \frac{n_1+n_2}{2\cos\frac{\theta}{2}}$$

3 **(c)**

The given equation of plane is $\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$

On comparing with $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, we get

$a=2, b=3, c=4$

Area of $\Delta\, ABC=\frac{1}{2}\sqrt{a^2b^2+b^2c^2+c^2a^2}$

$$\Delta=\frac{1}{2}\sqrt{4\times 9+9\times 16+16\times 4}$$

$$=\frac{1}{2}\sqrt{36+144+64}=\frac{1}{2}\sqrt{244}=\sqrt{61}$$

5 **(a)**

Let (u, v, w) be the centre of the sphere with radius r. Since, it passes through the origin

$\therefore u^2+v^2+w^2=r^2$. Equation of the diameter parallel to x-axis is

$\frac{x-u}{1}=\frac{y-v}{0}=\frac{z-w}{0}$ ...(i)

As it passes through u, v, w and direction ratios of x- axis are 1, 0, 0

The extremities of diameter are the points on Eq. (i) at a distance r from the centre (u, v, w)

$\therefore$ The required extremities are $P(r+u, v, w)$ and $Q(-r+u, v, w)$

P lies on the sphere $x^2+y^2+z^2-2rx=0$ as

$(r+u)^2+v^2+w^2-2r(r+u)=0$

Because $u^2+v^2+w^2=r^2$

and similarly Q lies on the sphere $x^2+y^2+z^2+2rx=0$

6 **(d)**

Distance of a point (1,1,1) from $x+y+z+k=0$ is

$$\left|\frac{1+1+1+k}{\sqrt{3}}\right|=\left|\frac{3+k}{\sqrt{3}}\right|$$

According to question

$$\left|\frac{3+k}{\sqrt{3}}\right|=\pm 2\sqrt{3}\ \Rightarrow k=3,-9$$

7 **(c)**

Since, given points divide the XOZ-plane.

$\therefore$ Required ratio $=-y_1:y_2=-3:7$

8 **(b)**

DC's of the given line are $\frac{1}{3},-\frac{2}{3},-\frac{2}{3}$

Hence, the equation of line can be point in the form

$$\frac{x-2}{1/3}=\frac{y+3}{-2/3}=\frac{z+5}{-2/3}=r$$

$\therefore$ Point is $\left(2+\frac{r}{3},-3-\frac{2r}{3},-5-\frac{2r}{3}\right)$

$\therefore r=\pm 6$

Points are $(4,-7,-9)$ and $(0,1,-1)$

9 **(a)**

The plane passes through $A(0,0,1), B(0,1,2)$ and $C(1,2,3)$. Therefore, a vector normal to the plane is given by

$$\vec{n}=\vec{A}B\times\vec{A}C=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 0 & 1 & 1\\ 1 & 2 & 2\end{vmatrix}=0\,\hat{\imath}+\hat{\jmath}-\hat{k}$$

Hence, direction ratios of normal to the plane are proportional to $0,1,-1$

10 **(b)**

Suppose xy-plane divides the join of (1, 2, 3) and (4, 2, 1) in the ratio $\lambda:1$. Then, the coordinates of the point of division are

$$\left(\frac{4\,\lambda+1}{\lambda+1},\frac{2\,\lambda+2}{\lambda+1},\frac{\lambda+3}{\lambda+1}\right)$$

This point lies on xy-plane

$\therefore$ z-coordinate $=0\Rightarrow\frac{\lambda+3}{\lambda+1}=0\Rightarrow\lambda=-3$

Hence, xy-plane divides the join of (1, 2, 3) and (4, 2, 1) externally in the ratio 3 : 1

ALTER We know that the XY-plane divides the segment joining $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ in the ratio $(-z_1):z_2$

$\therefore$ XY-plane divides the join of (1, 2, 3) and (4, 2, 1) in the ratio $-3:1$ i.e. $3:1$ externally

11 **(d)**

Let l,m,n be the direction cosines of $\vec{r}$. Then,

$l=m=n$ [Given]

$$\therefore l^2+m^2+n^2=1\Rightarrow 3\,l^2=1\Rightarrow l=\frac{1}{\sqrt{3}}=m=n$$

Now, $\vec{r}=|\vec{r}|\left(l\,\hat{\imath}+m\,\hat{\jmath}+n\,\hat{k}\right)$

$$\Rightarrow\vec{r}=6\left(\frac{1}{\sqrt{3}}\hat{\imath}+\frac{1}{\sqrt{3}}\hat{\jmath}+\frac{1}{\sqrt{3}}\hat{k}\right)=2\sqrt{3}(\hat{\imath}+\hat{\jmath}+\hat{k})$$

12 **(a)**

Since, direction ratio of given planes are $(2,-1,1)$ and $(1,1,2)$

$$\therefore\ \theta\cos^{-1}\left(\frac{2\times1-1\times1+1\times2}{\sqrt{4+1+1}\sqrt{1+1+4}}\right)$$

$= \cos^{-1}\left(\frac{3}{\sqrt{6}\sqrt{6}}\right)$

$= \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

13 **(b)**

The equation of a plane parallel to the plane $\vec{r} \cdot (4\hat{\imath} - 12\hat{\jmath} - 3\hat{k}) - 7 = 0$ is,

$\vec{r} \cdot (4\hat{\imath} - 12\hat{\jmath} - 3\hat{k}) + \lambda = 0$

This passes through $2\hat{\imath} - \hat{\jmath} - 4\hat{k}$

$\therefore (2\hat{\imath} - \hat{\jmath} - 4\hat{k}) \cdot (4\hat{\imath} - 12\hat{\jmath} - 3\hat{k}) + \lambda = 0$

$\Rightarrow 8 + 12 + 12 + \lambda = 0$

$\Rightarrow \lambda = -32$

So, the required plane is $\vec{r} \cdot (4\hat{\imath} - 12\hat{\jmath} - 3\hat{k}) - 32 = 0$

14 **(a)**

Equation of the give plane can be writer as

$(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) \cdot (x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\hat{\mathbf{k}}) = 8$

So, that the normal to the given plane is $3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and the required line being perpendicular to the plane is parallel to this normal and since, it passes through $3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$, its equation is

$\vec{\mathbf{r}} = 3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}} + \lambda(3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$

Where λ is a parameter

Since, this lie passes through the vector $3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$ *ie*, the point $(3, -5, 7)$ and is parallel to $3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$, its direction ratios are $3, -4, 5$

Its cartesian equation is $\frac{x-3}{3} = \frac{y+5}{-4} = \frac{z-7}{5}$

15 **(a)**

Given lines can be rewritten as

$$\frac{x - \frac{1}{3}}{1} = \frac{y - \frac{1}{3}}{2} = \frac{z - 1}{3}$$

This shows that DR's of given equation are (1, 2, 3).

16 **(d)**

Given line is parallel to $\vec{b} = -\hat{\imath} + \hat{\jmath} + \hat{k}$ and the given plane is normal to $\vec{n} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$

Let θ be the angle between the given line and given plane. Then,

$$\sin\theta = \frac{\vec{b} \cdot \vec{n}}{|\vec{b}|\,|\vec{n}|}$$

$$\Rightarrow \sin\theta = \frac{-3 + 2 - 1}{\sqrt{3}\,\sqrt{14}} \Rightarrow \theta = \sin^{-1}\left(\frac{-2}{\sqrt{42}}\right)$$

17 **(d)**

Let the source of light be situated at $A(a, 0, 0)$, where, $a \neq 0$

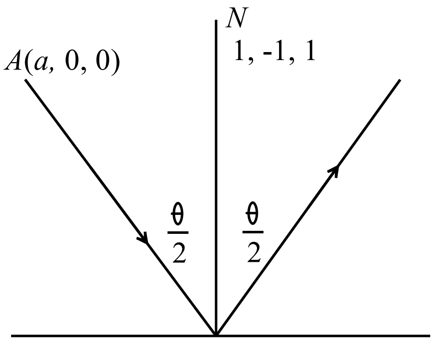
Let OA be the incident ray, OB be the reflected ray and ON be the normal to the mirror at O

$\therefore \angle AON = \angle NOB = \frac{\theta}{2}$ (say)

Direction ratios of $\vec{O}A$ are proportional to $a, 0, 0$ and so its direction cosines are 1, 0, 0

Direction cosines of ON are $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$\therefore \cos\frac{\theta}{2} = \frac{1}{\sqrt{3}}$



Let l, m, n be the direction cosines of the reflected ray OB. Then,

$$\frac{l+1}{2\cos\theta/2} = \frac{1}{\sqrt{3}}, \frac{m+0}{2\cos\theta/2} = -\frac{1}{\sqrt{3}} \text{ and }, \frac{n+0}{2\cos\theta/2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow l = \frac{2}{3} - 1, m = -\frac{2}{3}, n = \frac{2}{3}$$

$$\Rightarrow l = -\frac{1}{3}, m = -\frac{2}{3}, n = \frac{2}{3}$$

Hence, direction cosines of the reflected ray are $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

18 **(d)**

Given, $3lm - 4ln + mn = 0$ .....(i)

and $l + 2m + 3n = 0$ ...(ii)

From Eq. (ii), $l = -(2m + 3n)$ putting in Eq. (i)

$-3(2m + 3n)m + 4(2m + 3n)n + mn = 0$

$\Longrightarrow -6m^2 + 12n^2 = 0$

$\Longrightarrow m = \pm\sqrt{2}n$

Now, $m = \sqrt{2}n$

$\Longrightarrow l = -(2\sqrt{2}n + 3n) = -(2\sqrt{2} + 3)n$

$\therefore l : m : n = -(3 + 2\sqrt{2})n : \sqrt{2}n : n$

$= -(3 + 2\sqrt{2}) : \sqrt{2} : 1$

Also, $m = -\sqrt{2}n \Longrightarrow l = -(-2\sqrt{2} + 3)n$

$\therefore l : m : n = -(3 - 2\sqrt{2})n : -\sqrt{2} : n$

$= -(3 - 2\sqrt{2}) : -\sqrt{2} : 1$

$= \cos\theta$

$$= \frac{(3+2\sqrt{2})(3-2\sqrt{2}) + (\sqrt{2})(-\sqrt{2}) + 1 \cdot 1}{\sqrt{(3+2\sqrt{2})^2 + (\sqrt{2})^2 + 1^2}\sqrt{(3-2\sqrt{2})^2 + (-\sqrt{2})^2 + 1^2}}$$

$= 0$

$$\Longrightarrow \theta = \frac{\pi}{2}$$

19 **(a)**

Equation of plane passing through $(-1, 3, 0)$ is

$A(x + 1) + B(y - 3) + C(z - 0) = 0$ ....(i)

Also, plane (i) is passing through the points (2, 2, 1) and (1,1,3)

$3A - B + C = 0$ ...(ii)

And $2A - 2B + 3C = 0$ ...(iii)

On solving Eqs. (i) and (iii), we get

$$\frac{A}{-3+2} = \frac{B}{2-9} = \frac{C}{-6+2}$$

$\therefore A : B : C = -1 : -7 : -4$

$\Rightarrow A: B: C = 1: 7: 4$

From Eq. (i), $1(x+1) + 7(y-3) + 4(z) = 0$

$\Rightarrow x + 7y + 4z - 20 = 0$

$\therefore$ Distance from the plane to the point $(5, 7, 8)$

$$= \frac{1 \times 5 + 7 \times 7 + 4 \times 8 - 20}{\sqrt{1^2 + 7^2 + 4^2}}$$

$$= \frac{5 + 49 + 32 - 20}{\sqrt{66}} = \frac{66}{\sqrt{66}} = \sqrt{66}$$

20 **(a)**

The line of intersection of the plane $\vec{r} \cdot (3\hat{\imath} - \hat{\jmath} + \hat{k}) = 1$ and $\vec{r} \cdot (\hat{\imath} + 4\hat{\jmath} - 2\hat{k}) = 2$ is perpendicular to each of the normal vectors $\overrightarrow{n_1} = 3\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\overrightarrow{n_2} = \hat{\imath} + 4\hat{\jmath} - 2\hat{k}$ and hence it is parallel to the vector $\overrightarrow{n_1} \times \overrightarrow{n_2} = (3\hat{\imath} - \hat{\jmath} + \hat{k}) \times (\hat{\imath} + 4\hat{\jmath} - 2\hat{k}) = -2\hat{\imath} + 7\hat{\jmath} + 13\hat{k}$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	C	B	C	D	A	D	C	B	A	B
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	A	B	A	A	D	D	D	A	A