DPP

DAILY PRACTICE PROBLEMS

CLASS : XIIth | SOLUTIO | SUBJECT : MATHS

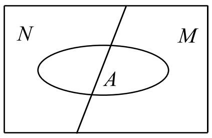
DATE : | | DPP NO. :3

Topic :-PROBABILITY

1 **(a)**

The total number of ways of selecting 3 integers from 20 natural numbers is $^{20}C_3 = 1140$. Their product is a multiple of 3 means at least one number is divisible by 3. The number which are divisible by 3 are 3, 6, 9,12, 15,18 and the number of ways of selecting at least one of them is $^6C_1 \times {^{14}C_2} + {^6C_2} \times {^{14}C_1} + {^6C_3} = 776$. Hence, the required probability is 776/1140=194/285

2 **(a)**



Let N be the event of picking up a normal die: $P(N) = 1/4$. Let M be the event of picking up a magnetic die : $P(M) = 3/4$. Let A be the event that die shows up 3

$\therefore P(A) = P(A \cap N) + P(A \cap M)$

$= P(N)P(A/N) + P(M)P(A/M)$

$= \frac{1}{4} \times \frac{1}{6} + \frac{3}{4} \times \frac{7}{24}$

$P(N/A) = \frac{P(N \cap A)}{P(A)} = \frac{(1/4)(1/6)}{7/24} = \frac{1}{7}$

3 **(a)**

$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$

$\therefore \; P(\overline{A}) = 1 - \frac{1}{2} - \frac{1}{2}, P(\overline{B}) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(\overline{C}) = 1 - \frac{1}{4} = \frac{3}{4}$

Therefore, the required probability is

$1 - P(\overline{A})P(\overline{B})P(\overline{C}) = 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$= 1 - \frac{1}{4} = \frac{3}{4}$

4 **(c)**

In the first 9 throws, we should have three sixes and six non-sixes; and a six in the 10th throw, and thereafter it does not matter whatever face appears. So, the required probability is

$^9C_3 \left(\frac{1}{6}\right)^3 \times \left(\frac{5}{6}\right)^6 = \frac{1}{6} \times 1 \times 1 \times 1 \times \ldots \times 1$

10 times

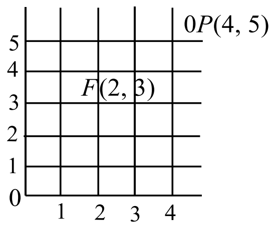
$= \frac{84 \times 5^6}{6^{10}}$

5 **(b)**

$n(S) = \frac{9!}{4!\,5!} = 126$

$n(A) = 0$ to F and F to P

$= \frac{5!}{2! \times 3!} \times \frac{4}{2! \times 2!} = 60$



$\Rightarrow P(A) = \frac{60}{126} = \frac{10}{21}$

6 **(d)**

Let A and B, respectively, be the events that urn A and urn B are selected. Let R be the event that the selected ball is red. Since the urn is chosen at random. Therefore,

$P(A) = P(B) = \frac{1}{2}$

And $P(R) = P(A)P(R/A) + P(B)P(R/B)$

$= \frac{1}{2} \times \frac{5}{10} + \frac{1}{2} \times \frac{4}{10}$

$= \frac{9}{20}$

7 **(d)**

Player should get $(HT, HT, HT, \dots)$ or $(TH, TH, \dots)$ at least $2n$ times. If the sequence starts from first place, then the probability is $1/2^{2n}$ and if starts from any other place, then the probability is $1/22^{2n+1}$. Hence, required probability is

$2\left(\frac{1}{2^{2n}} + \frac{m}{2^{2n+1}}\right) = \frac{m+2}{2^{2n}}$

8 **(c)**

$P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$

$\Rightarrow 0.7 = 0.4 + p - 0.4p$

$\therefore 0.6p = 0.3 \ \Rightarrow p = \frac{1}{2}$

9 **(c)**

Given limit,

$\lim_{x\to 0}\left(\frac{a^x + b^x}{2}\right)^{\frac{2}{x}}$

$= \lim_{x\to 0}\left(1 - \frac{a^x + b^x - 2}{2}\right)^{\frac{2}{a^x+b^x-2}\lim_{x\to 0}\left(\frac{a^x-1+b^x-1}{x}\right)}$

$= e^{\log ab} = ab = 6$

Total number of possible ways in which a, b can take values is $6 \times 6 = 36$. Total possible ways are (1, 6), (6, 1), (2, 3), (3, 2). The total number of possible ways is 4. Hence, the4 required probability is 4/36=1/9

10 **(c)**

Given that 5 and 6 have appeared on two of the dice, the sample space reduces to $6^4 - 2 \times 5^4 + 4^4$ (inclusion-exclusion principle). Also, the number of favourable cases are $4! = 24$. So, the required probability is 24/302=12/151

11 **(b)**

Let event A be drawing 9 cards which are not ace and B be drawing an ace card. Therefore, the required probability is

$P(A \cap B) = P(A) \times P(B)$

Now, there are four aces and 48 other cards. Hence,

$P(A) = \frac{^{48}C_9}{^{52}C_9}$

After having drawn 9 non-ace cards, the 10^{th} card must be ace. Hence,

$P(B) = \frac{^{4}C_1}{^{42}C_1} = \frac{4}{42}$

Hence,

$P(A \cap B) = \frac{^{48}C_9}{^{52}C_9}\frac{4}{42}$

12 **(b)**

Team totals must be 0, 1, 2, ..., 39. Let the teams be $T_1, T_2, \dots, T_{40}$, so that T_1 loses to T_1 for $i < j$. In other words, this order uniquely determines the result of every game. There are 40! Such orders and 780 games, so 2^{780} possible outcomes for the games. Hence, the probability is $40!/2^{780}$

13 **(a)**

The total number of ways in which $2n$ boys can be divided into two equal groups is

$\frac{(2n)!}{(n!)^2 2!}$

Now, the number of ways in which $2n - 2$ boys other than the two tallest boys can be divided into equal group is

$\frac{(2n-2)!}{(n-1!)^2 2!}$

Two tallest boys can be put in different groups in $^{2}C_1$ ways. Hence, the required probability is

$$\frac{2\,\frac{(2n-2)!}{\left((n-1)!\right)^2 2!}}{\frac{(2n)!}{(n!)^2 2!}} = \frac{n}{2n-1}$$

14 **(a)**

1. This question can also be solved by one student

2. This question can be solved by two students simultaneously

3. This question can be solved by three students all together

$P(A) = \frac{1}{2}, P(B) = \frac{1}{4}, P(C) = \frac{1}{6}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - [P(A)P(B) + P(B)P(C) + P(C)P(A)] + [P(A)P(B)P(C)]$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{6} - \left[\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{2}\right] + \left[\frac{1}{2} \times \frac{1}{4} \times \frac{1}{6}\right]$

$= \frac{33}{48}$

Alternative solution:

We have,

$P(\overline{A} = \frac{1}{2}, P(\overline{B}) = \frac{3}{4}, P(\overline{C}) = \frac{5}{6}$

Then the probability that the problem is not solved is

$P(\overline{A})P(\overline{B})P(\overline{C}) = \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{6}\right) = \frac{5}{16}$

Hence probability that problem is solved is $1 - 5/16 = 11/16$

15 **(d)**

Let p_1 denote the probability that out of 10 tosses, head occurs i times and no two heads occurs consecutively. It is clear that $i > 5$

For $i = 0$, i.e., no head, $p_0 = 1/2^{10}$

For $i = 1$, i.e., one head $p_1 = {}^{10}C_1(1/2)^1(1/2)^9 = 10/2^{10}$

Now for $i = 2$, we have 2 heads and 8 tails. Then, we have 9 possible places for heads. For example, see the constructive:

$xTxTxTxTxTxTxTxTx$

Here x represents possible places for heads

$\therefore p_2 = {}^9C_2\left(\frac{1}{2}\right)^2 (1/2)^8 = 36/2^{10}$

Similarly,

$p_3 = {}^8C_3/2^{10} = 56/2^{10}$

$p_4 = {}^7C_2/2^{10} = 35/2^{10}$

$p_5 = {}^6C_5/2^{10} = 6/2^{10}$

$\therefore p = p_0 + p_1 + p_3 + p_4 + p_5$

$= \frac{1 + 10 + 36 + 56 + 35 + 6}{2^{10}} = \frac{144}{2^{10}} = \frac{9}{64}$

16 **(c)**

The probability that A gets r heads in three tosses of a coin is

$P(X = r) = {}^3C_r\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{3-r} = {}^3C_r\left(\frac{1}{2}\right)^3$

The probability that A and B both get r heads in three tosses of a coin is

${}^3C_r\left(\frac{1}{2}\right)^3 {}^3C_r\left(\frac{1}{2}\right)^3 = ({}^3C_r)^2\left(\frac{1}{2}\right)^6$

Hence, the required probability is

$\sum_{r=0}^{3}({}^3C_r)^2\left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^6 = \{1 + 9 + 9 + 1\} = \frac{20}{64} = \frac{5}{16}$

17 **(c)**

If A draws card higher than B, then number of favourable cases is $(n-1) + (n-2) + \ldots + 3 + 2 + 1$ (as when B draws card from 2 to n and so on). Therefore, the required probability is

$\frac{\frac{n(n-1)}{2}}{n^2} = \frac{n-1}{2n}$

18 **(d)**

A: exactly one ace

B: both aces

E: $A \cup B$

$P(B/A \cup B)) = \frac{{}^4C_2}{{}^4C_1\ {}^{12}C_1 + {}^4C_2} = \frac{6}{54} = \frac{1}{9}$

19 **(b)**

The probability of winning of A the second race is $1/2$ (since both events are independent)

20 **(a)**

The number of composite numbers in 1 to 30 is $n(S) = 19$

The number of composite number when divided by 5 leaves a remainder is (E) =14. Therefore, the required probability is $14/19$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	A	C	B	D	D	C	C	C
Q.	11	12	13	14	15	16	17	18	19	20
A.	B	B	A	A	D	C	C	D	B	A