ELECTRIC CHARGES AND FIELDS

1 ELECTRIC CHARGE

Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects.

All bodies consist of atoms, which contain equal amount of positive and negative charges in the form of protons and electrons respectively. The number of electrons being equal to the number of protons as an atom is electrically neutral. If the electrons are removed from a body, it gets positively charged. If the electrons are transferred to a body, it gets negatively charged.

"Similar charges (charges of the same sign) repel one another; and dissimilar charges (charges of opposite sign) attract one another."

1.1 WAYS OF CHARGING A BODY

(i) Charging by friction

When two bodies are rubbed together, a transfer of electrons takes place from one body to another. The body from which electrons have been transferred is left with an excess of positive charge, so it gets positively charged. The body which receives the electrons becomes negatively charged.

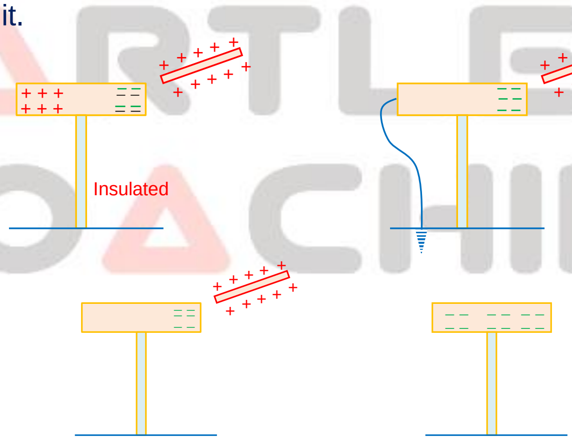
"The positive and negative charges produced by rubbing are always equal in magnitude."

When a glass rod is rubbed with silk, it loses its electrons and gets a positive charge, while the piece of silk acquires equal negative charges.

An ebonite rod acquires a negative charge, if it is rubbed with wool (or fur). The piece of wool (or fur) acquires an equal positive charge.

(ii) Charging by electrostatic induction

If a positively charged rod is brought near an insulated conductor, the negative charges (electrons) in the conductor will be attracted towards the rod. As a result, there will be an excess of negative charge at the end of the conductor near the rod and the excess of positive charge at the far end. This is known as 'electrostatic induction'. The charges thus induced are found to be equal and opposite to each other. Now if we touch the far end with a conductor connected to the earth, the positive charges here will be cancelled by negative charges coming from the earth through the conducting wire. Now, if we remove the wire first and then the rod, the induced negative charges which were held at the outer end will spread over the entire conductor. It means that the conductor has become negatively charged by induction. In the same way one can induce a positive charge on a conductor by bringing a negative charged rod near it.



Important points regarding electrostatic induction

- **(a)** Inducing body neither gains nor loses charges.
- **(b)** The nature of induced charge is always opposite to that of inducing charge.
- **(c)** Induced charge can be lesser or equal to inducing charge but it is never greater than the inducing charge.
- **(d)** Induction takes place only in bodies (either conducting or nor conducting) and not in particles.

(iii) Charging by conduction

Let us consider two conductors, one charged and the other uncharged. We bring the conductors in contact with each other. The charge (whether negative or positive) under its own repulsion will spread over both the conductors. Thus the conductors will be charged with the same sign. This is called 'charging by conduction (through contact)'.

1.2 UNIT OF CHARGE

In SI units as current is assumed to be fundamental quantity and $I = \left(\frac{q}{t}\right)$, charge is a derived physical quantity with dimensions [AT] and unit (ampere $\times$ second) called 'coulomb (C)'.

The coulomb is related to CGS units of charge through the basic relation

1 coulomb = 3×10^9 esu of charge = $\frac{1}{10}$ emu of charge

1.3 PROPERTIES OF CHARGE

(i) Charge is always associated with mass

The charge can not exist without mass though mass can exist without charge.

(ii) Charge is quantised

When a physical quantity can have only discrete values rather than any value, the quantity is said to be quantised.

Several experiments have established that the smallest charge that can exist in nature is the charge of an electron. If the charge of an election (= 1.6×10^{-19}C) is taken as the elementary unit, i.e., quanta of charge, and is denoted by *e*, the charge on any body will be some integral multiple of *e*, i.e.,

$q = \pm ne$; $n = 1, 2, 3, \ldots\ldots\ldots$ … (**1**)

charge on a body can never be $\left(\frac{2e}{3}\right)$, (17.2) *e* or ($10^{-5}$) *e* etc.

(iii) Charge is conserved

A large number of experiments show that in an isolated system, total charge does not change with time, though individual charges may change, i.e., charge can neither be created nor be destroyed. This is known as the principle of conservation of charge.

(iv) Charge is invariant

This means that charge is independent of frame of reference, i.e., charge on a body does not change whatever be its speed.

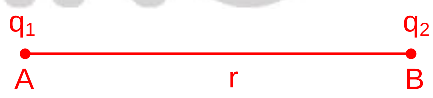
1.4 CONDUCTORS AND INSULATORS

The conductors are materials, which allow electricity (electric charge) to pass through them due to the presence of free elections. e.g., metals are good conductors.

The insulators are materials, which do not allow electric charge to pass through them as there is no free electrons in them. e.g. wood, plastics, glass etc.

2 COULOMB'S LAW

"Two stationary point charges repel or attract each-other with a force which is directly proportional to the product of the magnitudes of their charges and inversely proportional to the square of the distance between them."



Let '*r*' be the distance between two point charges q_1 and q_2.

According to **Coulomb's law**, we have $F \propto \frac{|q_1||q_2|}{r^2}$

where *F* is the magnitude of the mutual force that acts on each of the two charges q_1 and q_2.

or, $F = \frac{K|q_1||q_2|}{r^2}$, where *K* is a constant of proportionality

The value of *K* depends upon the medium in which two point charges are placed.

In the SI system. $K = \dfrac{1}{4\pi\varepsilon_0}$ for vacuum (or air)

The constant ε_0 (= 8.85 × 10^{-12} C^2/N-m^2) is called "permittivity" of the free space. Thus

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2} \approx 9 \times 10^9 \frac{|q_1||q_2|}{r^2} \qquad \dots (2)$$

2.1 PERMITTIVITY OF A MEDIUM

If the medium between the two point charges q_1 and q_2 is not a vacuum (or air). Then the electrostatic force between the two charges becomes

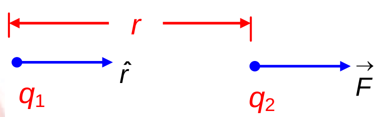
$$F = \frac{1}{4\pi\varepsilon}\frac{|q_1||q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{|q_1||q_2|}{r^2} \qquad \dots (3)$$

where $\varepsilon = \varepsilon_0\,\varepsilon_r$ is called the 'absolute permittivity' or 'permittivity' of the medium and ε_r is a dimensionless constant called 'relative permittivity' of the medium which is a constant for a given medium. ε_r is also sometimes called "dielectric constant" or 'specific inductive capacity' of the medium.

2.2 COLOUMB'S LAW IN VECTOR FORM

The vector form of Coulomb's law is $\vec{F} = \dfrac{Kq_1q_2}{r^2}\,\hat{r}$ … (4)

The unit vector $\hat{r}$ has its origin at the 'source of the force". For example, to find the force on q_2, the origin of $\hat{r}$ is at q_1. The signs of the charges must be explicitly included in equation (4). If F is the magnitude of the force, then $\vec{F} = +F\hat{r}$ means a repulsion, whereas $\vec{F} = -F\hat{r}$ means an attraction.

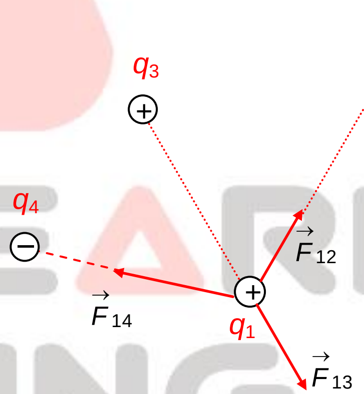


3 PRINCIPLE OF SUPERPOSITION

According to the principle of superposition, the force acting on one charge due to another is independent of the presence of charges. So, we can calculate the force separately for each pair of charges and then take their vector sum or find the net force on any charge.

The figure shows a charge q_1 interacting with other charges. Thus, to find the force on q_1, we first calculate the forces exerted by each of the other charges, one at a time. The net force $\vec{F}_1$ on q_1 is simply the vector sum

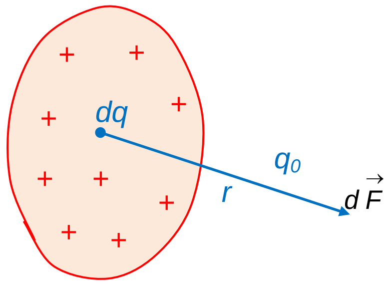
$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \qquad \dots (5)$$



where $\vec{F}_{12}$ is the force on the charge q_1 due to the charge q_2 and so on.

4 FORCE DUE TO CONTINUOUS CHARGE DISTRIBUTIONS

To find the force exerted by a continuous charge distribution on a point charge, we divide the charge into infinitesimal charge elements. Each infinitesimal charge element is then considered as a point charge.



The magnitude of the force dF exerted by the charge dq on the charge q_0 is given by

$$dF = \frac{1}{4\pi\varepsilon_0}\frac{|dq||q_0|}{r^2},$$

where r is the distance between dq and q_0. The total force is then found by adding all the infinitesimal force elements, which involves the integral

$$\vec{F} = \int d\vec{F}$$

Taking $d\vec{F} = dF_x\hat{i} + dF_y\hat{j} + dF_z\hat{K}$, we have

$$\left.\begin{aligned} F_x &= \int dF_x \\ F_y &= \int dF_y \\ F_z &= \int dF_y \end{aligned}\right] \qquad \ldots (\mathbf{6})$$

Because of the vector nature of the integration, the mathematical procedure must be carried out with care. The symmetry of charge distribution will usually result in a simplified calculation.

Each type of charge distribution is described (in the table given below) by an appropriate Greek letter parameter: λ, σ or ρ.

How we choose the charge element dq depends upon the particular type of the charge distribution.

Charge distribution	Relevant parameter	SI units	Charge element dq
Along a line	λ, charge per unit length	C/m	$dq = \lambda\, dx$
On a surface	σ, charge per unit area	C/m^2	$dq = \sigma\, dA$
Throughout a volume	ρ, charge per unit volume	C/m^3	$dq = \rho dV$

5 ELECTRIC FIELD

An electric field is defined as a region in which there should be a force on a charge brought into that region. Whenever a charge is being placed in an electric field, it experiences a force.

Electric fields that we will study are usually produced by different types of charged bodies – point charges, charged plates. Charged sphere etc. We can also define the electric field of a charged body as its region of influence within which it will exerts force on other charges.

If two point charges are placed as shown, we describe the forces on them in two ways

(i) The charge q_2 is in the electric field of charge q_1. Thus the electric field of charge q_1 exerts force on q_2.

(ii) The charge q_1 is in the electric field of charge q_2. Hence the electric field of charge q_2 exerts a force on q_1.

Electric field $\xrightarrow{\text{exerts force on}}$ charges inside it.

Electric field $\xrightarrow{\text{is created by}}$ charged bodies.

5.1 ELECTRIC INTENSITY OR ELECTRIC FIELD STRENGTH ($\vec{E}$)

The electric field intensity at a point in an electric field is the force experienced by a unit positive charge placed at that point, it is being assumed that the unit charge does not affect the field.

Thus, if a positive test charge q_0 experiences a force $\vec{F}$ at a point in an electric field, then the electric field intensity $\vec{E}$ at that point is given by

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad \ldots (7)$$

Important points regarding electric Intensity

(i) It is a vector quantity. The direction of the electric field intensity at a point inside the electric field is the direction in which the electric field exerts force on a (unit) positive charge.

(ii) Dimensions of the electric field intensity

$$E = \frac{F}{q_0} = \frac{[MLT^{-2}]}{[AT]} = [MLT^{-3}A^{-1}]$$

In S.I. systems, the unit of $\vec{E}$ is N/C or V/m as

$$\frac{\text{N}}{\text{C}} = \frac{\text{N}\times\text{m}}{\text{C}\times\text{m}} = \frac{\text{J}}{\text{C}\times\text{m}} = \frac{\text{V}}{\text{m}}$$

5.2 FORCED EXERTED BY A FIELD ON A CHARGE INSIDE IT

By definition as $\vec{E} = \frac{\vec{F}}{q_0}$, i.e.,

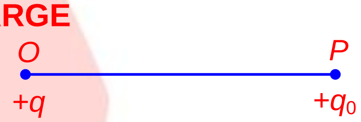
$$\vec{F} = q_0\vec{E}$$

If q_0 is a +ve charge, force $\vec{F}$ on it is in the direction of $\vec{E}$.

If q_0 is a –ve charge, $\vec{F}$ on it is opposite to the direction of $\vec{E}$

5.3 ELECTRIC FIELD INTENSITY DUE TO A POINT CHARGE

Let a positive test charge q_0 be placed at a distance r from a point charge q. The magnitude of force acting on q_0 is given by Coulomb's law,



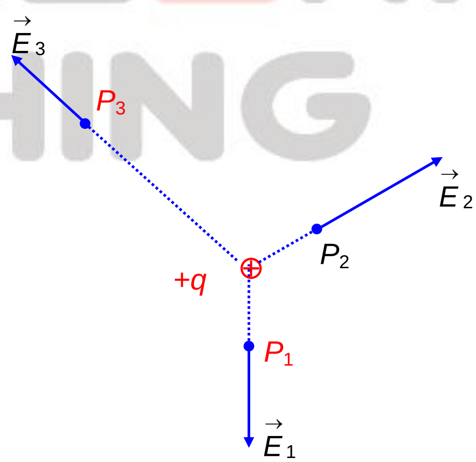
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_0 q}{r^2}$$

The magnitude of the electric field at the site of the charge is

$$E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \ldots (8)$$

The direction of $\vec{E}$ is the same as the direction of $\vec{F}$, along a radial line from q, pointing outward if q is positive and negative if q is negative.

The figure given below shows the direction of the electric field $\vec{E}$ at various points near a positive point charge.



5.4 ELECTRIC FIELD INTENSITY DUE TO A GROUP OF POINT CHARGES

Since the principle of linear superposition is valid for Coulomb's law, it is also valid for the electric field. To calculate the electric field strength at a point due to a group of N point charges. We first find the individual field strengths $\vec{E}_1$ due to Q_1, $\vec{E}_2$ due to Q_2, and so on

The resultant field strength is the vector sum of individual field strengths.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ \dots$$

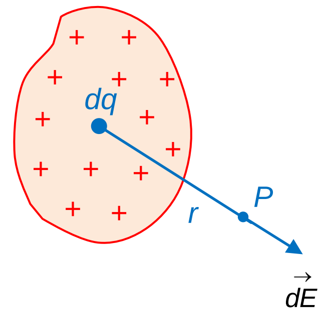
$$= \sum \vec{E}_n \ (n = 1, 2, 3, \dots N)$$

6 ELECTRIC FIELD OF CONTINUOUS CHARGE DISTRIBUTIONS

To find the field of a continuous charged distribution, we divide the charge into infinitesimal charge elements. Each infinitesimal charge element is then considered as a point charge and its field is given by

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

At a point distant r from the element, the net field is the summation of fields of all the elements.



$$\vec{E} = \int d\vec{E}$$

Taking $d\vec{E} = dE_x\,\hat{i} + dE_y\,\hat{j} + dE_z\,\hat{k}$, we have $E_x = \int dE_x$, $E_y = \int dE_y$ and $E_z = \int dE_z$

Because of the vector nature of the integration, the mathematical procedure must be carried out with care. Fortunately, in the cases we consider, the symmetry of the charge distribution usually results in a simplified calculation.

Each type of charge distribution is described (in the table given below) by an appropriate Greek-letter parameter: λ, σ or ρ. How we choose the charge element 'dq' depends upon the particular type of charge distribution.

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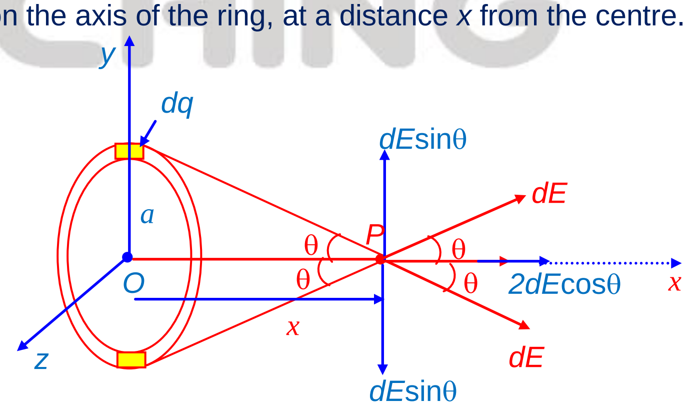
6.1 ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED RING AT A POINT ON THE AXIS OF THE RING

Let us consider a charge Q distributed uniformly on a thin, circular, non-conducting ring of radius a. We have to find electric field E at a point P on the axis of the ring, at a distance x from the centre.

From symmetry, we observe that every element dq can be paired with a similar element on the opposite side of the ring. Every component $dE\sin\theta$ perpendicular to the x-axis is thus cancelled by a component $dE\sin\theta$ in the opposite direction. In a summation process, all the perpendicular components add to zero. Thus we only add the dE_x components.



Now, $$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(d^2 + x^2)}$$

Hence, the resultant electric field at P is given by

$$E = \int dE_x$$
$$= \int dE\cos\theta$$
$$= \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{(a^2+x^2)}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{x}{(a^2+x^2)^{3/2}}\int dq$$

As we integrate around the ring, all the terms remain constant and $\int dq = Q$

So, the total field is

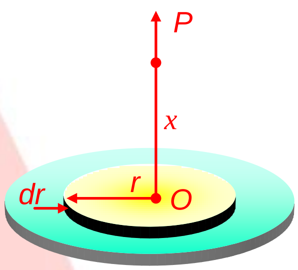
$$E = \frac{1}{4\pi\varepsilon_0}\frac{xQ}{(a^2+x^2)^{3/2}}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(a^2+x^2)^{3/2}} \qquad \dots (\mathbf{9A})$$

6.2 ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED DISC AT A POINT ON THE AXIS OF THE DISC

Let us consider a flat, circular, non-conducting thin disc of radius R having a uniform surface charge density σ c/m^2. We have to find the electric field intensity at an axial point at a distance x from the disc.

Let O be the centre of a uniformly charged disc of radius R and surface charge density σ. Let P be an axial point, distant x from O, at which electric field intensity is required.



From the circular symmetry of the disc, we imagine the disc to be made up of a large number of concentric circular rings and consider one such ring of radius r and an infinitesimally small width dr

The area of the elemental ring = circumference x width = $(2\pi\, rdr)$

The charge dq on the elemental ring = $(2\pi rdr)\,\sigma$

Therefore, the electric field intensity at P due to the elementary ring is given by

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{(2\pi rdr)\sigma x}{(r^2+x^2)^{3/2}} = \frac{\sigma x}{2\varepsilon_0}\frac{rdr}{(r^2+x^2)^{3/2}},$$

and is directed along the x-axis. Hence, the electric intensity E due to the whole disc is given by

$$E = \frac{\sigma x}{2\varepsilon_0}\int_O^R\frac{rdr}{(r^2+x^2)^{3/2}}$$
$$= \frac{\sigma x}{2\varepsilon_0}\left[-\frac{1}{(r^2+x^2)^{1/2}}\right]_0^R = \frac{\sigma x}{2\varepsilon_0}\left[-\frac{1}{(R^2+x^2)^{1/2}}+\frac{1}{x}\right]$$
$$= \frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{(R^2+x^2)^{1/2}}\right] \qquad \dots (\mathbf{9B})$$

7 MOTION OF A CHARGED PARTICLE IN AN UNIFORM ELECTRIC FIELD

A particle of mass m and charge q in an uniform electric field $\vec{E}$ experiences a force

$$\vec{F} = q\vec{E}$$

From Newton's second law of motion,

$$\vec{F} = m\vec{a}$$

Hence, the acceleration of the charged particle in the uniform electric field is

$$\vec{a} = \frac{q\vec{E}}{m}$$

Since the field is uniform, the acceleration is constant in magnitude and direction. So we can use the equation of kinematics for constant acceleration. Now, there are two possibilities.

(a) If the particle is initially at rest

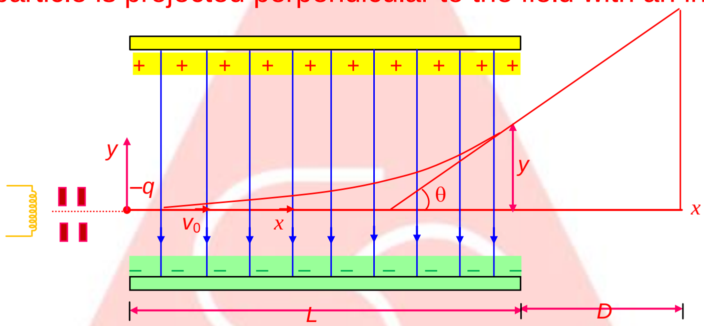
From equation $v = u + at$, we get

$$v = at = \frac{qE}{m}t \left[\because u = 0;\ a = \frac{qE}{m}\right]$$

From equation $S = ut + \frac{1}{2}at^2$, we have

$$S = \frac{1}{2}at^2 = \frac{qE}{2m}t^2$$

(b) (ii) If the particle is projected perpendicular to the field with an initial velocity v_0.



For motion along x-axis, we have $v_x = v_0$ = constant ($\because u = v_0$ and $a = 0$)

$\therefore\ x = v_0 t$ … (i)

for motion along y-axis, we have

$$y = \frac{1}{2}\left[\frac{qE}{m}\right]t^2 \quad \text{… (ii)}$$

$$\left[\because u = 0;\ a = \frac{qE}{m}\right]$$

Substituting the value of t from equation (i) in equation (ii),
we get

$$y = \frac{qE}{2m}\left[\frac{x}{v_0}\right]^2$$

$$= \frac{qE}{2mv_0^2}x^2$$

Which is the equation of the parabola.

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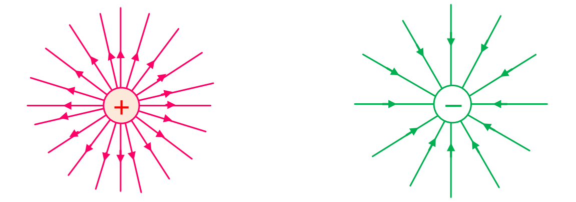
8 ELECTRIC LINES OF FORCE

The idea of electric lines of force or the electric field lines introduced by Michel Faraday is a way to visualize electrostatic field geometrically.

The properties of electric lines of force are the following:

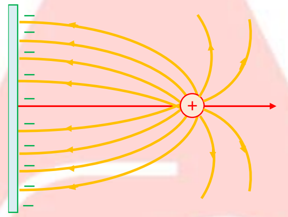
(i) The electric lines of force are continuous curves in an electric field starting from a positively charged body and ending on a negatively charged body.



(ii) The tangent to the curve at any point gives the direction of the electric field intensity at that point.

(iii) Electric lines of force never intersect since if they cross at a point, electric field intensity at that point will have two directions, which is not possible.

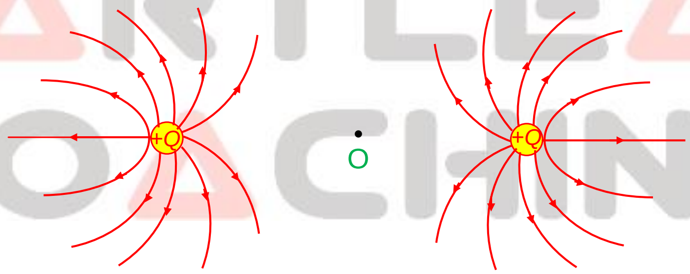
(iv) Electric lines of force do not pass but leave or end on a charged conductor normally. Suppose the lines of force are not perpendicular to the conductor surface. In this situation, the component of electric field parallel to the surface would cause the electrons to move and hence conductor will not remain equipotential which is absurd as in electrostatics conductor is an equipotential surface.



Fixed point charge near infinite metal plate

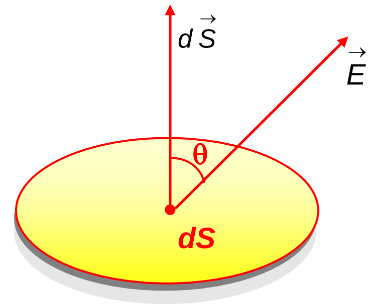
(v) The number of electric lines of force that originate from or terminate on a charge is proportional to the magnitude of the charge.

(vi) As number of lines of force per unit area normal to the area at point represents magnitude of intensity, crowded lines represent strong field while distant lines weak field. Further, if the lines of force are equidistant straight lines, the filed is uniform.



Electric lines of force due to two equal positive charges (field is zero at *O*). *O* is a null point.

9 FLUX OF AN ELECTRIC FIELD OR ELECTRIC FLUX



Let us consider a plane surface of area S placed in an electric field $\vec{E}$.

Electric flux through an elementary area $d\vec{S}$ is defined as the scalar product of $d\vec{S}$ and $\vec{E}$ i.e.

$d\phi_E = \vec{E}\,.\,d\vec{S}$, where $d\vec{S}$ is the area vector, whose magnitude is the area ds of the element and whose direction is along the outward normal to the elementary area.

Hence, the electric flux through the entire surface is given by

$$\phi_E = \int \vec{E}.d\vec{S} \qquad \ldots (\mathbf{10})$$

or, $\phi_E = \int EdS \cos\theta$

If the electric field is uniform, then $\phi_E = \int Eds \cos\theta = E\cos\theta \int ds$

When the electric flux through a closed surface is required, we use a small circular sign on the integration symbol;

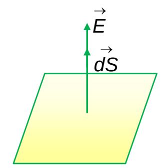
$$\phi_E = \vec{E}.d\vec{S} \qquad \ldots (\mathbf{11})$$

Important points regarding electric flux:

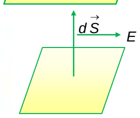
(i) The number of lines of force passing normally to the given area gives the measure of flux of electric field over the given area.

(ii) It is a real scalar physical quantity with units (volt × m).

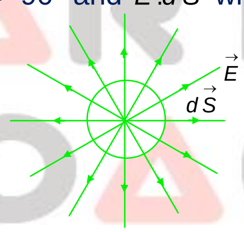
(iii) It will be maximum when cos θ = max. = 1, i.e., θ = 0°, i.e., electric field is normal to the surface with $(d\phi_E)_{max} = EdS$



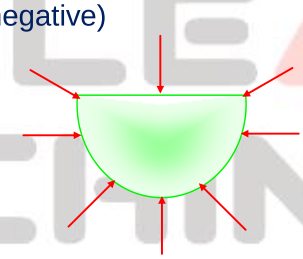
(iv) It will be minimum when |cos θ| = min = 0, i.e., θ = 90°, i.e., field is parallel to the area with $(d\phi_E)_{min} = 0$



(v) For a closed surface, ϕ_E is positive if the lines of force point outward everywhere ($\vec{E}$ will be outward everywhere, θ < 90° and $\vec{E}\,.\,d\vec{S}$ will be positive) and negative if they point inward ($\vec{E}$ is inward everywhere, θ > 90° and $\vec{E}.d\vec{S}$ will be negative)



Positive flux



Negative flux

10 GAUSS'S LAW

This law gives a relation between the electric flux through any closed hypothetical surface (called a **Gaussian surface**) and the charge enclosed by the surface. It states, " The electric flux (ϕ_E) through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the 'net' charge enclosed by the surface."

That is,

$$\phi_E = \vec{E}.d\vec{S} = \frac{\sum q}{\varepsilon_0}, \qquad \ldots (\mathbf{12})$$

where $\sum q$ denotes the algebraic sum of all the charges enclosed by the surface.

If there are several charges $+q_1$, $+q_2$, $+q_3$, $-q_4$, $-q_5$ … etc inside the Gaussian surface, then

$$\sum q = q_1 + q_2 + q_3 - q_4 - q_5 \ldots$$

It is clear from equation (11) that the electric flux linked with a closed body is independent of the shape and size of the body and position of charge inside it.

Applications of Gauss's Law

Gauss's law is useful when there is symmetry in the charge distribution, as in the case of uniformly charged sphere, long cylinders, and flat sheets. In such cases, it is possible to find a simple Gaussian surface over which the surface integral given by equation (10) can be easily evaluated.

These are steps to apply the Gauss's law

(i) Use the symmetry of the charge distribution to determine the pattern of the lines

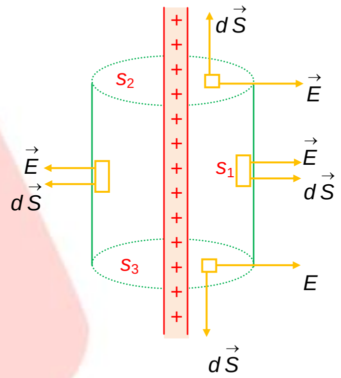
(ii) Choose a Gaussian surface for which $\vec{E}$ is either parallel to $d\vec{S}$ or perpendicular to $d\vec{S}$

(iii) If $\vec{E}$ is parallel to $d\vec{S}$, then the magnitude of $\vec{E}$ should be constant over this part of the surface.

The integral then reduces to a sum over area elements.

10.1 FIELD DUE TO AN INFINITE LINE OF CHARGE

Consider an infinite line of charge has a linear charge density λ. Using Gauss's law, let us find the electric field at a distance 'r' from the line.



The cylindrical symmetry tells us that the field strength will be the same at all points at a fixed distance r from the line. Since the line is infinite and uniform, for every charge element on one side, there is symmetrically located element on the other side. The component along the line of the fields due to all such elements cancel in pairs. Thus, the field lines are directed radially outward, perpendicular to the line of charge.

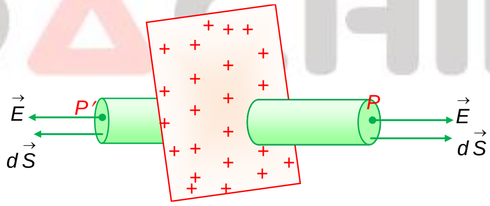
The appropriate choice of Gaussian surface is a cylinder of radius r and length L. On the flat end faces, S_2 and S_3, $\vec{E}$ is perpendicular $d\vec{S}$, which means no flux crosses them. On the curved surface S_1, $\vec{E}$ is parallel $d\vec{S}$, so that $\vec{E} \cdot d\vec{S} = EdS$.

The charge enclosed by the cylinder is $Q = \lambda L$.

Applying Gauss's law to the curved surface, we have

$$E.dS = E(2\pi rL) = \frac{\lambda L}{\varepsilon_0} \text{ or, } E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad \ldots \textbf{(13A)}$$

10.2 FIELD DUE TO AN INFINITE PLANE SHEET OF CHARGE



Let us consider a thin non-conducting plane sheet of charge, infinite in extent, and having a surface charge density (charge per unit area) σ C/m^2. Let P be a point, distant r from the sheet, at which the electric intensity is required.

Let us choose a point P' symmetrical with P, on the other side of the sheet. Let us now draw a Gaussian cylinder cutting through the sheet, with its plane ends parallel to the sheet and passing through P and P'. Let A be the area of each plane end.

By symmetry, the electric intensity at all points on either side near the sheet will be perpendicular to the sheet, directed outward (if the sheet is positively charged). Thus $\vec{E}$ is perpendicular to the plane ends of the cylinder and parallel to the curved surface. Also its magnitude will be the same at P and P'. Therefore, the flux through the two plane ends is

$$\phi_E = \int \vec{E}.d\vec{S} + \int \vec{E}.d\vec{S} = \int EdS + \int EdS = EA + EA = 2EA$$

The flux through the curved surface of the Gaussian cylinder is zero because $\vec{E}$ and $d\vec{S}$ are at right angles everywhere on the curved surfaces.

Hence, the total flux through the Gaussian cylinder is

$$\phi_E = 2EA$$

The charge enclosed by the Gaussian surface $q = \sigma A$

Applying **Gauss's law**, we have

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

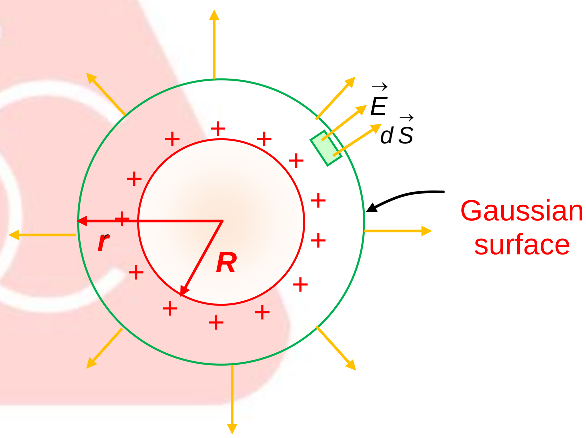
$$\Rightarrow \quad E = \frac{\sigma}{2\varepsilon_0} \qquad \ldots \textbf{(13B)}$$

10.3 ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED SPHERICAL SHELL

Using Gauss's law, let us find the intensity of the electric field due to a uniformly charged spherical shell or a solid conducting sphere at

Case I: At an external point

At all points inside the charged spherical conductor or hollow spherical shell, electric field $\vec{E} = 0$, as there is no charge inside such sphere. In an isolated charged spherical conductor any excess charge on it is distributed uniformly over its outer surface same as that of charged spherical shell or hollow sphere. Since the charge lines must point radially outward. Also, the field strength will have the same value at all points on any imaginary spherical surface concentric with the charged conducting sphere or the shell. This symmetry leads us to choose the Gaussian surface to be a sphere of radius $r > R$.



Any arbitrary element of area $d\vec{S}$ is parallel to the local $\vec{E}$, so $\vec{E}.d\vec{S} = EdS$ at all points on the surface.

According to Gauss's law, $dS = E(4\pi r) = \dfrac{Q}{\varepsilon_0}$

$$\text{Therefore,} \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad \ldots \textbf{(13C)}$$

For points outside the charged conducting sphere or the charged spherical shell, the field is same as that of a point charge at the centre.

Case II: At an Internal Point ($r < R$)

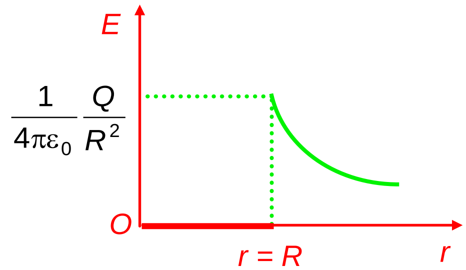
The field still has the same symmetry and so we again pick a spherical Gaussian surface, but now with radius r less than R. Since the enclosed charge is zero, from Gauss's law we have

$$E(4\pi r^2) = 0$$

$$\therefore \quad E = 0 \qquad \ldots \textbf{(13D)}$$

Thus, we conclude that $E = 0$ at all points inside a uniformly charged conducting sphere or the charged spherical shell.

Variation of E with The Distance from the centre (r)

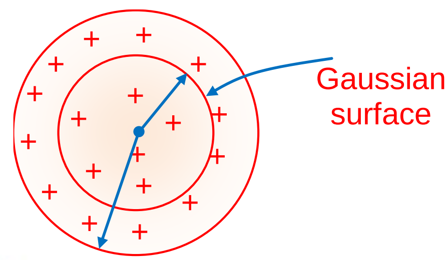


10.4 ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED SPHERE

A non-conducting uniformly charged sphere of radius R has a total charge Q uniformly distributed throughout its volume. Using the Gauss's Law, Let us find the field

Case I: at an internal point ($r < R$)

Positive charge Q is uniformly distributed throughout the volume of sphere of radius R. For finding the electric field at a distance ($r < R$) from the centre, we choose a spherical Gaussian surface of radius r, concentric with the charge distribution. From symmetry the magnitude E of the electric field has the same value at every point on the Gaussian surface, and the direction of $\vec{E}$ is radial at every point on the surface.



So, applying Gauss's law

$$= E\,(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

Here, $Q = \left(\frac{4}{3}\pi r^3\right)\rho = \left(\frac{4}{3}\pi r^3\right) \times \frac{Q}{4\pi R^3} = \frac{Qr^3}{R^3}$

where ρ is volume density of charge.

Therefore

$$E\,(4\pi r^2) = \frac{Qr^3}{R^3\varepsilon_0}$$

or, $$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r \qquad \text{... (13E)}$$

The field increases linearly with distance from the centre

Case II: At an External point ($r > R$)

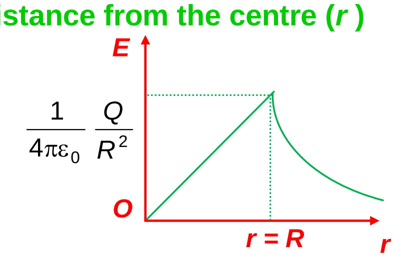
To find the electric field outside the charged sphere, we use a spherical Gaussian surface of radius r (> R). This surface encloses the entire charged sphere. So, from Gauss's law, we have

$$E\,(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

or, $$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad \text{... (13F)}$$

The field at points outside the sphere is the same as that of a point charge at the centre.

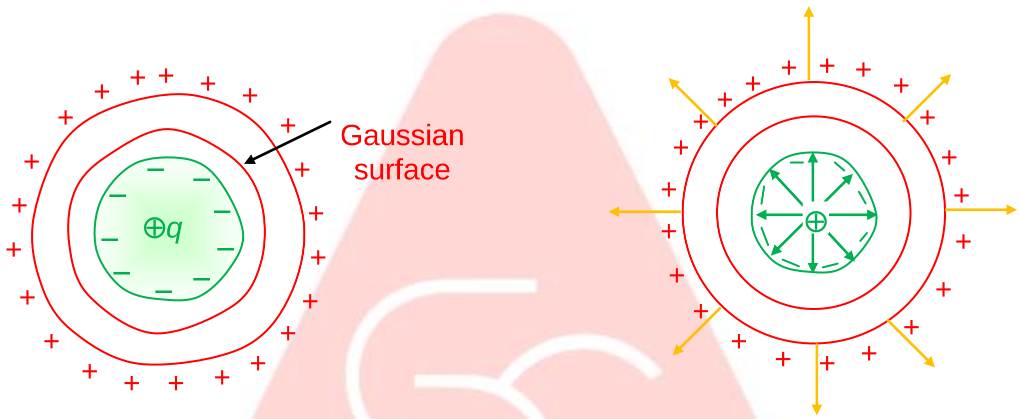
Variation of E with the distance from the centre (r)



A conductor contains “free” electrons, which can move freely in the material, but cannot leave it. Now, when an excess charge is given to an insulated conductor, it sets up electric field inside the conductor. The free electrons will redistribute themselves and within a fraction of a second (approx. 10^{-12} s) the internal field will vanish. Thus, in electrostatic equilibrium the value of $\vec{E}$ at all points within a conductor is zero. This idea, together with the Gauss’s law can be used to prove interesting facts regarding a conductor.

11.1 CAVITY INSIDE A CONDCUTOR

Consider a charge + q_0 suspended in a cavity in a conductor. Consider a Gaussian surface just outside the cavity and inside the conductor. $\vec{E}$ = 0 on this Gaussian surface as it is inside the conductor. Hence from Gauss’s law



$\vec{E} \cdot d\vec{S} = \dfrac{\sum q}{\varepsilon_0}$, we have

$\sum q = 0$

This concludes that a charge of $-q$ must reside on the metal surface of the cavity so that the sum of this induced charge $-q$ and the original charge $+q$ within the Gaussian surface is zero. In other words, a charge q suspended inside a cavity in a conductor induces an equal and opposite charge $-q$ on the surface of the cavity. Further as the conductor is electrically neutral, a charge $+q$ is induced on the outer surface of the conductor. As field inside the conductor is zero.

The field lines coming from q cannot penetrate into the conductor, as shown in the above figure.

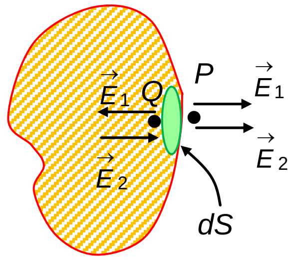
The same line of approach can be used to show that the field inside the cavity of a conductor is zero when no charge is kept inside it.

11.2 ELECTROSTATIC SHEILDING

Suppose we have a very sensitive electronic instrument that we want to protect from external fields that might cause wrong measurements. We surround the instrument with a conducting box or we keep the instrument inside the cavity of a conductor. By doing this, the charge in the conductor is so distributed that the net electric field inside the cavity becomes zero and so instrument is protected from the external fields. This is called electrostatic shielding.

12 FORCE ON THE SURFACE OF A CHARGED CONDUCTOR

In a charged conductor the charge resides entirely on the surface. This shows that every element of the surface of the conductor experiences a normal outward force, which holds its charge there. This force is produced as a result of repulsion of the charge on the element by the similar charge on the rest of the surface of the conductor. Let us calculate this force.



Let dS be a small element of the surface of a charged conductor. Let σ be the surface density of charge. Let us consider a point P just outside the surface. The magnitude of the electric intensity at P is given by

$E = \frac{\sigma}{\varepsilon_0}$, and is directed along the outward drawn normal to the element.

The intensity E can be considered as made up of two parts: (i) an intensity E_1 due to the charge on the element dS, and (ii) an intensity E_2 due to the charge on the rest of the surface of the conductor. Since their directions are the same, we have

$$E_1 = E_1 + E_2 = \frac{\sigma}{\varepsilon_0} \qquad \dots \text{(i)}$$

Let us now consider a point Q just inside the surface. The intensity at Q may again be considered as made up of two parts. The intensity due to the charge on the element dS is equal and opposite to that at P i.e., $-E_1$, since Q is very close to P but on the opposite side of the surface. The intensity due to the charge on the rest of the surface is same in magnitude and direction as at P i.e, E_2 (since Q is very close to P). But the resultant intensity at Q must be zero, since Q lies inside the conductor. Hence

$$-E_1 + E_2 = 0$$

or $\quad E_1 = E_2$.

Substituting this in equation (i), we get

$$2E_2 = \frac{\sigma}{\varepsilon_0}$$

$$\therefore \quad E_2 = \frac{\sigma}{2\varepsilon_0}$$

This gives the outward force experienced by a unit positive charge on the elements dS due to the charge on the rest of the surface. Since the charge on the element is $\sigma\, dS$, the force on dS is

$$F = E_2(\sigma dS) = \frac{\sigma^2 dS}{2\varepsilon_0}$$

Hence the force per unit area of the surface is

$$\frac{F}{dS} = \frac{\sigma^2}{2\varepsilon_0} \qquad \dots \textbf{(14A)}$$

Whatever the sign of σ, this force acts outward along the normal to the surface.

Now, from equation (i) $E = \frac{\sigma}{\varepsilon_0}$, so that $\sigma = \varepsilon_0 E$.

Substituting this value of σ in equation (14A), the outward force per unit area of the surface

$$= \frac{(\varepsilon_0 E)^2}{2\varepsilon_0}$$

$$= \frac{\varepsilon_0 E^2}{2} \qquad \dots \textbf{(14B)}$$

Hence the force per unit area (or electrostatic pressure) experienced by a charged conductor is $\sigma^2/2\varepsilon_0$ or $\varepsilon_0 E^2/2$ newton/meter^2 directed along the outward drawn normal to the surface.

13 ENERGY DENSITY OF AN ELECTRIC FIELD

Let us consider an electrostatic field E around a charged conductor. The charged conductor experiences a force of $\varepsilon_0 E^2/2$ Newton per meter^2 area, which is everywhere, directed along the outward drawn normal to the surface. If the conductor is placed in a medium of dielectric constant K, the normal force is $K\varepsilon_0 E^2/2$ newton.

Suppose the surface is displaced through a small distance of dl meter everywhere along its outward normal. The work done per meter^2 area of the surface

$$= \frac{K\,\varepsilon_0 E^2}{2}\,dl \text{ joule.}$$

The volume swept out by 1 meter^2 area is dl meter^3. Thus the work done in producing dl meter^3 of the field

$$\frac{K\varepsilon_0 E^2}{2}\,dl \text{ joule.}$$

Hence the work done is producing unit volume of the field.

$$= \frac{K\varepsilon_0 E^2}{2} \text{ joule}$$

This work is stored as energy of strain in the field. Hence the energy per meter^3, or the energy density u, of the field is

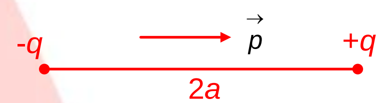
$$u = \frac{1}{2}K\varepsilon_0 E^2 \qquad \ldots (\mathbf{15})$$

18 ELECTRIC DIPOLE

Two equal and opposite point charges placed at a short distance apart constitute an electric dipole.

18.1 ELECTRIC DIPOLE MOMENT

Electric dipole moment is a vector $\vec{p}$ directed along the axis of the dipole, from the negative to the positive charge.



The magnitude of dipole moment is

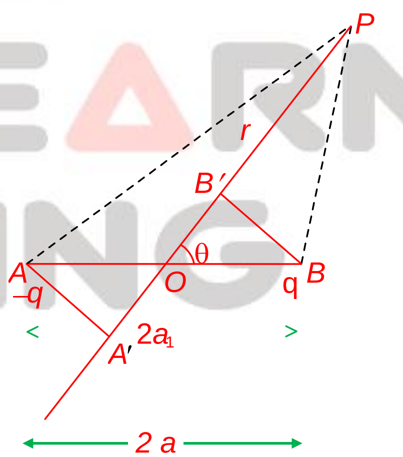
$p = (2a)\,q$

where $2a$ is the distance between the two charges. … (**24**)

18.2 ELECTRIC POTENTIAL DUE TO AN ELECTRIC DIPOLE

Suppose, the negative charge $-q$ is placed at a point A and a positive charge q is placed at a point B. The separation $AB = 2a$

The middle point of AB is O. The potential is to be evaluated at a point P where $OP = r$ and $P\hat{O}B = \theta$. Let AA' be the perpendicular from A to PO and BB' be the perpendicular from B to PO. As $2a$ is very small compared to r,



$$AP \simeq A'P = OP + OA' = r + a\cos\theta$$

Similarly, $BP \simeq B'P = OP - OB' = r - a\cos\theta$

The potential at P due to the charge $-q$ is

$$V_1 = -\frac{1}{4\pi\varepsilon_0}\frac{q}{AP} \simeq -\frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a\cos\theta)}$$ and that due to the charge $+q$ is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP} \simeq \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a\cos\theta)}$$

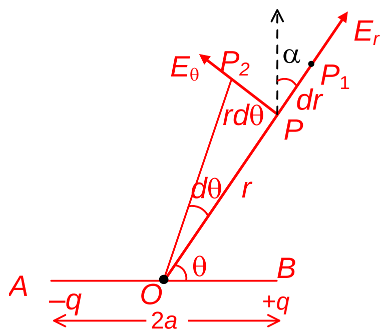
The net potential at P due to the dipole is

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r - a\cos\theta} - \frac{q}{r + a\cos\theta}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2qa\cos\theta}{(r^2 - a^2\cos^2\theta)} \simeq \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}. \qquad \ldots (25)$$

18.3 ELECTRIC FIELD DUE TO AN ELECTRIC DIPOLE



Consider a point *P* at a distance *r* from *O* making an angle θ with *AB*. PP_1 is a small displacement in the direction of *OP* and PP_2 is a small displacement perpendicular to *OP*. Thus PP_1 is in radial direction and PP_2 is in transverse direction. In going from *P* to P_1, the angle θ does not change and the distance *OP* changes from *r* to *r* + *dr*. Thus $PP_1 = dr$. In going from *P* to P_2, the angle θ changes from θ to θ + *d*θ while the distance *r* remains almost constant so $PP_2 = r\,d\theta$.

The component of the electric field at P in the radial direction PP_1 is

$$E_r = -\frac{dV}{dr} = -\frac{\partial V}{\partial r} = -\frac{1}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\frac{p\cos\theta}{r^2}\right) = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}.$$

The component of the electric field at P in the transverse direction PP_2 is

$$E_\theta = -\frac{dV}{PP_2} = -\frac{dV}{r\,d\theta} = -\frac{1}{r}\frac{\partial V}{\partial\theta} = -\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}\right) = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}$$

The resultant electric field at P, $E = \sqrt{E_r^2 + E_\theta^2}$

$$= \frac{1}{4\pi\varepsilon_0}\sqrt{\left(\frac{2p\cos\theta}{r^3}\right)^2 + \left(\frac{p\sin\theta}{r^3}\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{3\cos^2\theta + 1} \qquad \ldots (26)$$

If the resultant field makes an angle α with the radial direction *OP*, we have

$$\tan\alpha = \frac{E_\theta}{E_r} = \frac{\frac{p\sin\theta}{r^3}}{\frac{2p\cos\theta}{r^3}} = \frac{\tan\theta}{2}$$

or $\alpha = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$ … (27)

Now consider some special cases

Case I: θ = 0. In this case, the point *P* is on the axis of the dipole

$$V = \frac{p}{4\pi\varepsilon_0 r^2} \qquad \ldots (28A)$$

$$E = \frac{2p}{4\pi\varepsilon_0 r^3} \qquad \ldots (28B)$$

Such a position of the point is called an end-on position.

Case II: θ = 90°. In this case, the point *P* is on the perpendicular bisector of the dipole

$V = 0,$ … (28C)

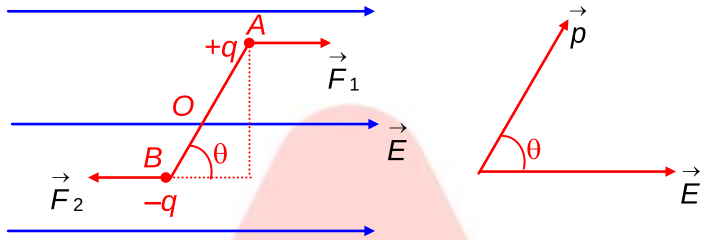
$$E = \frac{p}{4\pi\varepsilon_0 r^3} \quad \ldots \textbf{(28D)}$$

$$\tan\alpha = \frac{\tan\theta}{2} = \infty$$

$$\alpha = 90^\circ$$

The field is anti-parallel to the dipole axis. Such a position of the point *P* is called a broad side on position.

18.4 DIPOLE IN AN EXTERNAL UNIFORM ELECTRIC FIELD



Suppose an electric dipole of dipole moment $|\vec{p}| = 2aq$ is placed in a uniform electric field $\vec{E}$ at an angle θ. A force $\vec{F}_1 = q\vec{E}$ will act on positive charge and $\vec{F}_2 = -q\vec{E}$ on the negative charge. Since $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude but opposite in direction, we have

$$\vec{F}_1 + \vec{F}_2 = 0$$

Thus, the net force on a dipole in a uniform electric field is zero.

The torque of $\vec{F}_1$ about *O*,

$$\vec{\tau}_1 = \vec{OA} \times \vec{F}_1 = q\,(\vec{OA} \times \vec{E})$$

The torque of $\vec{F}_2$ about *O*,

$$\vec{\tau}_2 = \vec{OB} \times \vec{F}_2 = -q\,(\vec{OB} \times \vec{E})$$

$$= (\vec{BO} \times \vec{E})$$

The net torque acting on the dipole is

$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 = q\,(\vec{OA} \times \vec{E}) + q\,(\vec{BO} \times \vec{E})$$

$$= q\,(\vec{OA} + \vec{BO}) \times \vec{E}$$

$$= q\,(\vec{BA} \times \vec{E})$$

or, $$\vec{\tau} = \vec{p} \times \vec{E} \quad \ldots \textbf{(29)}$$

Thus, the magnitude of the torque is $\tau = pE\sin\theta$. The direction of torque is perpendicular to the plane of paper and inwards. Further this torque is zero at $\theta = 0°$ or $\theta = 180°$, i.e., when the dipole is parallel or antiparallel to $\vec{E}$ and maximum at $\theta = 90°$.

18.5 POTENTIAL ENERGY OF DIPOLE

When an electric dipole is placed in an electric field $\vec{E}$, a torque $\vec{\tau} = \vec{p} \times \vec{E}$ acts on it. If we rotate the dipole through a small angle dθ, the work done by the torque is

$$dW = \tau\, d\theta$$

or, $$dW = -pE\sin\theta\, d\theta$$

The work is negative as the rotation $d\theta$ is opposite to the torque. The change in electric potential energy of the dipole is therefore

$dU = -dW = pE \sin\theta \, d\theta$

Now at angle $\theta = 90°$, the electric potential energy of the dipole may be assumed to be zero as net work done by the electric forces in brining the dipole from infinity to this position will be zero.

$dU = pE\sin\theta \, d\theta$ from 90° to θ, we have

$$\int_{90°}^{\theta} dU = \int_{90°}^{\theta} pE \sin\theta \, d\theta$$

or, $U(\theta) - U(90°) = pE \, [-\sin\theta]^{\theta}_{90°}$

or $U(\theta) = -pE\cos\theta = -\vec{p}\cdot\vec{E}$ … (**30**)

It the dipole is rotated from an angle θ_1 to θ_2, then

work done by external forces = $U(\theta_2) - U(\theta_1)$

or, $W_{ext} = -pE\cos\theta_2 - (pE\cos\theta_1)$

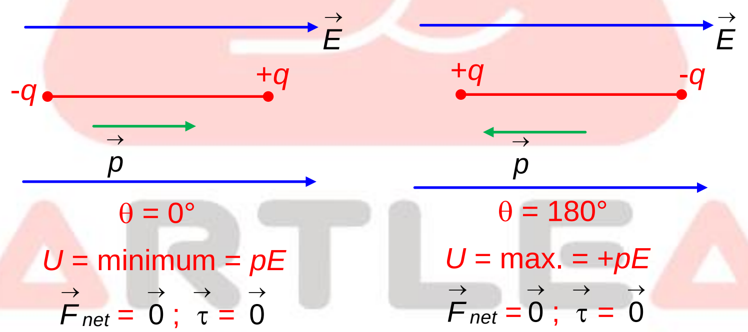
or, $W_{ext} = pE(\cos\theta_1 - \cos\theta_2)$ … (**31**)

Work done by electric force

$W_{electric\ force} = -W_{ext} = pE\,[\cos\theta_2 - \cos\theta_1]$ … (**32**)

18.6 EQUILIBRIUM OF DIPOLE

When an electric dipole is placed in a uniform electric field, the net force on it is zero for any position of the dipole in the electric field. But torque acting on it is zero only at $\theta = 0°$ and 180°. Thus, we can say that at these two positions of the dipole, net force or torque on it is zero or the dipole is in equilibrium. Of this $\theta = 0°$ is the 'stable equilibrium' position of the dipole because potential energy in this position is minimum ($U = -pE\cos\theta° = -pE$) and when displaced from this position, a torque starts acting on it which is restoring in nature and which has a tendency to bring the dipole back in its equilibrium position. On the other hand, at $\theta = 180°$, the potential energy of the dipole is maximum ($U = -pE \cos 180° = +pE$) and when it is displaced from this position, the torque has a tendency to rotate it in other direction. This torque is not restoring in nature. So this equilibrium is known as 'unstable equilibrium position'.



18.7 ANGULAR SHM OF DIPOLE

When a dipole is suspended in a uniform electric field, it will align itself parallel to the field. Now if it is given a small angular displacement θ about its equilibrium, the (restoring) couple will be

$C = -pE\sin\theta$

or, $C = -pE\theta$ [as $\sin\theta \approx \theta$, for small θ)

or, $I\dfrac{d^2\theta}{dt^2} = -pE\theta$

or, $\dfrac{d^2\theta}{dt^2} = -\dfrac{pE}{I}\theta$

or, $\dfrac{d^2\theta}{dt^2} = -\omega^2\theta$

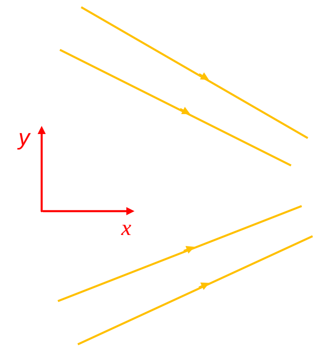
where $\omega^2 = \dfrac{pE}{I}$

This is standard equation of angular simple harmonic motion with time-period $T\left(=\frac{2\pi}{\omega}\right)$. So the dipole will execute angular SHM with time-period

$$T = 2\pi\sqrt{\frac{I}{pE}} \quad \dots (33)$$

18.8 FORCE ACTING ON A DIPOLE IN AN EXTERNAL NON-UNIFORM FIELD

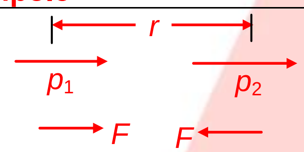
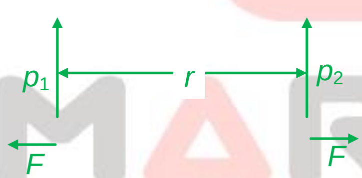
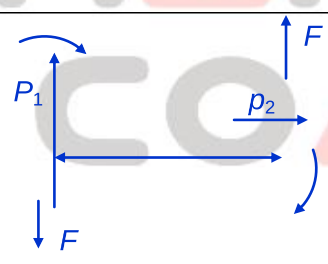
When dipole lies in a non-uniform electric field the charges of the dipole experience unequal forces. Therefore, the net force on the dipole is not equal to zero. The magnitude of the force is given by the negative derivative of the potential energy with respect to distance along the axis of the dipole



$$\vec{F} = -\frac{dU}{dl} = \vec{p}\cdot\frac{d\vec{E}}{dl}$$

18.8 INTERACTION BETWEEN DIPOLES

In this situation, one dipole is in the field of other dipole. Depending on the positions of dipoles relative to each other, force, couple and potential energy are different.

S. No.	Relative Position of Dipole	Potential energy (U)	Force (F)	Couple (C)
1.		$-\frac{1}{4\pi\varepsilon_0}\frac{2p_1p_2}{r^3}$	$-\frac{1}{4\pi\varepsilon_0}\frac{6p_1p_2}{r^4}$ (along r)	O
2.		$\frac{1}{4\pi\varepsilon_0}\frac{p_1p_2}{r^3}$	$+\frac{1}{4\pi\varepsilon_0}\frac{3p_1p_2}{r^4}$ (along r)	O
3.		O	$\pm\frac{1}{4\pi\varepsilon_0}\frac{3p_1p_2}{r^4}$ (perpendicular + or)	on $p_1 = \frac{1}{4\pi\varepsilon_0}\frac{2p_1p_2}{r^3}$ (CW) on $p_2 = \frac{1}{4\pi\varepsilon_0}\frac{p_1p_2}{r^3}$ (CW)

19 EARTHING A CONDUCTOR

Potential of earth is often taken to be zero. If a conductor is connected to the earth, the potential of the conductor becomes equal to that of the earth, i.e., zero. If the conductor was at some other potential, charges will l flow from it to the earth or from the earth to it to bring its potential to zero. The figure given below shows the symbol for earthing.