ELECTROMAGNETIC INDUCTION

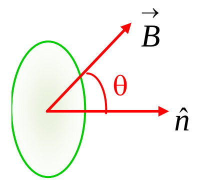
1 MAGNETIC FLUX

Consider a closed curve enclosing an area A (as shown in the figure). Let there be a uniform magnetic field $\vec{B}$ in that region. The magnetic flux through the area $\vec{A}$ is given by

$$\phi = \vec{B} \cdot \vec{A} \qquad \dots (1)$$

$$= BA \cos\theta$$

where θ is the angle which the vector B makes with the normal to the surface. If $\vec{B} \perp$ to $\vec{A}$, then the flux through the closed area $\vec{A}$ is zero. SI unit of flux is weber (Wb).



2 FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Faraday in 1831 discovered that whenever magnetic flux linked with a closed conducting loop changes, an e.m.f is induced in it. If the circuit is closed, a current flows through it as long as magnetic flux is changing. This current is called induced current, the emf induced in the loop is called induced emf and the phenomenon is known as electromagnetic induction.

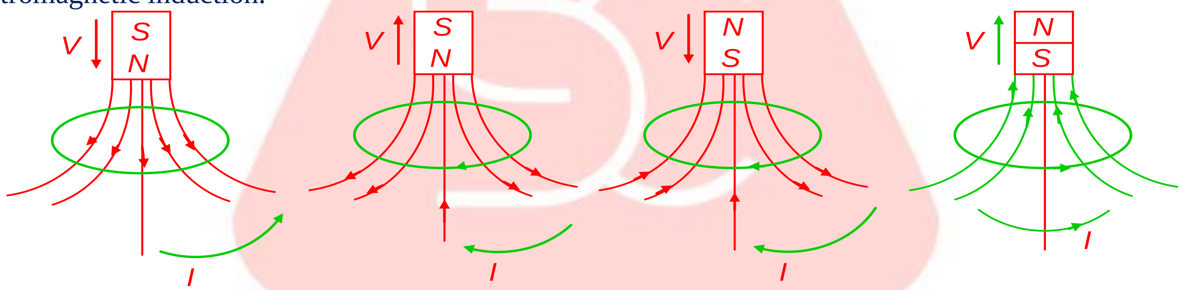


Figure shows a bar magnet placed along the axis of a conducting loop connected with a galvanometer. In this case there is no current in the galvanometer. Now if we move the magnet towards the loop there is a deflection in the galvanometer showing that there is an electric current inside the loop. If the magnet is moved away from the loop a current flows again in the loop but in the opposite direction. This current exists as long as the magnet is moving. Faraday studied this behaviour in detail and discovered the following law of nature.

Whenever the flux of magnetic field through the area bounded by a closed conducting loop changes, an emf is produced in the loop. The emf is given by

$$\varepsilon = -\frac{d\phi}{dt} \qquad \dots (2)$$

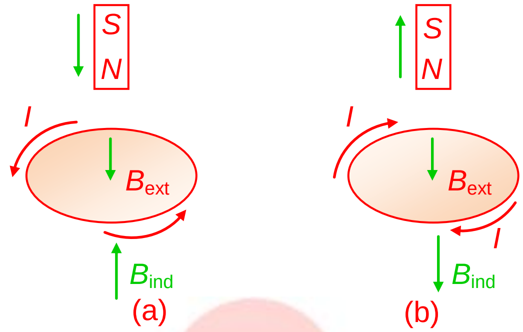
where $\phi = \int \vec{B}.d\vec{A}$ the flux of magnetic field through the area.

The emf so produced drives an electric current through the loop. If the resistance of the loop is R, then the current

$$i = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{d\phi}{dt}. \qquad \dots (3)$$

3 LENZ'S LAW

The effect of the induced emf is such as to oppose the change in flux that produces it.



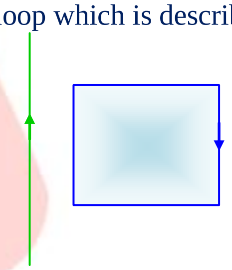
In figure (a) as the magnet approaches the loop, the positive flux through the loop increases. The induced current sets up an induced magnetic field. B_{ind} whose negative flux opposes this change. The direction of B_{ind} is opposite to that of external field B_{ext} due to the magnet.

In figure (b) the flux through the loop decreases as the magnet moves away from the loop, the flux due to the induced magnetic field tries to maintain the flux through the loop, so the direction of induced current is opposite to that first case. The direction of B_{ind} is same as that of B_{ext} due to magnet.

Lenz's law is closely related to law of conservation of energy and is actually a consequence of this general law of nature. As the north pole of the magnet moves towards the loop which opposes the motion of N-pole of the bar magnet. Thus, in order to move the magnet toward the loop with a constant velocity an external force is to be applied. The work done by this external force gets transformed into electric energy, which induces current in the loop.

There is another alternative way to find the direction of current inside the loop which is describe below.

Figure shows a conducting loop placed near a long, straight wire carrying a current i as shown. If the current increases continuously, then there will be an emf induced inside the loop. Due to this induced emf an electric current is induced. To determine the direction of current inside the loop we put an arrow as shown. The right hand thumb rule shows the positive normal to the loop is going into the plane. Again the same rule shows that the magnetic field at the site of the loop is also going into the plane of the diagram.



Thus $\vec{B}$ and $d\vec{A}$ are in same direction. Therefore $\int \vec{B}.d\vec{A}$ is positive if i increases, the magnitude of ϕ increases. Since ϕ is positive and its magnitude increases, $\dfrac{d\phi}{dt}$ is positive. Thus ε is negative and hence the current is negative. Thus the current induced is opposite, to that of arrow.

4 CALCULATION OF INDUCED EMF

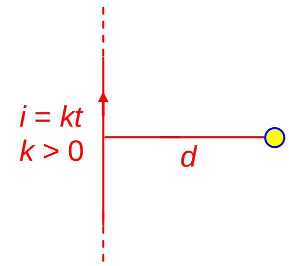
As we know that magnetic flux (ϕ) linked with a closed conducting loop = $B.A\cos\theta$

where B is the strength of the magnetic field, A is the magnitude of the area vector and θ is the angle between magnetic field vector and area vector.

Hence flux will be affected by change in any of them, which is discussed below.

4.1 BY CHANGING THE MAGNETIC FIELD

Consider a long infinite wire carrying a time varying current $i = kt$ ($k > 0$), since current in the wire is continuously increasing therefore we conclude that magnetic field due to this wire in the region is also increasing. Let a circular loop of radius a ($a << d$) is placed at a distance d from the wire. The resistance of the loop is R.



Now magnetic field B due to wire = $\dfrac{\mu_0 i}{2\pi d}$ going into and perpendicular to the plane of the paper

Flux through the circular loop

$$\phi = \frac{\mu_0 i}{2\pi d} \times \pi a^2$$

$$\phi = \frac{\mu_0 a^2 kt}{2d}$$

Induced e.m.f. in the loop.

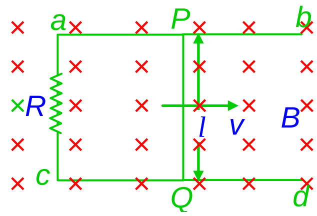
$$\therefore \quad \varepsilon = -\frac{d\phi}{dt} = \frac{-\mu_0 a^2 k}{2d}$$

Induced current in the loop $I = \frac{|\varepsilon|}{R} = \frac{\mu_0 a^2 k}{2dR}$

Direction of induced current is anticlockwise

4.2 BY CHANGING THE AREA

Figure shows a conductor kept in contact with two conducting wires *ab* and *cd* which are connected with the help of a resistance *R*. The entire setup is placed inside a region of uniform magnetic field. Let the conductor start moving with uniform velocity *v* away from resistance as shown. As the conductor moves away the area of closed loop increases causing an induced e.m.f. in the closed loop.



Let at any time *t* the wire *PQ* is at a distance *x* from *ac*

So flux ϕ of magnetic field through closed loop is

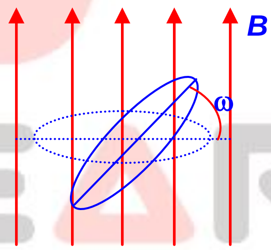
$$\varepsilon = -\frac{d\phi}{dt} = -\frac{Bldx}{dt}$$

$$I = \frac{|\varepsilon|}{R} = \frac{Blv}{R}$$

direction of current is anticlockwise which can be found either by Faraday's law or by Lenz's law.

4.3 BY CHANGING THE ANGLE

Let us consider the case when the magnitude of the magnetic field strength and the area of the coil remains constant. When the coil is rotated relative to the direction of the field, an induced current is produced which lasts as long as coil is rotating.



We have, $\phi = BA\cos\theta$ [where *B* is the magnetic field strength, *A* is the magnitude of the area vector & θ is the angle between them]

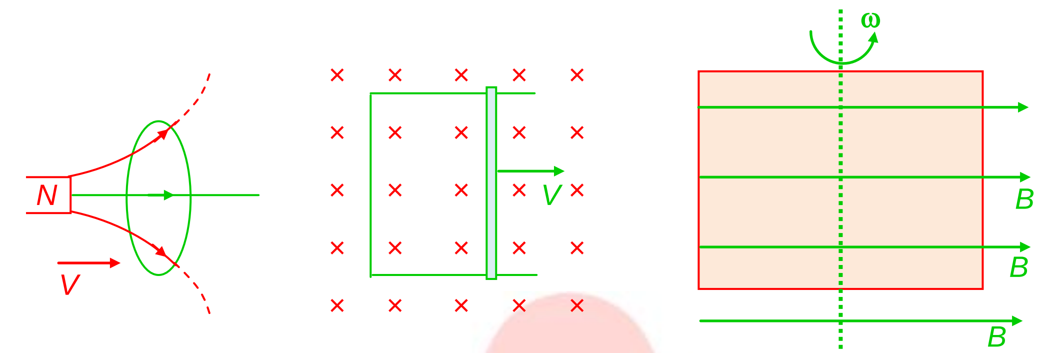
If the angular velocity with which the coil is rotating is ω then $\theta = \omega t$

Induced e.m.f in the coil

$$\varepsilon = -\frac{d\phi}{dt} = BA\omega \sin \omega t$$

Induced current in the coil $= I = \frac{|\varepsilon|}{R} = \frac{B\omega A}{R}\sin\omega t$

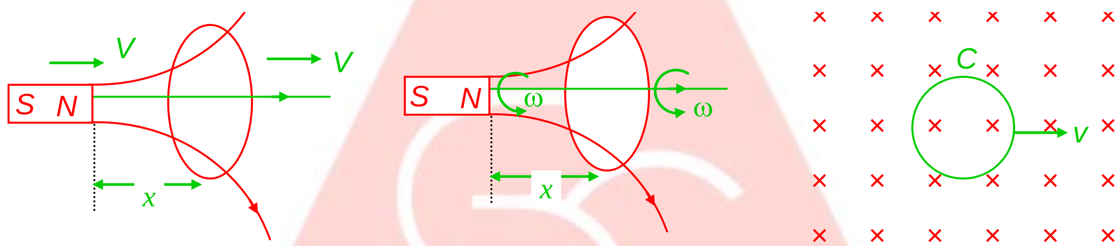
So we conclude that induced current in any circuit is caused either by the change of magnetic field vector or by the change of area vector.



Flux changes due to change of magnetic field

Flux changes due to change of magnitude of area

Flux changes due to change in the angle between magnetic field vector and area vector.



(a) (b) (c)

In figure (a), (b) and (c) the flux linked with the loop *C* will not change with time as here none of the quantities i.e *B*, *A* and θ changes.

Hence, $\phi \neq 0$ but $\frac{d\phi}{dt} = 0$

So e.m.f. induced inside the loop will be zero.

5 MECHANISM OF THE INDUCED EMF ACROSS THE ENDS OF A MOVING ROD

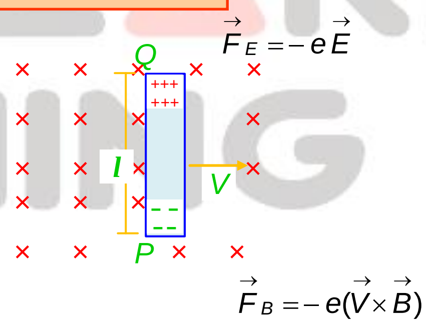


Figure shows a conducting rod of length *l* moving with a constant velocity *v* in a uniform magnetic field. The length of the rod is perpendicular to magnetic field, and velocity is perpendicular to both the magnetic field and the length of the rod. An electron inside the conductor experiences a magnetic force $\vec{F}_B = -e(\vec{v} \times \vec{B})$ directed downward along the rod. As a result electrons migrate towards the lower end and leave unbalanced positive charges at the top. This redistribution of charges sets up an electric field *E* directed downward. This electric field exerts a force on free electrons in the upward direction. As redistribution continues electric field grows in magnitude until a situation, when

$$|q\vec{V} \times \vec{B}| = |q\vec{E}|$$

After this, there is no resultant force on the free electrons and the potential difference across the conductor is

$$\int d\varepsilon = -\vec{E}.d\vec{l} = \int(\vec{v}\times\vec{B}).d\vec{l} \qquad \dots (4)$$

Thus it is the magnetic force on the moving free electrons that maintains the potential difference. So e.m.f. developed across the ends of the rod moving perpendicular to magnetic field with velocity perpendicular to the rod,

$$\varepsilon = vBl \qquad \dots (5)$$

As this emf is produced due to the motion of the conductor, it is called motional emf.

In the problems related to motional e.m.f. we can replace the rod by a battery of e.m.f. *vBl.*

6 INDUCED ELECTRIC FIELD DUE TO A TME VARYING MAGNETIC FIELD

Consider a conducting loop placed at rest in a magnetic field $\vec{B}$. Suppose, the field is constant till $t = 0$ and then changes with time. An induced current starts in the loop at $t = 0$.

The free electrons were at rest till $t = 0$ (we are not interested in the random motion of the electrons). The magnetic field cannot exert force on electrons at rest. Thus, the magnetic force cannot start the induced current. The electrons may be forced to move only by an electric field. So we conclude that an electric field appears at time $t = 0$. This electric field is produced by the changing magnetic field and not by charged particles. The electric field produced by the changing magnetic field is nonelectrostatic and nonconservative in nature. We cannot define a potential corresponding to this field. We call it induced electric field. The lines of induced electric field are closed curves. There are no starting and terminating points of the field lines.

If $\vec{E}$ be the induced electric field, the force on the charge q placed in the field is $q\vec{E}$. The work done per unit charge as the charge moves through $d\vec{l}$ is $E.d\vec{l}$.

The presence of a conducting loop is not necessary to have an induced electric field. As long as $\vec{B}$ keeps changing, the induced electric field is present. If a loop is there, the free electrons start drifting and consequently an induced current results.

7 SELF INDUCTANCE

When the current flows in a coil, it gives rise to a magnetic flux through the coil itself. When the strength of current changes, the flux also changes and an e.m.f. is induced in the coil. This e.m.f. is called self induced e.m.f. and the phenomenon is known as self induction.

The flux through the coil is proportional to current through it, i.e.,

$$\phi \propto i$$

$$\therefore \quad \varepsilon = -\frac{d\phi}{dt} \text{ i.e., } \varepsilon \propto \frac{di}{dt}$$

$$\therefore \quad \varepsilon = -L\frac{di}{dt} \qquad \dots (7)$$

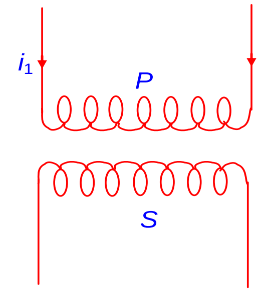
Where L is a constant of proportionality and is called self-inductance or simply inductance of the coil. The unit of inductance is Henry. If the coil is wound over an iron core, the inductance is increased by a factor μ_r (relative permeability of iron).

8 MUTUAL INDUCTANCE

Consider two coils P and S placed close to each other as shown in the figure. When current passing through a coil increases or decreases, the magnetic flux linked with the other coil also changes and an induced e.m.f. is developed in it. This phenomenon is known as mutual induction. This coil in which current is passed is known as primary and the other in which e.m.f. is developed is called as secondary.

Let the current through the primary coil at any instant be i_1. Then the magnetic flux ϕ_2 in the secondary at any time will be proportional to i_1 i.e.,

$$\phi_2 \propto i_1$$



Therefore the induced e.m.f. in secondary when i_1 changes is given by

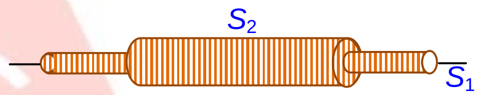
$$\varepsilon = -\frac{d\phi_2}{dt} \text{ i.e., } \varepsilon \propto -\frac{di_1}{dt}$$

$$\therefore \quad \varepsilon = -M\frac{di_1}{dt} \qquad \ldots (8)$$

where M is the constant of proportionality and is known as mutual inductance of two coils. It is defined as the e.m.f. Induced in the secondary coil by unit rate of change of current in the primary coil. The unit of mutual inductance is henry (H).

8.1 MUTUAL INDUCTANCE OF A PAIR OF SOLENOIDS ONE SURROUNDING THE OTHER COIL

Figure shows a coil of N_2 turns and radius R_2 surrounding a long solenoid of length l_1, radius R_1 and number of turns N_1.



To calculate M between them, let us assume a current i_1 through the inner solenoid S_1

There is no magnetic field outside the solenoid and the field inside has magnitude,

$$B = \mu_0\left(\frac{N_1}{l_1}\right)i_1$$

and is directed parallel to the solenoid's axis. The magnetic flux ϕ_{B2} through the surrounding coil is, therefore,

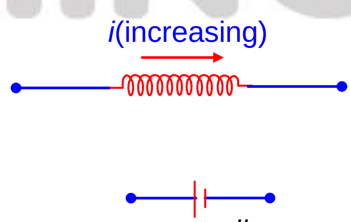
$$\phi_{B2} = B(\pi R_1^2) = \frac{\mu_0 N_1 i_1}{l_1}\pi R_1^2$$

Now, $$M = \frac{N_2\phi_{B2}}{i_1} = \left(\frac{N_2}{i_1}\right)\left(\frac{\mu_0 N_1 i_1}{l_1}\right)\pi R_1^2 = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}, \quad M = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}$$

Notice that M is independent of the radius R_2 of the surrounding coil. This is because solenoid's magnetic field is confined to its interior coil.

9 ENERGY STORED IN AN INDUCTOR

The energy of a capacitor is stored in the electric field between its plates. Similarly an inductor has the capability of storing energy in its magnetic field.



$$\varepsilon = L\frac{di}{dt}$$

An increasing current in an inductor causes an emf between its terminals.

The work done per unit time is power.

$$P = \frac{dW}{dt} = -\varepsilon i = -Li\frac{di}{dt}$$

As $\quad dW = -dU$

or, $$\frac{dW}{dt} = -\frac{dU}{dt}$$

we have, $\frac{dU}{dt} = Li\frac{di}{dt}$

or, $dU = Lidi$

The total energy U supplied while the current increases from zero to a final value i is,

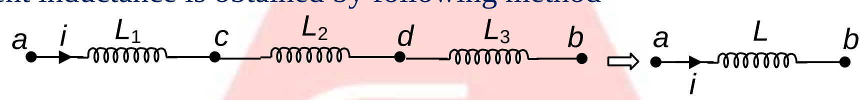
$$U = L\int_0^i idi = \frac{1}{2}Li^2$$

$$U = \frac{1}{2}Li^2 \qquad \dots (9)$$

10 COMBINATION OF INDUCTANCES

10.1 SERIES COMBINATION

In series: If several inductances are connected in series and there are no interactions between them then their equivalent inductance is obtained by following method



(a) (b)

Refer figure (a)

$V_a - V_c = L_1\frac{di}{dt}$

$V_c - V_d = L_2\frac{di}{dt}$ and $V_d - V_b = L_3\frac{di}{dt}$

adding all these equations, we have

$$V_a - V_b = (L_1 + L_2 + L_3)\frac{di}{dt} \qquad \dots \text{(i)}$$

Refer figure (b)

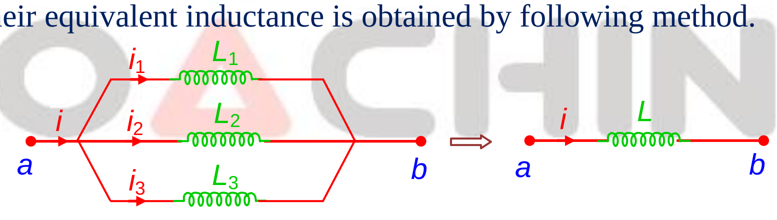
$$V_a - V_b = L\frac{di}{dt} \qquad \dots \text{(ii)}$$

Here L = equivalent inductance.

From equations (i) and (ii), we have $L = L_1 + L_2 + L_3$ … (**10**)

10.2 PARALLEL COMBINATION

If several inductors are connected in parallel combination such that there are no interactions between them, then their equivalent inductance is obtained by following method.



(a) (b)

Refer figure (a)

$i = i_1 + i_2 + i_3$

or $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt}$

or $\frac{di}{dt} = \frac{V_a - V_b}{L_1} + \frac{V_a - V_b}{L_2} + \frac{V_a - V_b}{L_3}$ … (i)

Refer figure (b)

$$\frac{di}{dt}=\frac{V_a-V_b}{L} \qquad \dots \text{(ii)}$$

From equation (i) and (ii),

$$\frac{1}{L}=\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3} \qquad \dots \textbf{(11)}$$

MIND MAP

1. Magnetic flux

$\phi = B.A \cos\theta$

It can be changed by changing B, A or θ with time

2. Faraday's law:

The emf induced in a closed loop is given by rate of change of magnetic flux with time

$$\varepsilon = -\frac{d\phi}{dt}$$

3. Lenz's law: effect of the induced emf is such as to oppose the change in flux that produces it.

4. Motional Emf:

Emf induced across a conductor of length l moving in a uniform magnetic field is

$$\varepsilon = \int (\vec{v} \times \vec{B}).d\vec{l}$$

ELECTROMAGNETIC INDUCTION

5. (a) Self induction

$\phi = Li$

$$\varepsilon = -L\frac{di}{dt}$$

where L is called self inductance of the coil

(b) Mutual induction

$\phi = M\,i$

$$\varepsilon = -M\frac{di}{dt}$$

Mutual inductance of pair of solenoids

$$M = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}$$

(c) Energy stored in an inductor

$$E = \frac{1}{2}Li^2$$

(d) Grouping of inductors

(i) In series combination

$L = L_1 + L_2 + L_3 \ldots\ldots$

(ii) In parallel combination

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \ldots\ldots$$