# ELECTROSTATIC POTENTIAL AND CAPACITANCE

## 14 ELECTRIC POTENTIAL

The electric potential at a point in an electric field is the external work needed to bring a unit positive charge, with constant, from infinity (point of zero potential) to the given point. Thus,

$$V = \frac{W_{ext}}{q_0} \quad \text{... (16)}$$

Where $W_{ext}$ is work done in moving a charge $q_0$ from infinity to that point.

**Important points regarding electric potential**

**(i)** As electric field is conservative, $W_{ext} = U$ .

So,

$$V = \frac{U}{q_0}$$

or, $U = q_0 V$

Thus, the electric potential at a point is numerically equal to the potential energy per unit charge at that point.

**(ii)** It is a scalar having SI unit (J/C) called volt (V).

$$1\text{V} = \frac{1\text{J}}{1\text{C}}$$

**(iii)** If $V_A$ and $V_B$ are the electric potentials of two points $A$ and $B$, the potential difference between $A$ and $B$ is equal to $V_B - V_A$.

Thus the potential difference between two points, $A$ and $B$, is defined as

$$V_B - V_A = \frac{W_{AB}}{q_0}$$

where $W_{A \to B}$ is the work done by an external agent in moving a positive test charge $q_0$ from $A$ to $B$ .

**(iv)** We know that

$$\Delta V = \frac{W_{ext}}{q_0}$$

Now, $W_{ext} = \int \vec{F}_{ext} . d\vec{l}$

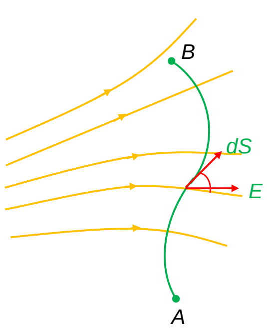
Since the external force is equal and opposite of the electrostatic force, we have

$$\vec{F}_{ext} = -q\vec{E}$$

or $W_{ext} = \int q\vec{E} . d\vec{l}$

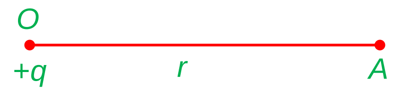
The figure shows a curved path in a non-uniform field. The potential difference between the points $A$ and $B$ is given by

$$V_B - V_A = -\int_A^B \vec{E} . d\vec{l} \quad \text{... (17)}$$




Since the electrostatic field is conservative, the value of this line integral depends only on the end points $A$ and $B$ and not on the path taken. So the electric potential at a point can be interpreted as the negative of the work done by the field in displacing a unit positive charge from some reference point (usually taken at infinity) to the given point.

## 14.1 ELECTRIC POTENTIAL AT A POINT DUE TO A POINT CHARGE




As $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}$ and $V = -\int_{\infty}^{r} \vec{E}.d\vec{r}$

$$\therefore \quad V = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}\cdot d\vec{r} = -\frac{1}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{q}{r^2}dr$$

or $$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}, \qquad \text{... (18)}$$

where $r$ is the distance of $A$ from the point charge $q$.

The electric potential at $A$ ($V_A$) is positive if the point charge $q$ is positive. $V_A$ will be negative if the point charge $q$ is negative.

## 14.2 ELECTRIC POTENTIAL DUE TO A GROUP OF POINT CHARGES

The potential at any point due to a group of point charges is the algebraic sum of the potentials contributed at the same point by all the individual point charges.

$$V = V_1 + V_2 + V_3 + \ldots \qquad \text{... (19)}$$

## 14.3 ELECTRIC POTENTIAL DUE TO A CONTINUOUS CHARGE DISTRIBUTION

The electric potential due to a continuous charge distribution is the sum of potentials of all the infinitesimal charge elements in which the distribution may be divided.

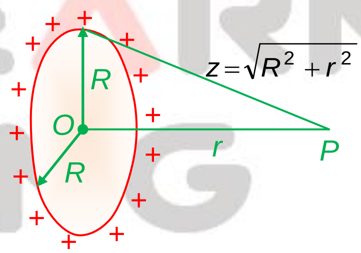
$$V = \int dV$$

$$V = \int \frac{dq}{4\pi\varepsilon_0 r}$$

### 14.3.1 Electric Potential due to a charged ring

A charge $Q$ is uniformly distributed over the circumference of a ring. Let us calculate the electric potential at an axial point at a distance $r'$ from the centre of the ring.

The electric potential at $P$ due to the charge element $dq$ of the ring is given by




$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{Z} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(R^2+r^2)^{1/2}}$$

Hence, the electric potential at $P$ due to the uniformly charged ring is given by

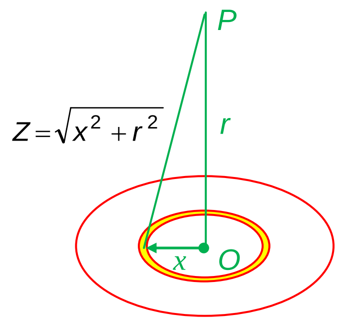
$$V = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{(R^2+r^2)^{1/2}} = \frac{1}{4\pi\varepsilon_0}\frac{1}{(R^2+r^2)^{1/2}}\int dq$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{(R^2+r^2)}} \qquad \text{... (19A)}$$

### 14.3.2 Electric potential due to a charged disc at a point on the axis

A non-conducting disc of radius '$R$' has a uniform surface charge density $\sigma$ C/m$^2$. Let us calculate the potential at a point on the axis of the disc at a distance '$r$' from its centre

The symmetry of the disc tells us that the appropriate choice of element is a ring of radius $x$ and thickness $dx$. All points on this ring are at the same distance $Z = \sqrt{x^2 + r^2}$ , from the point $P$. The charge on the ring is $dq = \sigma dA = \sigma\,(2\pi x dx)$ and so the potential due to the ring is




$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{Z} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi x dx)}{\sqrt{x^2 + r^2}}$$

Since potential is scalar, there are no components to worry about.

The potential due to the whole disc is given by

$$V = \frac{\sigma}{2\varepsilon_0}\int_O^R \frac{x dx}{\sqrt{x^2 + r^2}} \quad = \frac{\sigma}{2\varepsilon_0}\left[(x^2 + r^2)^{1/2}\right]_0^R$$

$$= \frac{\sigma}{2\varepsilon_0}\left[(R^2 + r^2)^{1/2} - r\right] \qquad \text{... (19B)}$$

☞ Let us see how this expression behaves at large distances, when $r >> R$. We use binomial theorem $(1 + x)^n \approx 1 + nx$ for small $x$ to expand the first term

$$(R^2 + r^2)^{1/2} = r\,[1 + \frac{R^2}{r^2}]^{1/2}$$

$$\approx r\left[1 + \frac{R^2}{2r^2} + \ldots\right]$$

Substituting this into the expression for $V$, we find

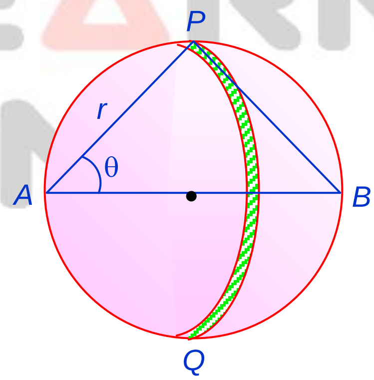
$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$, where $Q = \pi r^2$ is the total charge on the disc.

Thus, we conclude that at large distances, the potential due to the disc is the same as that of a point charge $Q$.

### 14.3.3 Electric Potential due to a closed disc at a point on the edge.

Let us calculate the potential at the edge of a thin disc of radius '$R$' carrying a uniformly distributed charge with surface density $\sigma$

Let $AB$ be a diameter and $A$ be a point where the potential is to be calculated. From $A$ as centre, we draw two arcs of radii r and $r + dr$ as shown. The infinitesimal region between these two arcs is an element whose area is $dA = (2r\theta)\,dr$, where $2\theta$ is the angle subtended by this element $PQ$ at the point $A$. Potential at $A$ due to the element $PQ$ is




$$dV = \frac{\sigma dA}{4\pi\varepsilon_0 r} = \frac{2\sigma r\theta dr}{4\pi\varepsilon_0 r} = \frac{2\sigma\theta dr}{4\pi\varepsilon_0}$$

From $\Delta\,APB$, we have

$$r = 2R\cos\theta$$

or, $\quad dr = -2R\sin\theta\,d\theta$

Hence

$$dV = \frac{-4\sigma\theta R\sin\theta\,d\theta}{4\pi\varepsilon_0}$$

$$V = -\int_{\pi/2}^{0} \frac{\sigma R\theta \sin\theta d\theta}{\pi\varepsilon_0}$$

$$V = -\frac{\sigma R}{\pi\varepsilon_0}\left|-\theta\cos\theta + \sin\theta\right|_{\pi/2}^{0} \quad V = \frac{\sigma R}{\pi\varepsilon_0} \qquad \text{... (19C)}$$

### 14.3.4 Electric Potential due to a shell

A shell of radius $R$ has a charge $Q$ uniformly distributed over its surface. Let us calculate the potential at a point

(a) outside the shell; ($r > R$) (b) inside the shell ($r < R$).

(a) At points outside a uniform spherical distribution, the electric field is $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$

since $\vec{E}$ is radial, $\vec{E}.d\vec{r} = Edr$

since $V(\infty) = 0$, we have

$$V(r) - V(\infty) = \int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r^2} dr = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{r}$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \; (r > R) \qquad \text{... (19D)}$$

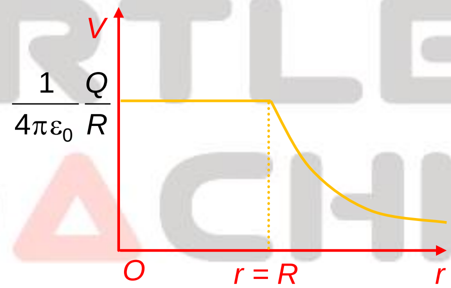
We see that the potential due to a uniformly charged shell is the same as that due to a point charge $Q$ at the centre of the shell.

(b) **At an Internal Point**

At points inside the shell, $E = 0$. So, the work done in bringing a unit positive charge from a point on the surface to any point inside the shell is zero. Thus, the potential has a fixed value at all points within the spherical shell and is equal to the potential at the surface.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \qquad \text{... (19E)}$$

Variation of electric potential with the distance from the centre ($r$)




☞ All the above results hold for a "conducting sphere' also whose charge lies entirely on the outer surface.

### 14.3.5 Electric Potential due to a non-conducting charged sphere

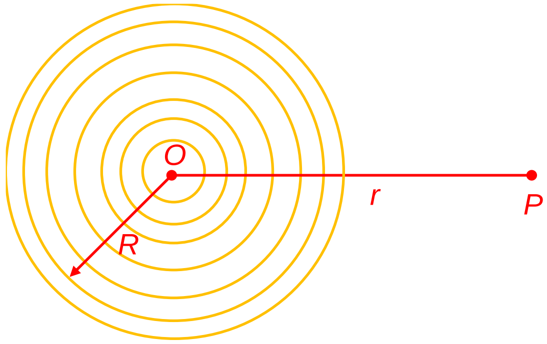
A charge $Q$ is uniformly distributed throughout a non-conducting spherical volume of radius $R$. Let us find expressions for the potential at an (a) external point ($r > R$); (b) internal point ($r < R$); where $r$ is the distance of the point from the centre of the sphere.

**(a) At an external point**

Let $O$ be the centre of a non-conducting sphere of radius $R$, having a charge $Q$ distributed uniformly over its entire volume.

Let $P$ be a point distant $r$ ( > $R$) from $O$ at which potential is required. Let $\rho$ be the charge density

Let us divide the sphere into a large number of thin concentric shells carrying charges $q_1$, $q_2$, $q_3$ … etc. The potential at the point $P$ due to the shell of charge $q_1$ is $\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$




Now, potential is a scalar quantity. Therefore the potentials $V$ due to the whole sphere is equal to the sum of the potentials due to all the shells.

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r} + \ldots$$

$$= \frac{1}{4\pi\varepsilon_0 r}[q_1 + q_2 + q_3 + \ldots..]$$

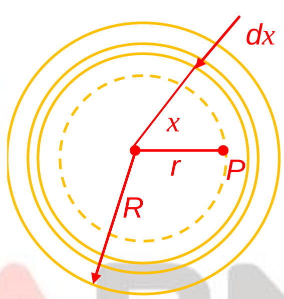
But $q_1 + q_2 + q_3 + \ldots = Q$, the charge on the sphere.

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \qquad \ldots (\textbf{19F})$$

**(b) Potential at an internal point**

Suppose the point $P$ lies inside the sphere at a distance $r$ from the centre $O$, If we draw a concentric sphere through the point $P$, the point $P$ will be external for the solid sphere of radius $r$, and internal for the outer spherical shell of internal radius $r$ and external radius $R$.

The charge on the inner solid sphere is $\frac{4}{3}\pi r^3\rho$. Therefore, the potential $V_1$ at $P$ due to this sphere is given by




$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{4/3\ \pi r^3\rho}{r} = \frac{r^2\rho}{3\varepsilon_0}$$

Let us now find the potential at $P$ due to the outer spherical shell. Let us divide this shell into a number of thin concentric shells and consider one such shell of radius $x$ and infinitesimally small thickness $dx$. The volume of this shell = surface area × thickness = $4\pi x^2 dx$. The charge on this shell, $dq = 4\pi x^2\, dx\rho$. The potential at $P$ due to this shell

$$dV_2 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi x^2(dx)\rho}{x}$$

$$= \frac{\rho x dx}{\varepsilon_0}$$

The potential $V_2$ at $P$ due to the whole shell of internal radius $r$ and external radius $R$ is given by

$$V_2 = \int_r^R \frac{\rho x dx}{\varepsilon_0} = \frac{\rho}{\varepsilon_0}\left|\frac{x^2}{2}\right|_r^R$$

$$= \frac{\rho(R^2 - r^2)}{2\varepsilon_0}$$

Since the potential is a scalar quantity, the total potential $V$ at $P$ is given by

$$V = V_1 + V_2$$

$$= \frac{r^2\rho}{3\varepsilon_0} + \frac{\rho(R^2 - r^2)}{2\varepsilon_0}$$

$$= \frac{\rho(3R^2 - r^2)}{6\,\varepsilon_0}$$

But $\rho = \dfrac{Q}{\frac{4}{3}\pi R^3}$

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R^3}[3R^2 - r^2] \quad \dots (19G)$$

## 15 CALCULATION OF ELECTRIC FIELD FROM ELECTRIC POTENTIAL

In rectangular components, the electric field is

$$\vec{E} = E_x\,\hat{i} + E_y\,\hat{j} + E_z\,\hat{k};$$

and an infinitesimal displacement is $d\vec{S} = dx\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

Thus,

$$dV = -\vec{E}.d\vec{S} \quad \dots (20)$$

$$= -[E_x\,dx + E_y\,dy + E_z\,dz]$$

for a displacement in the $x$-direction,

$dy = dz = 0$ and so

$dV = -E_x\,dx$. Therefore,

$$E_x = -\left(\frac{dV}{dx}\right)_{y,z\ \text{constant}}$$

A derivative in which all variables except one are held constant is called partial derivative and is written with $\partial$ instead of $d$. The electric field is, therefore,

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} \quad \dots (21)$$

## 16 EQUIPOTENTIAL SURFACES

If we join the points in an electric field, which are at same potential, the surface (or curve) obtained is known as equipotential surface (curve).

**Important Points Regarding Equipotential surfaces**

(i) The lines of forces are always normal to equipotential surfaces

(ii) The net work done in taking a charge from $A$ to $B$ is zero if $A$ and $B$ are on same equipotential surface.

***Examples***

***(i) In the field of a point charge, the equipotential surfaces are spheres centered on the point charge.***

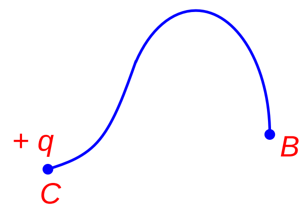
***(ii) In a uniform electric field, the equipotential surfaces are planes which are perpendicular to the field lines.***

*(iii)* ***In the field of an infinite line charge, the equipotential surfaces are co-axial cylinders having their axes at the line charge.***

*(iv)* ***The surface of a conductor is an equipotential surface and the inside of conductor is equipotential space. Hence there is no electric field (and charge) inside the conductor's surface. The lines of forces are always normal to the surface of a conductor.***

## 17 ELECTRIC POTENTIAL ENERGY

**If a charge is moved between two points in an electric field, work is usually done against the field or by the field. In the figure, if a charge +*q* is moved from *B* to *C* in the electric field of charge +*Q*, the work will have to be done by some outside agency in pushing the charge +*q* against the force of field of +*Q*.**




This situation is very similar to that of a mass moved in gravitational field of earth away from it. Work done against the gravitational pull of earth is stored in Gravitational potential energy and can be recovered back. Similarly in electric field, work done against an electric field is stored in the form of electric potential energy & can be recovered back. If the charge +*q* is taken back from *C* to *B*, the electric force will try to accelerate the charge and hence to recover the potential stored in the form of kinetic energy.

As the work done against an electric field can be recovered back, electrostatic forces and fields fall under the category of conservative forces and fields. Another property of these fields is that the work done is independent of path taken from one point to the another.

### 17.1 POTENTIAL ENERGY OF A SYSTEM OF TWO POINT CHARGES

**The potential energy possessed by a system of two-point charges $q_1$ and $q_2$ separated by a distance *r* is the work done required to bring them to this arrangements from infinity. This electrostatic potential energy is given by**

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r} \quad \text{... (22)}$$

### 17.2 ELECTRIC POTENTIAL ENERGY OF A SYSTEM OF POINT CHARGES

**The electric potential energy of such a system is the work done in assembling this system starting from infinite separation between any two-point charges.**

For a system of point charges $q_1$, $q_2$ …. $q_n$, the potential energy is

$$U = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}} \; (i \neq j) \quad \text{... (23)}$$

If simply means that we have to consider all the pairs that are possible.

**Important points regarding Electrostatic potetnail energy**

**(i)** Work done required by an external agency to move a charge *q* from *A* to *B* in an electric field with constant speed

$W_{A\to B} = q\,[V_B - V_A]$

**(ii)** When a charge *q* is let free in an electric field, it loses potential energy and gains kinetic energy, if it goes from *A* to *B*, then loss in potential energy = gain in kinetic energy or, $q\,(V_B - V_A) = \frac{1}{2}\,mV_B^2 - \frac{1}{2}\,mV_A^2$

## 20 CAPACITY OF AN ISOLATED CONDUCTOR

When charge is given to an isolated body, its potential increases i.e.,

$$Q \propto V$$

or, $Q = CV$ ... (**34**)

where $C$ is a constant called capacity of the body.

if $V = 1$ then $C = Q$,

So Capacity of a body is numerically equal to the charge required to raise its potential by unity.

In SI system, the unit of capacity is (Coulomb/volt) and is called farad (F).

$$1\text{F} = \frac{1C}{1V} = \frac{3\times10^9 \text{ esu of charge}}{\left(\frac{3}{300}\right)\text{esu of potential}} = 9 \times 10^{11} \text{ esu of capacity.}$$

The capacity of a body is independent of the charge given to it and depends on its shape and size only.

## 21 CAPACITOR

Capacitor is an arrangement of two conductors carrying charges of equal magnitude and opposite sign and separated by an insulating medium. The following points may be carefully noted

**(i)** The net charge on the capacitor as a whole is zero. When we say that a capacitor has a charge $Q$, we mean that positively charged conductor has a charge $+Q$ and the negatively charged conductor has a charge $-Q$.

**(ii)** The positively charged conductor is at a higher potential than negatively charged conductor. The potential difference $V$ between the conductors is proportional to the magnitude of charge $Q$ and the ratio $Q/V$ is known as capacitance C of the capacitor.

$$C = \frac{Q}{V}$$

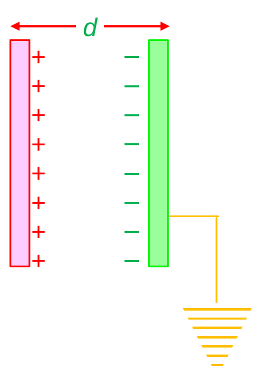
Unit of capacitance is farad (F). The capacitance is usually measured in microfarad $\mu F$.

$$1\mu F = 10^{-6} F$$

**(iii)** In a circuit, a capacitor is represented by the symbol:– 

### 21.1 PARALLEL PLATE CAPACITOR

A parallel plate capacitor consists of two metal plates placed parallel to each other and separated by a distance $d$ that is very small as compared to the dimensions of the plates. The area of each plate is $A$. The electric field between the plates is given by




$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Where $\sigma$ is surface charge density on either plate.

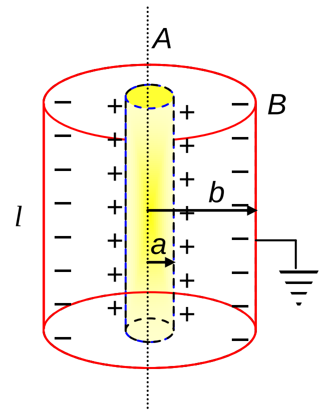
the potential difference ($V$) between plates is given by $V = Ed$.

or, $V = \frac{\sigma}{\varepsilon_0} d = \frac{Q}{A\varepsilon_0} d$

Hence, $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$ ... (**35**)

## 21.2 CYLINDRICAL CAPACITOR

Cylindrical capacitor consists of two co-axial cylinders of radii *a* and *b* and length *l*. If a charge *q* is given to the inner cylinder, induced charge –*q* will reach to the inner surface of the outer cylinder. By symmetry, the electric field in region between the cylinders is radially outward.




By Gauss's theorem, the electric field at a distance *r* from the axis of the cylinders is given by

$$E = \frac{1}{2\pi\varepsilon_0 l} \frac{q}{r}$$

The potential difference between the cylinders is given by

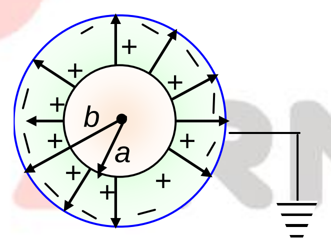
$$V = -\int_b^a \vec{E}.d\vec{r} = -\frac{1}{2\pi\varepsilon_0 l} q \int_b^a \frac{dr}{r}$$

$$= \frac{-q}{2\pi\varepsilon_0 l}\left(ln\frac{a}{b}\right)$$

or, $V = \frac{q}{V} = \frac{2\pi\varepsilon_0 l}{ln\left(\frac{b}{a}\right)}$ ... (**36**)

## 21.3 SPHERICAL CAPACITOR

A spherical capacitor consists of two concentric spheres of radii *a* and *b* as shown. The inner sphere is positively charged to potential *V* and outer sphere is at zero potential.




The inner surface of the outer sphere has an equal negative charge.

The potential difference between the spheres is

$$V = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$$

Hence, capacitance

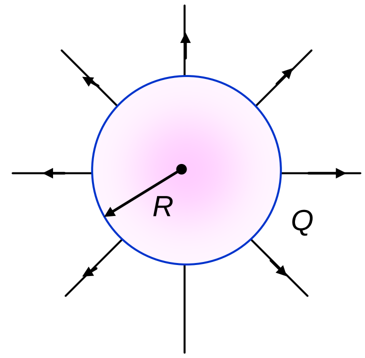
$$C = \frac{Q}{V} = \frac{4\pi\varepsilon_0 ab}{b-a}$$ ... (**37**)

## 21.4 ISOLATED SPHERE AS A CAPACITOR

A conducting sphere of radius $R$ carrying a charge $Q$ can be treated as a capacitor with high potential conductor as the sphere itself and the low potential conductor as a sphere of infinite radius. The potential difference between these two spheres is

$$V = \frac{Q}{4\pi\varepsilon_0 R} - 0$$

Hence, capacitance $C = \frac{Q}{V} = 4\pi\varepsilon_0 R$ ... (**38**)




## 22 ENERGY STORED IN CHARGED CAPACITOR

If $dq$ charge is given to a capacitor at potential $V$

$$dW = dq\,(V)$$

or, $$E = \int_0^q \left(\frac{q}{C}\right) dq \quad [\because q = CV]$$

or, $$W = \frac{q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}qV$$

This work is stored as electrical potential energy i.e., a capacitor stores electrical energy

$$U = \frac{1}{2}CV^2 = \frac{q^2}{2C} = \frac{1}{2}qV$$ ... (**39**)

### 22.1 ENERGY DENSITY OF A CHARGED CAPACITOR

This energy is note localized on the charges or the plates but is distributed in the field. Since in case of a parallel plate capacitor, the electric field is only between the plates, i.e., in a volume (A × d), the energy density

$$U_E = \frac{U}{\text{volume}} = \frac{\frac{1}{2}CV^2}{A \times d} = \frac{1}{2}\left[\frac{\varepsilon_0 A}{d}\right]\frac{V^2}{Ad}$$

or, $$U_E = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2 \quad \left[\because \frac{V}{d} = E\right]$$ ... (**40**)

### 22.2 FORCE BETWEEN THE PLATES OF A CAPACITOR

In a capacitor as plates carry equal and opposite charges, there is a force of attraction between the plates. To calculate this force, we use the fact that the electric field is conservative and in a conservative field $F = -\frac{dU}{dx}$. In case of parallel plate capacitor

$$U = \frac{q^2}{2C} = \frac{1}{2}\frac{q^2 x}{\varepsilon_0 A} \quad \left[\text{as } C = \frac{\varepsilon_0 A}{x}\right]$$

SO, $$F = -\frac{d}{dx}\left[\frac{q^2}{2\varepsilon_0 A}x\right] = \frac{-1}{2}\frac{q^2}{\varepsilon_0 A}$$ ... (**41**)

The negative sign implies that the force is attractive.

## 23 EFFECT OF DIELECTRIC

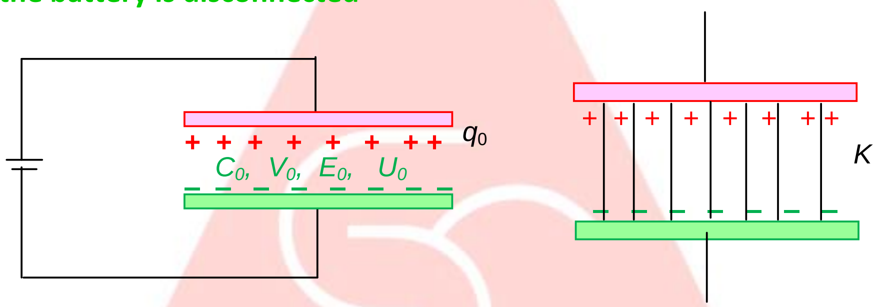
When certain non-conducting materials such as glass, paper or plastic are introduced between the plates of a capacitor, its capacity increases. These materials are called 'dielectrics' and the ratio of capacity of a capacitor when completely filled with dielectric $C$ to that without dielectric $C_0$ is called 'dielectric constant $K$, or relative permitivity $\varepsilon_r$ or specific inductive capacity (S. I. C) i.e.,

$$K = \frac{C}{C_0} \qquad \text{... (42)}$$

The effect of dielectric on other physical quantities such as charges, potential difference, field and energy associated with a capacitor depends on the fact that whether the charge capacitor is isolated (i.e., charge held constant) or battery attached (i.e., potential is held constant).

### 23.1 INTRODUCTION OF A DIELECTRIC SLAB OF DIELECTRIC CONSTANT *K* BETWEEN THE PLATES

**(a) When the battery is disconnected**




Let $q_0$, $C_0$, $V_0$, $E_0$ and $U_0$ represents the charge, capacity, potential difference electric field and energy associated with charged air capacitor respectively. With the introduction of a dielectric slab of dielectric constant $K$ between the plates and the battery disconnected.

**(i)** Charge remains constant, i.e., $q = q_0$ , as in an isolated system charge is conserved.

**(ii)** Capacity increases, i.e., $C = KC_0$, as by the presence of a dielectric capacity becomes $K$ times.

**(iii)** potential difference between the plates decreases, i.e., $V = \left(\frac{V_0}{K}\right)$,as

$$V = \frac{q}{C} = \frac{q_0}{KC_0} = \frac{V_0}{K} \quad [\because\ q = q_0 \text{ and } C = KC_0]$$

**(iv)** Field between the plates decreases, i.e., $E = \frac{E_0}{K}$, as

$$E = \frac{V}{d} = \frac{V_0}{Kd} = \frac{E_0}{K} \quad [\text{as } V = \frac{V_0}{K}]$$

and $E_0 = \frac{V_0}{d}$

**(v)** Energy stored in the capacitor decreases i.e.,

$$U = \left(\frac{U_0}{K}\right), \text{ as}$$

$$U = \frac{q^2}{2C} = \frac{q_0^2}{2KC_0} = \frac{V_0}{K} \quad (\text{as } q = q_0 \text{ and } C = KC_0]$$

**(b) When the battery remains connected (potential is held constant)**

**(i)** Potential difference remains constant, i.e., $V = V_0$, as battery is a source of constant potential difference.

**(ii)** Capacity increases, i.e., $C = KC_0$, as by presence of a dielectric capacity becomes $K$ times.

**(iii)** Charge on capacitor increases, i.e., $q = Kq_0$ , as

$$q = CV = (KC_0)V = Kq_0 \; [\because \; q_0 = C_0 v]$$

**(iv)** Electric field remains unchanged, i.e., $E = E_0$, as

$$E = \frac{V}{d} = \frac{V_0}{d} = E_0 \; [\text{as } V = V_0 \text{ and } \frac{V_0}{d} = E_0]$$

**(v)** Energy stored in the capacitor increases,

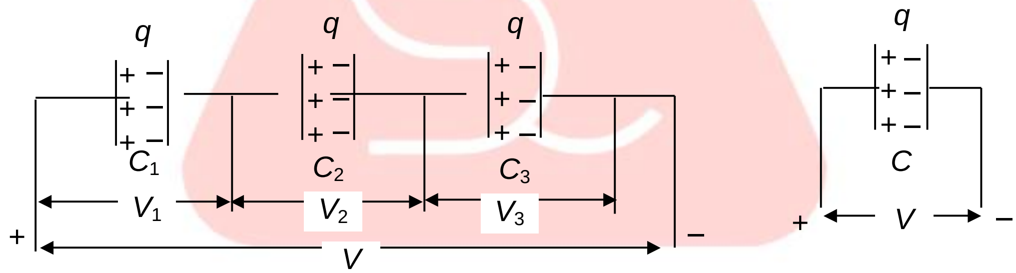
i.e., $U = KU_0$, as $U = \frac{1}{2} CV^2 = \frac{1}{2} (KC_0)(V_0)^2 = \frac{1}{2} KU_0$

$[\text{as } C = KC_0 \text{ and } U_0 = \frac{1}{2} C_0 V_0^{\,2}]$

## 24 GROUPING OF CAPACITORS

Replacing a combination of capacitors by a single equivalent capacitor is called 'grouping of capacitors'. It simplifies the problem and is divided into two types

### 24.1 SERIES COMBINATION OF CAPACITORS.




Capacitors are said to be in series if charge on each individual capacitor is same.

In this situation,

$$V = V_1 + V_2 + V_3$$

We know, $V = \left(\frac{q}{C}\right)$, so

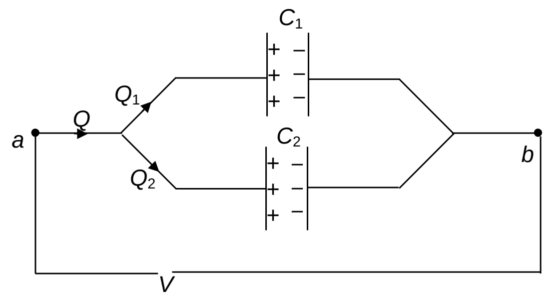
$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

or, $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \text{... (43)}$$

In case the two capacitors connected in series, we have

$$V_1 = \left(\frac{C_2}{C_1 + C_2}\right)V; \; V_2 = \left(\frac{C_1}{C_1 + C_2}\right)V$$

## 24.2 PARALLEL COMBINATION OF CAPACITORS




When capacitors are connected in parallel, the potential difference $V$ across each is same and the charge on $C_1$, $C_2$ is different, i.e., $Q_1$ and $Q_2$,

The total charge $Q$ is given as

$$Q = Q_1 + Q_2$$

$$Q = C_1V + C_2V \text{ or } \frac{Q}{V} = C_1 + C_2$$

Hence, the equivalent capacitance between $a$ and $b$ is

$$C = C_1 + C_2$$

The charges on capacitors is given as

$$Q_1 = \left(\frac{C_1}{C_1 + C_2}\right)Q$$

$$Q_2 = \left(\frac{C_2}{C_1 + C_2}\right)Q$$

In case of more than two capacitors.

$$C = C_1 + C_2 + C_3 + \ldots.. \qquad \ldots \textbf{(44)}$$

## 25 REDISTRIBUTION OF CHARGE

If there are two spherical conductors of radii $R_1$ and $R_2$ at potentials $V_1$ and $V_2$ respectively, far apart from each-other (so that charge on one does not affect the potential of the other). The charge on them will be

$$q_1 = C_1V_1 \text{ and } q_2 = C_2V_2$$

The total charge of the system

$$q = q_1 + q_2$$

The total capacity of the system

$$C = C_1 + C_2$$

Now if they are connected through a wire, charge will flow from conductor at higher potential to that at lower potential till both acquire same potential.

$$V = \frac{(q_1 + q_2)}{(C_1 + C_2)} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{R_1V_1 + R_2V_2}{R_1 + R_2} \quad (\because C \propto R)$$

If $q_1'$ and $q_2'$ are the charges on two conductors after sharing, then

$$q_1' = C_1V \text{ and } q_2' = C_2V \text{, where}$$

$$q_1' + q_2' = (q_1 + q_2) = q$$

So, $$\frac{q_1'}{q_2'} = \frac{C_1}{C_2} = \frac{R_1}{R_2} \quad [\text{ as } C \propto R)$$

i.e., charge is shared in proportion to capacity.

### 25.1 LOSS OF ENERGY DURING REDISTRIBUTION OF CHARGE

Initial potential energy of the system is

$$U_I = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

Final potential energy = $U_F = \frac{1}{2}(C_1 + C_2)V^2$

Putting $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ and simplifying, we get

$$U_F - U_I = -\frac{C_1C_2}{2(C_1 + C_2)}(V_1 \sim V_2)^2$$

Now as $C_1$, $C_2$ and $(V_1 \sim V_2)^2$ are always positive, there is decrease in energy of the system, i.e., in sharing energy is lost. This energy is lost mainly as heat when charge flows from one body to the other through the connecting wire and also as light and sound if sparking takes place.

## 26 VAN DE GRAFF GENERATOR

Van de graff generator is a machine that can build up high voltages of the order of a few million volts. The resulting large electric fields are used to accelerate charged particles like electrons, protons and ions to high energies needed for nuclear transmutation of one element into other. The high energy particles can be used to probe the small scale structure of matter.

### PRINCIPLE

Let us consider a large spherical conducting shell of radius $R$ with uniform charge density. Let the charge on the shell is $Q$. The corresponding potential due to the shell at various points having distance $r$ from the centre of the shell is given by

$$v = \frac{Q}{4\pi\varepsilon_0 r} \qquad r \geq R$$

$$v = \frac{Q}{4\pi\varepsilon_0 R} \qquad r \leq R$$

Next imagine that a sphere of radius $r_0(r_0 < R)$ carrying a total charge $q$ (uniformly distributed over it) is introduced in the larger conducting shell and placed at its centre. Potential due to inner sphere at all points distant $r$ from centre is given by

$$v = \frac{q}{4\pi\varepsilon_0 r} \qquad r \geq r_0$$

Applying superposition theorem, the potential due to the system (sphere inside the shell) is

$$v(R) = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R} + \frac{q}{r}\right],$$

$$v(r_0) = \frac{1}{4\pi\varepsilon_0 r}\left[\frac{Q}{R} + \frac{q}{r_0}\right]$$

the potential difference between the points $r = r_0$ *and* $r = R$ is

$$v(r_0)-v(R)=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_0}-\frac{1}{R}\right] \quad \text{... (45)}$$

Thus for positive value of *q*, whatever be the magnitude and sign of *Q*, the small sphere is at a higher potential than the shell. If inner and outer conductors are connected, charge would flow from the small sphere to the shell. By repeating the process, a large amount of charge can be piled up on the shell, thereby raising its potential.

## CONSTRUCTION AND WORKING

To implement the principle in practice a large spherical conducting shell with radius of few metres is supported on an insulating column several metres high. The conducting shell is made highly polished and is known as metal dome. There are two pulleys, one at the centre of the shell and the other at the ground. A long narrow belt made of insulating material passes over the pulleys. Charge is sprayed on to the belt at the lower pulley by means of a discharge through a metallic brush with sharp points connected to a high voltage source. The belt is moved rapidly by a motor driving the lower pulley. The positive charge is carried upward by the belt and collected by a metallic brush connected to the shell. As more and more charge is transferred to the sphere *S*, its potential goes on rising. In this way, the shell builds up a huge voltage of the order of mega volts.