1 THE MAGNETIC FIELD

In earlier lessons we found it convenient to describe the interaction between charged objects in terms of electric fields. Recall that an electric field surrounds an electric charge. The region of space surrounding a moving charge includes a magnetic field in addition to the electric field. A magnetic field also surrounds a magnetic substance.

In order to describe any type of field, we must define its magnitude, or strength, and its direction.

Magnetic field is the region surrounding a moving charge in which its magnetic effects are perceptible on a moving charge (electric current). Magnetic field intensity is a vector quantity and also known as magnetic induction vector. It is represented by $\vec{B}$.

Lines of magnetic induction may be drawn in the same way as lines of electric field. The number of lines per unit area crossing a small area perpendicular to the direction of the induction being numerically equal to $\vec{B}$. The number of lines of $\vec{B}$ crossing a given area is referred to as the magnetic flux linked with that area. For this reason $\vec{B}$ is also called magnetic flux density.

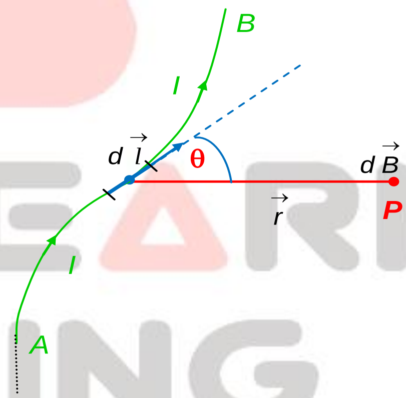
There are two methods of calculating magnetic field at some point. One is **Biot Savart law** which gives the magnetic field due to an infinitesimally small current carrying wire at some point and the another is **Ampere's law,** which is useful in calculating the magnetic field of a symmetric configuration carrying a steady current.

The unit of magnetic field is weber / m^2 and is known as tesla (T) in the SI system.

2 BIOT-SAVART LAW

Biot-Savart law gives the magnetic induction due to an infinitesimal current element.

Let *AB* be a conductor of an arbitrary shape carrying a current *I*, and *P* be a point in vacuum at which the field is to be determined. Let us divide the conductor into infinitesimal current–elements. Let $\vec{r}$ be a displacement vector from the element to the point *P*.



According to 'Biot-Savart Law', the magnetic field induction $d\vec{B}$ at *P* due to the current element $d\vec{l}$ is given by

$$d\vec{B} \propto \frac{I(d\vec{l} \times \vec{r})}{r^3}$$

or, $$d\vec{B} = k\frac{I(d\vec{l} \times \vec{r})}{r^3},$$

where *k* is a constant of proportionality. Here $d\vec{l}$ vector points in the direction of current *I*.

In S.I units, $k = \frac{\mu_0}{4\pi} = 10^{-7}\ \frac{\text{Wb}}{\text{amp} \times \text{metre}}$

$$\therefore \qquad d\vec{B} = \frac{\mu_0}{4\pi}\frac{I(d\vec{l} \times \vec{r})}{r^3} \qquad \dots (1)$$

Equation (1) is the vector form of the Biot Savart Law. The magnitude of the field induction at P is given by

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2},$$

where θ is the angle between $d\vec{l}$ and $\vec{r}$.

If the medium is other than air or vacuum, the magnetic induction is

$$d\vec{B} = \frac{\mu_0\mu_r}{4\pi}\frac{I(d\vec{l} \times \vec{r})}{r^3}, \qquad \dots (2)$$

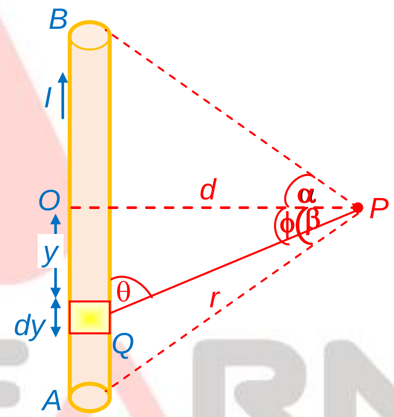
where μ_r is relative permeability of the medium and is a dimensionless quantity.

3 FIELD DUE TO A STRAIGHT CURRENT CARRYING WIRE

3.1 FIELD DUE TO A STRAIGHT WIRE OF FINITE LENGTH

Consider a straight wire segment carrying a current I and there is a point P at which magnetic field is to be calculated is as shown in the figure. This wire segment makes angle α and β at that point with normal OP. Consider an element of length dy at a distance y from O and distance of this element from point P is r and line joining P to Q makes an angle θ with the direction of current as shown in figure. Using Biot-Savart Law magnetic field at point P due to small current element is given by

$$dB = \frac{\mu_0 I}{4\pi}\left(\frac{dy\sin\theta}{r^2}\right)$$



As every element of the wire contributes to $\vec{B}$ in the same direction, we have

$$B = \frac{\mu_0 I}{4\pi}\int_A^B \frac{dy\sin\theta}{r^2} \qquad \dots (i)$$

From the triangle OPQ as shown in diagram, we have

$$y = d\tan\phi$$

or, $\quad dy = d\sec^2\phi\, d\phi$

and in same triangle,

$r = d\sec\phi$ and $\theta = (90° - \phi)$, where ϕ is angle between line OP and PQ

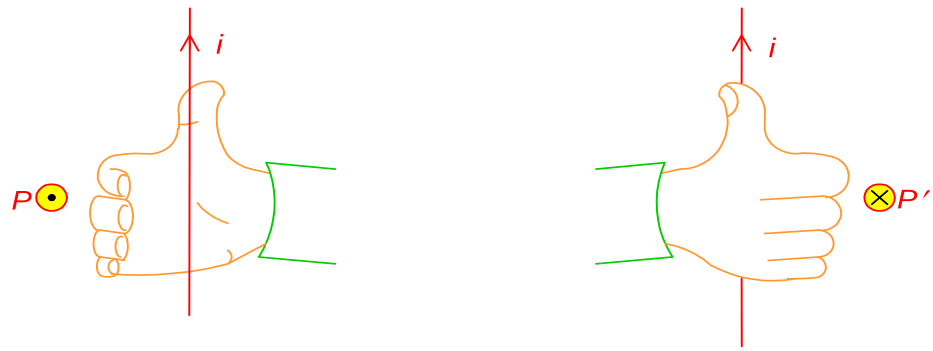
Now equation (i) can be written in this form

$$\therefore \qquad B = \frac{\mu_0}{4\pi}\frac{I}{d}\int_{-\beta}^{\alpha}\cos\phi\, d\phi$$

or, $\quad B = \frac{\mu_0}{4\pi}\frac{I}{d}$ **$[\sin\alpha + \sin\beta]$** $\qquad$ **… (3)**

Direction of $\vec{B}$: The direction of magnetic field is determined by the cross product of the vector $id\vec{l}$ with $\vec{r}$. Therefore, at point P, the direction of the magnetic field due to the whole conductor will be perpendicular to the plane of paper and going into the plane.

Right-hand Thumb Rule: The direction of B at a point P due to a long, straight wire can be found by the right-hand thumb rule. The direction of magnetic field is perpendicular to the plane containing wire and perpendicular from the point. The orientation of magnetic field is given by the direction of curl fingers if we stretch thumb along the wire in the direction of current. Refer figure.



Conventionally, the direction of the field perpendicular to the plane of the paper is represented by ⊗ if into the page and by ⊙ if out of the page.

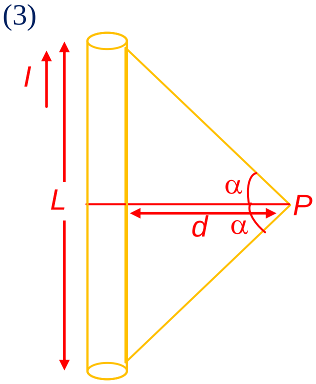
Now consider some special cases involving the application of equation (3)

Case I: When the point *P* is on perpendicular bisector

In this case angle $\alpha = \beta$, using result of equation (3), the magnetic field is

$$B = \frac{\mu_0}{4\pi}\frac{2I}{d}\sin\alpha$$

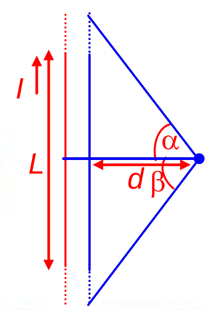
where $\sin\alpha = \dfrac{L}{\sqrt{L^2 + 4d^2}}$



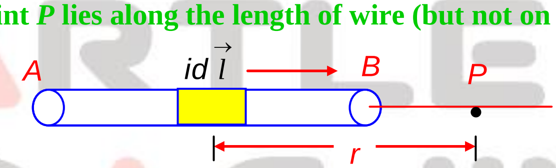
Case II: When wire is of infinite length

In this case $\alpha = \beta = 90°$, using result of equation (3), the magnetic field is

$$B = \frac{\mu_0}{4\pi}\frac{2I}{d}$$



Case III: When the point *P* lies along the length of wire (but not on it):

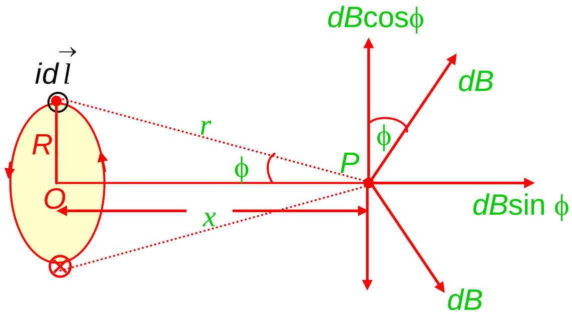


If the point is along the length of the wire (but not on it), then as $d\vec{l}$ and $\vec{r}$ will either be parallel or antiparallel, i.e., $\theta = 0$ or π, so $id\vec{l} \times \vec{r} = 0$ and hence using equation (1)

$$\vec{B} = \int_A^B d\vec{B} = 0$$

4 MAGNETIC FIELD AT AN AXIAL POINT OF A CIRCULAR COIL

Consider a circular loop of radius *R* and carrying a steady current *I*. We have to find out magnetic field at the axial point *P*, which is at distance *x* from the centre of the loop.



Consider an element $Id\vec{l}$ of the loop as shown in figure, and the distance of point *P* from current element is *r*. The magnetic field at *P* due to this current element from the equation (1) can be given by,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3}$$

In case of point on the axis of a circular coil, as for every current element there is a symmetrically situated opposite element, the component of the field perpendicular to the axis cancel each-other while along the axis add up.

$$\therefore \qquad B = \int dB\sin\phi = \frac{\mu_0}{4\pi}\int\frac{Idl\sin\theta}{r^2}\sin\phi$$

Here, θ is angle between the current element $Id\vec{l}$ and $\vec{r}$, which is $\frac{\pi}{2}$ everywhere and

$$\sin\phi = \frac{R}{r} = \frac{R}{\sqrt{R^2+x^2}}$$

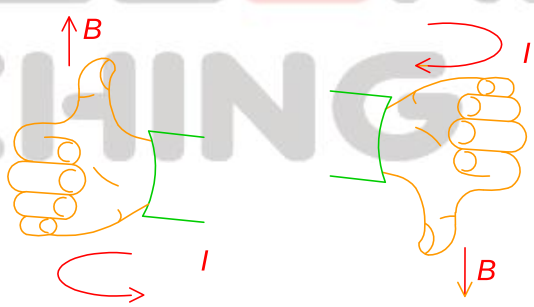
$$\therefore \qquad B = \frac{\mu_0}{4\pi}\frac{IR}{(R^2+x^2)^{3/2}}\int_0^{2\pi R} dL$$

or, $$B = \frac{\mu_0}{4\pi}\frac{IR}{(R^2+x^2)^{3/2}}(2\pi R)$$

or, $$B = \frac{\mu_0}{4\pi}\frac{2\pi IR^2}{(R^2+x^2)^{3/2}} \qquad \text{… (4)}$$

If the coil has *N* turns, then $B = \frac{\mu_0}{4\pi}\frac{2\pi NIR^2}{(R^2+x^2)^{3/2}}$

Direction of $\vec{B}$: Direction of magnetic field at a point the axis of a circular coil is along the axis and its orientation can be obtained by using the right-hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field.



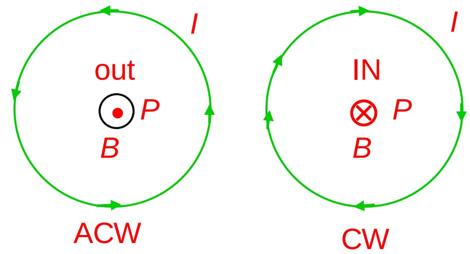
Magnetic field will be out of the page for anticlockwise current while into the page for clockwise current as shown in the figure given.

Now consider some special cases involving the application of equation (4)

Case I: Field at the centre of the coil

In this case distance of the point P from the centre $(x) = 0$, the magnetic field

$$B = \frac{\mu_0}{4\pi}\frac{2\pi I}{R} = \frac{\mu_0}{2}\frac{I}{R}$$



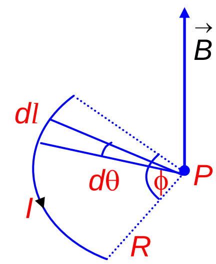
Case II: Field at a point far away from the centre

It means $x >> R$, $B = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{x^3}$

5 FIELD AT THE CENTRE OF A CURRENT ARC

Consider an arc of radius R carrying current I and subtending an angle ϕ at the centre.

According to Biot-Savart Law, the magnetic field induction at the point P is given by



$$B = \frac{\mu_0}{4\pi}\int_0^{\phi}\frac{I\,dl}{R^2}$$

Here, $dl = Rd\theta$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\int_0^{\phi}\frac{IRd\theta}{R^2}$$

or, $\quad B = \frac{\mu_0}{4\pi}\frac{I\phi}{R}$ **... (5)**

If 'l' is the length of the circular arc, we have

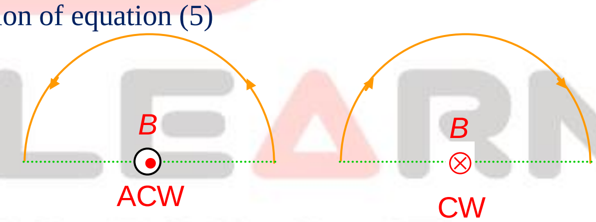
$$B = \frac{\mu_0}{4\pi}\frac{Il}{R^2} \quad \textbf{... (6)}$$

Consider some special cases involving the application of equation (5)

Case I: If the loop is the semi-circular,

In this case $\phi = \pi$, so

$$B = \frac{\mu_0}{4\pi}\frac{\pi I}{R}$$



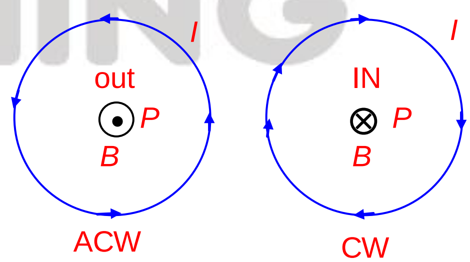
and will be out of the page for anticlockwise current while into the page for clockwise current as shown in the figure.

Case II: If the loop is a full circle with N turns,

In this case $\phi = 2\pi$ so,

$$B = \frac{\mu_0}{4\pi}\frac{2\pi NI}{R}$$

and will be out of the page for anticlockwise current while into the page for clockwise current as shown in the figure.



6 AMPERE'S LAW

This law is useful in finding the magnetic field due to currents under certain conditions of symmetry. Consider a closed plane curve enclosing some current-carrying conductors.

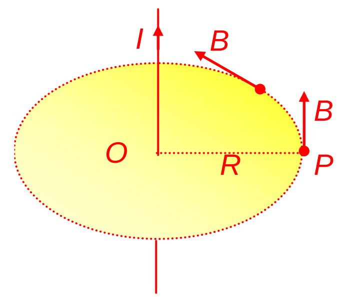
The line integral $\oint \vec{B}\cdot\vec{dl}$ taken along this closed curve is equal to μ_0 times the total current crossing the area bounded by the curve.

i.e., $\oint \vec{B}\cdot\vec{dl} = \mu_0 I$ **… (7)**

where I = total current (algebraic sum) crossing the area.

As a simple application of this law, we can derive the magnetic induction due to a long straight wire carrying current I.
Suppose the magnetic induction at point P, distant R from the wire is required.

Draw the circle through P with centre O and radius R as shown in figure.



The magnetic induction $|\vec{B}|$ at all points along this circle will be the same and will be tangential to the circle, which is also the direction of the length element $\vec{dl}$.

The current crossing the circular area is I.

Thus, by Ampere's law, $B \times 2\pi R = \mu_0 I$

$\Rightarrow$ $$B = \frac{\mu_0 I}{2\pi R}$$ **… (8)**

7 THE MAGNETIC FIELD OF A SOLENOID

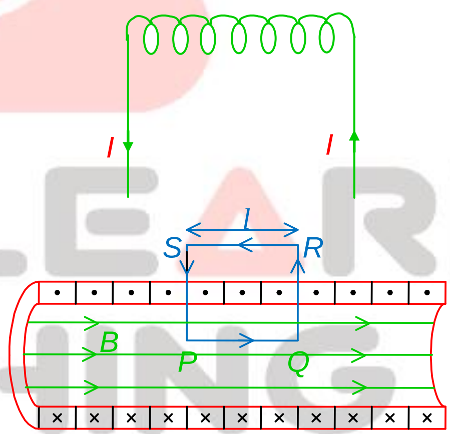
7.1 MAGNETIC FIELD INSIDE A LONG SOLENOID

A solenoid is a wire wound closely in the form of a helix, such that the adjacent turns are electrically insulated.

The magnetic field inside a very tightly wound long solenoid is uniform everywhere along the axis of the solenoid and is zero outside it.

To calculate the magnetic field at a point P inside the solenoid, let us draw a rectangle $PQRS$ as shown in figure. The line PQ is parallel to the solenoid axis and hence parallel to the magnetic field $\vec{B}$ inside the solenoid.

$$\Rightarrow \int_P^Q \vec{B}\cdot\vec{dl} = Bl$$



On the remaining three sides, $\vec{B}\cdot\vec{dl}$ is zero everywhere as $\vec{B}$ is either zero (outside the solenoid) or perpendicular to $\vec{dl}$ (inside the solenoid).

If n is the number of turns per unit length along the length of solenoid, total nl turns cross the rectangle $PQRS$. Each turn carries a current I.

$\Rightarrow$ Net current crossing $PQRS = nlI$

Using Ampere's law,

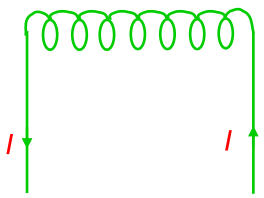
$\Rightarrow$ $Bl = \mu_0 nIl$

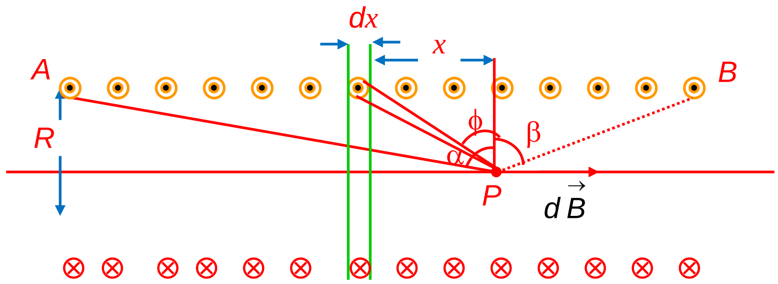
$\Rightarrow$ $\boldsymbol{B = \mu_0 nI}$ **… (9)**

7.2 MAGNETIC FIELD AT A POINT ON THE AXIS OF A SHORT SOLENOID

Consider a solenoid of length l and radius r containing N closely spaced turns and carrying a steady current

I. We have to find out an expression for the magnetic field at an axial point *P* lying in the space enclosed by the solenoid as shown in the figure below.





The field at a point on the axis of a solenoid can be obtained by the superposition of fields due to a large number of identical coils all having their centre on the axis of the solenoid.

Let us consider a coil of width *dx* at a distance *x* from the point *P* on the axis of the solenoid as shown in the above diagram.

The field at *P* due to this coil is given by

$$dB = \frac{\mu_0}{4\pi}\frac{2\pi dNIR^2}{(R^2+x^2)^{3/2}}$$

If *n* be the number of turns per unit length, $dN = ndx$.

From the above figure,

$$x = R\tan\phi$$

or, $$dx = R\sec^2\phi\, d\phi$$

$$\therefore \quad dB = \frac{\mu_0}{4\pi}\frac{2\pi(ndx)IR^2}{(R^2+R^2\tan^2\phi)^{3/2}} = \frac{\mu_0}{4\pi}(2\pi nI)\cos\phi\, d\phi$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}(2\pi nI)\int_{-\alpha}^{\beta}\cos\phi\, d\phi$$

or, $$B = \frac{\mu_0}{4\pi}(2\pi nI)[\sin\alpha+\sin\beta] \qquad \textbf{... (10)}$$

Now consider some cases involving the application of equation (10)

Case I: If the solenoid is of infinite length and the point is well inside the solenoid,

In this case, $\alpha = \beta = \dfrac{\pi}{2}$

$$\therefore \quad B = \frac{\mu_0}{4\pi}(2\pi nI)[1+1] = \mu_0 nI$$

Case II: If the solenoid is of infinite length and the point is near one end

In this case, $\alpha = 0$ and $\beta = \dfrac{\pi}{2}$

$$\therefore \quad B = \frac{\mu_0}{4\pi}(2\pi nI)[1+0] = \frac{1}{2}(\mu_0 nI)$$

Case III: If the solenoid is of finite length and the point is on the perpendicular bisector of its axis

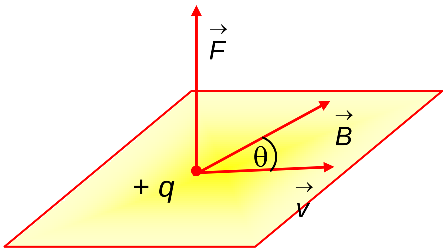
In this case, $\alpha = \beta$

$$\therefore \quad B = \frac{\mu_0}{4\pi}(4\pi nI)\sin\alpha,$$

where, $\sin\alpha = \dfrac{L}{\sqrt{L^2+4R^2}}$

8 FORCE ON A CHARGED PARTICLE IN A MAGNETIC FIELD

When a charge q moves with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ as shown in the figure, it experiences a magnetic force $\vec{F}$ given by



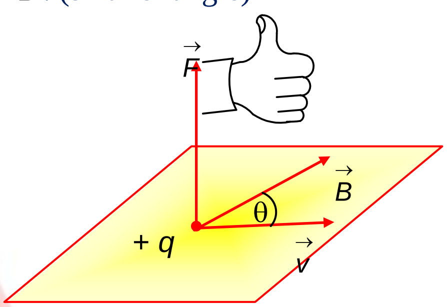
$$\vec{F} = q\,(\vec{v} \times \vec{B}) \qquad \ldots \textbf{(11)}$$

The magnitude of force is given by

$$|\vec{F}\,| = qv\,B \sin\theta$$

where θ is the angle between velocity $\vec{v}$ and magnetic field $\vec{B}$. (smaller angle)

The force is directed at right angle to the plane containing the vectors $\vec{v}$ and $\vec{B}$.



The right hand thumb rule: For determining the direction of the cross product $\vec{v} \times \vec{B}$, you point the fore fingers of your right hand along the direction of $\vec{v}$, and palm in the direction of magnetic field $\vec{B}$ then curl the fingers. The thumb then points in the direction of $\vec{v} \times \vec{B}$.

Since $F = q\vec{v} \times \vec{B}$, $\vec{F}$ is in the direction of $\vec{v} \times \vec{B}$ if q is positive and opposite the to direction of $\vec{v} \times \vec{B}$ if q is negative.

Some important points

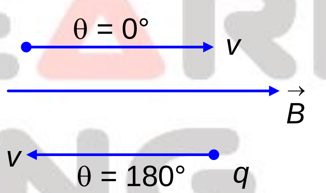
(1) The magnetic force will be maximum when sinθ = 1

⇒ θ = 90°, i.e., the charge is moving perpendicular to the field.

$F_{max} = qvB$

In this situations $\vec{F}$, $\vec{v}$ and $\vec{B}$ are mutually perpendicular to each other.

(2) The magnetic force will be minimum when sinθ = 0, i.e., θ = 0° or 180°. It means the charge is moving parallel to the field.



Thus, $F_{min} = 0$

(3) In case of motion of charged particle in a magnetic field, as the force is always perpendicular to motion,

$$W = \int \vec{F} . d\vec{s} = \int F ds \cos 90^\circ = 0$$

So work done by the force due to magnetic field on a moving charged particle is always zero. According to work-energy theorem,

$W = \Delta$ KE, so the kinetic energy will not change and hence the speed v will remain constant. However, in this situation, the force changes the direction of the motion. therefore., the velocity $\vec{v}$ of the charged particle changes.

8.1 DIFFERENCE BETWEEN MAGNETIC FORCE AND ELECTRIC FORCE

(1) Magnetic force is always perpendicular to the field while electric force is collinear with the field.

(2) Magnetic force is velocity dependent, i.e., acts only when the charged particle is in motion while electric force (qE) is independent of the state of rest or motion of the charged particle.

(3) Magnetic force does no work when the charged particle is displaced while the electric force does work in displacing the charged particle.

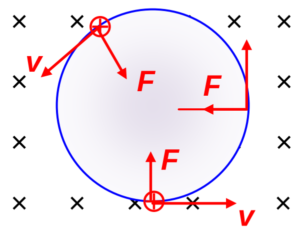
(4) Magnetic force is always non-central while the electric force may or may not be.

9 MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

9.1 WHEN THE CHARGED PARTICLE IS GIVEN VELOCITY PERPENDICULAR TO THE FIELD

Let a particle of charge q and mass m is moving with a velocity v and enters at right angles to a uniform magnetic field $\vec{B}$ as shown in figure.

The force on the particle is qvB and this force will always act in a direction perpendicular to v. Hence, the particle will move on a circular path. If the radius of the path is r then



$$\frac{mv^2}{r} = Bqv \quad \text{or,} \quad r = \frac{mv}{qB} \qquad \dots (12)$$

Thus, radius of the path is proportional to the momentum mv of the particle and inversely proportional to the magnitude of magnetic field.

Time period: is the time taken by the charge particle to complete one rotation of the circular path which is given by,

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} \qquad \dots (13)$$

The time period is independent of the speed v.

Frequency: The frequency is number of revolution of charged particle in one second, which is given by

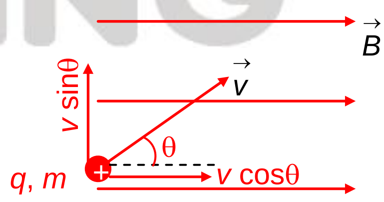
$$\nu = \frac{1}{T} = \frac{qB}{2\pi m} \qquad \dots (14)$$

9.2 WHEN THE CHARGED PARTICLE IS MOVING AT AN ANGLE TO THE FIELD

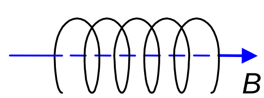
In this case the charged particle having charge q and mass m is moving with velocity v and it enters the magnetic field B at angle θ as shown in figure. Velocity can be resolved in two components, one along magnetic field and the other perpendicular to it. Let these components are $v_{\parallel}$ and $v_{\perp}$

$$v_{\parallel} = v \cos\theta$$

and $$v_{\perp} = v \sin\theta$$



The parallel component $v_{\parallel}$ of velocity remains unchanged as it is parallel to $\vec{B}$. Due to the $v_{\perp}$ the particle will move on a circular path. So the resultant path will be combination of straight-line motion and circular motion, which will be helical as shown in figure.



The radius of path is $(r) = \frac{mv_{\perp}}{qB} = \frac{mv\sin\theta}{qB} \qquad \dots (15)$

Time period (T) $= \dfrac{2\pi r}{v_\perp} = \dfrac{2\pi m v \sin\theta}{v \sin\theta\, qB} = \dfrac{2\pi m}{qB}$ … (16)

Frequency (f) $= \dfrac{Bq}{2\pi m}$ … (17)

Pitch: Pitch of helix described by charged particle is defined as the displacement of the particle in the time in which it completes one revolution.

Pitch $= (v_{\|})$ (time period)

$$= v\cos\theta\,\frac{2\pi m}{Bq} = \frac{2\pi m v\cos\theta}{qB} \quad \ldots (18)$$

10 MOTION OF A CHARGED PARTICL IN COMBINED ELECTRIC AND MAGNETIC FIELD

When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, the moving charged particle will experience electric force $\vec{F}_e = q\vec{E}$ and magnetic force $\vec{F}_m = q(\vec{v}\times\vec{B})$, so the net force on it will be

$$\vec{F} = q[\vec{E} + (\vec{v}\times\vec{B})] \quad \ldots (19)$$

Which is '**Lorentz force equation**'.

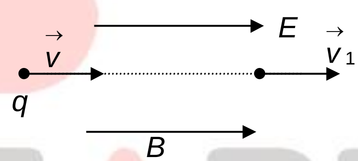
Now let us consider two special cases involving the application of above equation.

Case I: When $\vec{v}$, $\vec{E}$ and $\vec{B}$ all the three are collinear:

In this situation as the particle is moving parallel or anti-parallel to the field, the magnetic force on it will be zero and only electric force will act, so

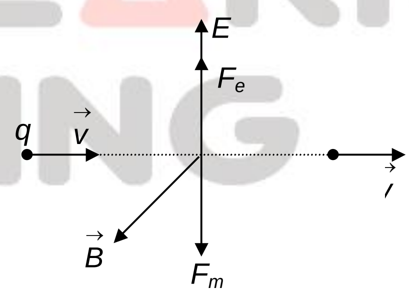
$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

Hence the particle will pass through the field following a straight-line path (parallel to the field) with change in its speed. So in this situation speed, velocity, momentum and kinetic energy all will change without change in direction of motion as shown in the figure given below.



$\vec{v}$, $\vec{E}$ and $\vec{B}$ are collinear.

Case II: $\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular



$\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular. In case situation of $\vec{E}$ and $\vec{B}$ are such that

$$\vec{F} = \vec{F}_e + \vec{F}_m = 0$$

or, $\vec{a} = \left(\dfrac{\vec{F}}{m}\right) = 0$, then the particle will pass through the field with the same velocity.

In this situation,

$F_e = F_m$ or, $qE = qvB$

or, $v = \dfrac{E}{B}$

This principle is used in velocity-selector to get a charged beam having a specific velocity.

11 CYCLOTRON

The cyclotron is a device used to accelerate positively charged particle to high energies. Such charged particles are required to carry out nuclear reactions.

11.1 PRINCIPLE

It works on the fact that

(i) When a charged particle moves at right angle to a uniform magnetic field it describes a circular path and

(ii) If the charged particle simultaneously and repeatedly crosses an electric field while moving in the direction of the electric field, it gets accelerated to a sufficiently high energy. Such an electric field is called an oscillating electric field.

11.2 CONSTRUCTION

The cyclotron consists of two D shaped hollow metallic semi-cylindrical chambers D_1 and D_2 called the dees, enclosed in an evacuated steel box. The dees are kept horizontally with a small gap separating them. An oscillator which produces an alternating potential difference of the order of 10^3 volts and frequency of the order of mega cycles/sec is applied across the dees. A strong magnetic field produced by a strong electromagnet acts perpendicular to the plane of the dees. A sources S of ions or positively charged particles is kept in the gap between the dees.

11.3 THEORY AND WORKING

Let m and q be the mass and charge of the ion or the particle to be accelerated. Let D_2 be negative and D_1 be positive when source (s) produces the particle.

The particle, therefore, gets attracted towards D_2, the electric field becomes zero. It is because the electric field inside a charged conductor is always zero. Thus, inside D_2,it moves with constant speed v at right angles to the magnetic field acting downward. As a result, the particle takes a semi-circular path inside D_2. Let r be the radius of the semicircular path. Then

$$\frac{mv^2}{r} = qvB \qquad \therefore \qquad r = \frac{mv}{qB}$$

$\therefore$ Time taken by particle to complete the semi-circular path

$t = \frac{\pi r}{v}$. But $v = \frac{qBr}{m}$

$$t = \frac{\pi rm}{qBr} = \frac{\pi m}{qB}$$

Above relation shows that t is independent of both the radius of the circular path and the speed of the charged particle.

As the particle reaches the gap just after completing the semi-circular path inside D_2, the polarity of the dees is reversed i.e. D_1 become negative and D_2 positive. The particle accelerates due to the electric field and enters D_1 with greater speed, say v_1. It then moves along a semi-circular path of larger radius inside D_1 due to the magnetic field (B) acting perpendicular to v_1. As it comes out of D_1 after completing the semi-circular path, the polarity of dees again gets reversed. The particle again accelerates and enters D_2 with a greater speed and then describes a semi-circular path of larger radius. The whole process keeps repeating time and again.

As a result, the particle keeps on accelerating every time it crosses the gap between the dees and keeps describing semi-circular paths of increasing radii. The result is that a spiral path is followed by the particle till it comes out of the dees through a window W with a very high speed i.e. high energy. The particles are deflected to come out through the windows by employing an electric field.

11.4 CYCLOTRON FREQUENCY OR MAGNETIC RESONANCE FREQUENCY (ν)

The cyclotron works when the frequency of the applied alternating potential difference (ν_0) is equal to the frequency of the revolving charged particle (ν)

$$\therefore \quad \nu_0 = \nu = \frac{1}{T} = \frac{1}{2\times t} = \frac{qB}{2\pi m}$$

∴ **Cyclotron angular frequency** $\omega = 2\pi\nu = \dfrac{qB}{m}$ **… (20)**

11.5 MAXIMUM VELOCITY AND MAXIMUM ENERGY OF THE ACCELERATED PARTICLE

Let v_m be the maximum velocity of the particle when it comes out of the dees following semi-circular path of maximum radius i.e. radius of dees, then

$$v_m = \frac{qBR}{m}$$

∴ Maximum kinetic energy acquired by the particle

$$E_{\max} = \frac{1}{2}mv_m^2 = \frac{1}{2}m\left(\frac{qBR}{m}\right)^2$$

$$\therefore \quad E_{\max} = \frac{1}{2}\frac{B^2q^2R^2}{m} \qquad \text{… (21)}$$

11.6 LIMITATIONS OF CYCLOTRON

As the positive ion accelerates in the cyclotron, it moves with larger and speed. As this speed becomes comparable with the speed of light, the mass of the ion increases in accordance with the relation.

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

where, $m_0 \rightarrow$ the rest mass of the ion

$v \rightarrow$ velocity of the ion

$m \rightarrow$ mass of the ion when it moves with velocity v

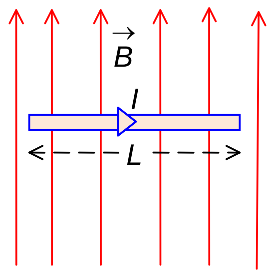
$c \rightarrow$ velocity of light

Thus, the time taken by the ion to complete a semi-circular path $c = \left(\dfrac{\pi m}{qB}\right)$ also increases.

It means that the polarity of dees changes before the ion completes the semi-circular path. The ion, therefore, gets out of phase with the oscillating electric field.

12 FORCE ON A CURRENT CARRYING WIRE IN A MAGNETIC FIELD

Let L be the length of a straight conductor carrying a current I and placed perpendicular to a uniform magnetic field of induction B.



A current in a conductor is due to the movement of electrons and the direction of the conventional current is opposite to that of the direction of motion of electrons. If n be the number of moving charges per unit volume, each charge is of q, and travelling with drift velocity v_d, the charge passing through any cross-section per second is nqv_dA

$I = nqv_dA$

where A is the cross-sectional area.

The number of charges in length L of a conductor

$N = nLA$

The force on each charge

$F = Bqv_d$

The force on all charges i.e., the force on the conductor

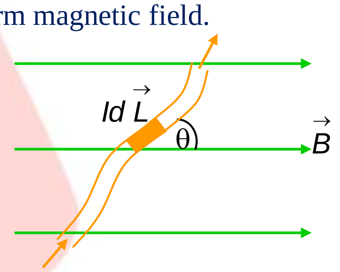
$F = Bqv_d \times nLA$

or $F = BIL$

In vector form $\vec{F} = I(\vec{L} \times \vec{B})$ ... **(22)**

where L is a vector in the direction of the current I; the magnitude of $\vec{L}$ equals the length L of the segment. Note that this expression applies only to a straight segment of wire in a uniform magnetic field.

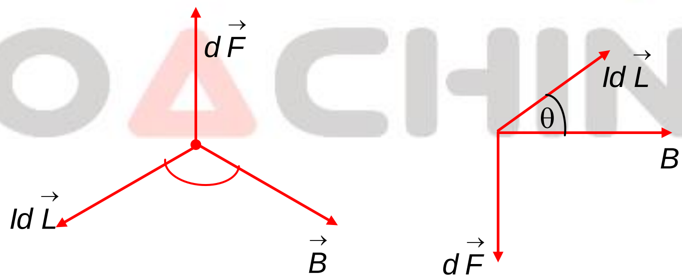
Now consider an arbitrarily shaped wire segment of uniform cross-section in a magnetic field $\vec{B}$, as shown in figure. Then the magnetic force on a very small segment dL in the presence of magnetic field $\vec{B}$ is given by



$$d\vec{F} = I\,d\vec{L} \times \vec{B}$$

The magnitude of force is $dF = BIdL \sin\theta$, where θ is the angle between the vectors $Id\vec{L}$ and $\vec{B}$.

Direction of force: The direction of force is always perpendicular to the plane containing $Id\vec{L}$ and $\vec{B}$ and is same as that of cross-product of two vectors $(\vec{a} \times \vec{b})$ with $\vec{a} = Id\vec{L}$ and $\vec{b} = \vec{B}$.



The direction of force when current element $Id\vec{L}$ and $\vec{B}$ are perpendicular to each-other can also be determined by applying either of the following rules.

(a) Fleming's Left-hand Rule: Strech the fore-finger, central finger and thumb of the left hand mutually perpendicular. Then if the fore-finger points in the direction of the field $(\vec{B})$ and the central in the direction of current I, the thumb will point in the direction of force (or motion).

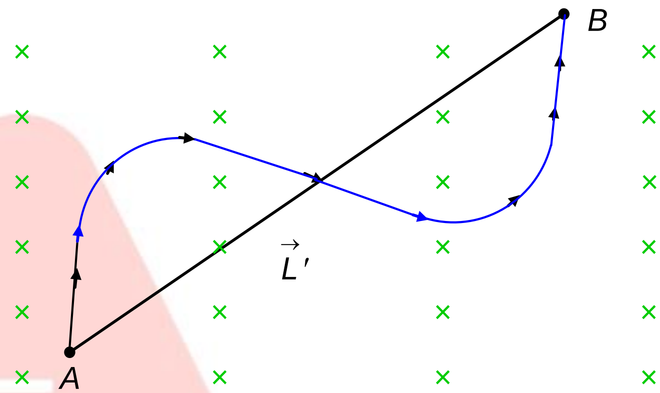
(b) Right-hand Palm rule: Stretch the fingers and thumb of the right hand at right angles to each-other. If the fingers point in the direction of current *I*, and the palm in the direction of the field $\vec{B}$ then thumb will point in the direction of force.

12.1 FORCE ON A CURVED CURRENT CARRYING WIRE

In this case a current-carrying conductor is placed in a uniform magnetic field $\vec{B}$. the force is given by $\vec{F} = \int I d\vec{L} \times \vec{B} = I \int d\vec{L} \times \vec{B}$

and for a conductor $\int d\vec{L}$ represents the vector sum of all the lengths elements from initial to final point, which in accordance with the law of vector addition is equal to the length vector $\vec{L}$ joining initial to the final point. So a current-carrying conductor of any arbitrary shape in a uniform field experiences a force

$$\vec{F} = I\left[\int d\vec{L}\right] \times \vec{B} = I\,\vec{L} \times \vec{B}$$



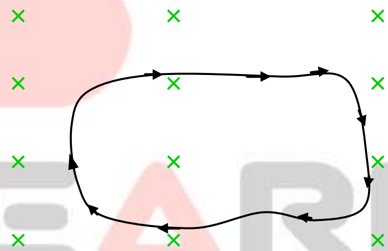
12.2 FORCE ON A CLOSED LOOP OF AN ARBITRARILY SHAPED CONDUCTOR

Consider a current-carrying conductor in the form of a loop of any arbitrary shape is placed in a uniform field $\vec{B}$. In this case the vector sum of the current element must be taken over the closed loop.

for a closed loop, the vector sum of $d\vec{L}$ is always zero.

$\therefore \quad \vec{F} = 0$

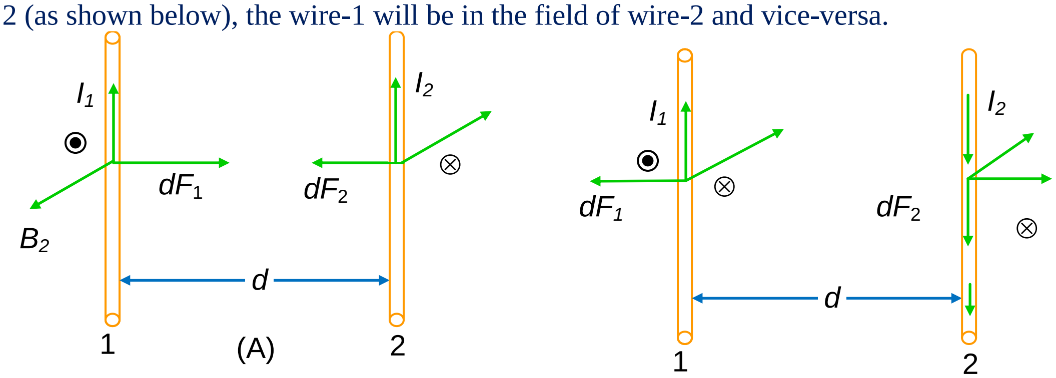
i.e., the magnetic force on a current loop in a uniform magnetic field is always zero.



12.3 FORCE BETWEEN TWO LONG STRAIGHT PARALLEL CURRENT CARRYING CONDUCTORS

Let us consider two very long parallel straight wires carrying currents I_1 and I_2.

Each wire is placed in the region of magnetic induction of other and hence will experience a force. The net force on a current-carrying conductor due to its own field is zero. So if there are two long parallel current-carrying wires 1 and 2 (as shown below), the wire-1 will be in the field of wire-2 and vice-versa.



The force on dl_2 length of wire-2 due to field of wire-1, $dF_2 = I_2\, dL_2\, B_1$

$$= \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d}dL_2 \; [\because B_1 = \frac{\mu_0}{4\pi}\frac{2I_1}{d}]$$

or, $$\frac{dF_2}{dL_2} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d} \qquad \ldots (23)$$

It will be true for wire-1 in the field of wire-2. The direction of force in accordance with the right-hand screw rule will be as shown above.

So the force per unit length in case of two parallel current-carrying wires separated by a distance 'd' is

$$\frac{dF}{dL} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d}$$

If I_1 and I_2 are along the same direction, the forces between the wires is attractive in nature and if I_1 and I_2 are oppositely directed the force is repulsive. The direction of forces is given by Fleming's left hand rule.

Definition of 'ampere'

We have $\frac{dF}{dL} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d}$

If $I_1 = I_2 = 1A$; $d = 1$m; $dL = 1$m; then

$dF = 2 \times 10^{-7}$ N

Hence, 'ampere' is defined as the current which when passing though each of two parallel infinitely long straight conductors placed in free space at a distance of 1 m from each-other produces between them force of 2×10^{-7} N for one metre of their length.

13 CURRENT LOOP IN A UNIFORM MAGNETIC FIELD

13.1 MAGNETIC MOMENT

According to magnetic effects of current, in case of current-carrying coil for axial point,

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\pi NIR^2}{(R^2+x^2)^{3/2}}$$

when $x >> R$, $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\pi NIR^2}{x^3}$

If we compare this result with the field due to a small bar magnet for a distant axial point, i.e.,

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{x^3},$$

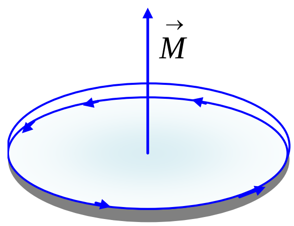
where M is magnetic moment of the bar magnet

we find that a current-carrying coil for a distant point behaves as a magnetic dipole of moment

$$\vec{M} = NI\pi R^2 = NIA \qquad \ldots (24)$$

where A is area of the loop. So the magnetic moment of a current carrying coil is defined as the product of current in the coil with the area of coil in the vector form.

Magnetic moment of a current loop is a vector quantity and direction is perpendicular to the plane of the loop. Its dimensions are $[L^2A]$ and units are $A\text{-}m^2$.



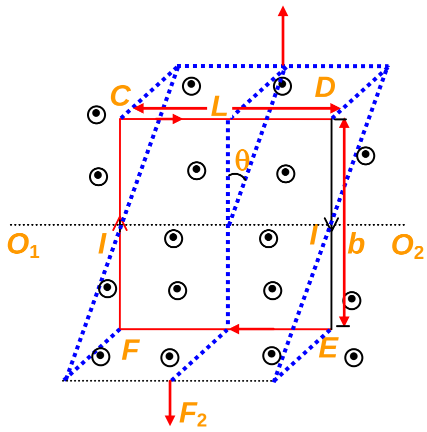
Magnetic moment in case of a charged particle having charge q and moving in a circle of radius R with speed v is given by $\frac{1}{2}qvR$

As we know, $I = qf = q\frac{v}{2\pi R}$ and $|A| = \pi R^2$

$$\therefore \qquad M = I\,|\vec{S}| = \frac{1}{2}qvR$$

13.2 TORQUE ON A CURRENT LOOP

Consider a rectangular coil *CDEF* of length *L* and width *b* is placed vertically, while a uniform magnetic induction *B* passes normally through it as shown. The coil is capable of rotation about an axis O_1O_2.



If the loop is oriented in the magnetic field such that the normal to the plane of the coil makes an angle θ with the direction of $\vec{B}$, then the torque experienced by the loop

$$\tau = \frac{b}{2}(ILB)\sin\theta + \frac{b}{2}(ILB)\sin\theta$$

i.e., $\tau = ILbB\sin\theta = IAB\sin\theta$

where $A = Lb$ is the area of the loop.

The maximum torque experienced is $\tau = IAB$, when $\theta = 90°$ and for a coil of *N* turns

$$\tau = NIAB$$

Here $NIA = M$ = Magnetic moment of the loop.

In vector notation $\vec{\tau} = \vec{M} \times \vec{B}$. **… (25)**

This result holds good for plane loops of all shapes rectangular, circular or otherwise.

13.3 WORK DONE IN ROTATING A CURRENT LOOP

When a current loop is rotated in a uniform magnetic field through an angle θ about an axis then work done will be

$$\int_0^W dW = \int \tau\, d\theta = \int_0^\theta MB\sin\theta\, d\theta$$

$$W = -\left[MB\cos\theta\right]_0^\theta = MB(1-\cos\theta) \qquad \dots (26)$$

14 MOVING COIL GALVANOMETER

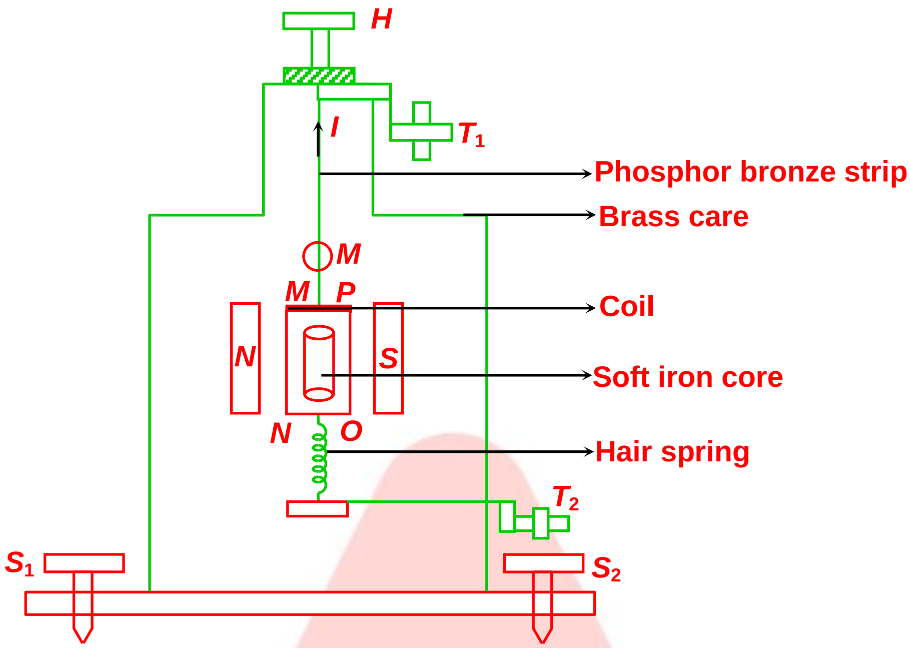
It is a sensitive instrument used for detecting and measuring small electric currents.

Principle " **It works on the fact that when a current carrying coil is kept in a magnetic field, it experiences a torque**"

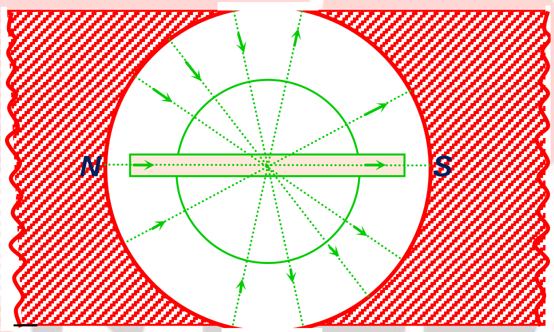
14.1 CONSTRUCTION

It consists of a coil *MNOP* having a larger number of turns of insulated copper wires and wound over a frame made of non-magnetic material such as copper. The coil may be rectangular or circular in shape.

The coil is suspended vertically from a movable screw (called torsion head) *M* by means of a phosphor bronze wire in uniform magnetic field which is produced by strong cylindrical pole pieces *N* and *S* shown in the figure.



A soft iron core is held within the coil in such a manner that the coil rotates without touching the poles and the core. The core is spherical in shape for a circular coil and cylindrical for a rectangular coil. The lower end of the coil is connected to a hair spring (a very light but highly elastic spring made of quartz or phosphor bronze). The other end of the spring is connected to a binding terminal T_2. The torsion head is also connected to binding terminal T_1. A small light mirror M is attached to the suspension wire to measure the deflection of the coil by a lamp and scale arrangement.



The entire apparatus is enclosed in a brass case (with glass window at the front) to avoid disturbances due to air etc.

14.2 THEORY AND WORKING

Let a current I be passed through the coil by connecting a cell between T_1 and T_2. A torque acts on the coil and the coil turns by a certain angle. Therefore, suspension wire gets twisted through the same angle. As a result, a restoring torque comes into play in the suspension wire. The restoring torque tends to oppose the deflecting torque.

As the coil turns more, the restoring torque also increases till stage comes in which it becomes equal to the deflecting torque.

Let θ be the angle by which the suspension wire gets twisted.

This is called the equilibrium position of the coil

$\therefore$ In equilibrium position

Deflecting torque = Restoring torque

or $NIAB\cos\alpha = C\theta$

Here α is the angle which the plane of the coil makes with the direction of the magnetic field.

The soft iron core placed symmetrically between the cylindrical pole pieces produces a radial magnetic field. In such a field, the lines of force pass through the axis of the cylindrical gap between *N* and *S*. Hence, the plane of the coil in all position will remain parallel to the lines of force.

$\therefore \quad \alpha = 0^0$

C is the torsion constant of the suspension wire and θ is the angle of twist.

$\therefore \quad NIAB = C \times \theta$

$\therefore \quad I \propto \theta$

Thus deflection of the coil is directly proportional to the current flowing through the coil.

14.3 CURRENT SENSITIVITY AND VOLTAGE SENSITIVITY OF A GALVANOMETER

Current sensitivity of a galvanometer is defined as the deflection produced in a galvanometer when unit current is passed through it.

From $NIAB = C\theta$

$$\therefore \quad \text{current sensitivity} = \frac{\theta}{I} = \frac{NBA}{C}$$

It is measured in rad A^{-1}.

Voltage sensitivity is defined as the deflection produced in the galvanometer when a unit potential difference is applied across the two terminals of the galvanometer.

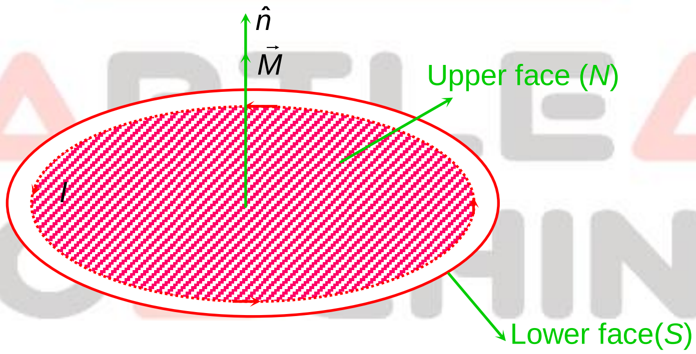
$$\therefore \quad \text{voltage sensitivity} = \frac{\theta}{V} = \frac{\theta}{IG} = \frac{NBA}{CG}$$

$$= \frac{\text{current sensitivity}}{\text{resistance of galvanometer}}$$

The unit of voltage sensitivity is rad/volt.

15 CURRENT LOOP AS A MAGNETIC DIPOLE

ent carrying loop thus behaves as a system of two equal and opposite magnetic poles and hence is a magnetic dipole.



The magnetic dipole moment of the current loop (*M*) is directly proportional to (i) strength of current (*I*) through the loop and (ii) area (*A*) enclosed by the loop.

i..e $M \propto I$ and $M \propto A$

$\therefore \quad M = KIA$

where *K* is a constant of proportionality .

If we define unit magnetic dipole moment as that of a small one turn loop of unit area carrying unit current.

$1 = K \times 1 \times 1$ or $K = 1$

$M = IA$

For *N* such turns, $M = NLA$

The SI unit of *M* is ampere metre^2. It is the magnetic moment of one turn loop of area one square meter carrying a current of one ampere.

In vector form, we can rewrite equation as

$\vec{M} = NA\hat{n}$

where $\hat{n}$ is unit vector perpendicular to the plane of the loop in a direction given by right handed screw rule.

16 MAGNETIC DIPOLE MOMENT OF AN ATOM DUE TO REVOLVING ELECTRON

In every atom, electrons revolve around the nucleus. A revolving electron is like a loop of current, which has a definite magnetic dipole moment.

If e is the change on an electron revolving in an orbit of radius r with a uniform angular velocity ω, then equivalent current

$i = \frac{\text{charge}}{\text{time}} = \frac{e}{T}$ where,

T = the period of revolution of electron = $2\pi/\omega$

$$\therefore \qquad i = \frac{e}{2\pi/\omega} = \frac{\omega e}{2\pi} \qquad \ldots \text{(i)}$$

Area of the orbit $A = \pi r^2$

Magnetic moment of the atom is given by

$$M = iA = \frac{\omega e}{2\pi}\pi r^2$$

or $$M = \frac{1}{2}e\omega r^2 \qquad \ldots \text{(ii)}$$

Also $mvr = \frac{nh}{2\pi}$, where n = 1, 2, 3 …… denotes the number of the orbit. Using $v = r\omega$, we get

$m(r\omega)r = \frac{nh}{2\pi}$ or $\omega r^2 = \frac{nh}{2\pi m}$

Put in (ii)

$$M = \frac{1}{2}e\frac{nh}{2\pi m} = n\frac{eh}{4\pi m} = n(\mu_B) \qquad \ldots \text{(iii)}$$

where $\mu_B = eh/4\pi m$

We may define Bohr magneton as the magnetic dipole moment associated with an atom due to orbital motion of an electron in the first orbit of hydrogen atom.

17 BAR MAGNET AS AN EQUIVALENT SOLENOID

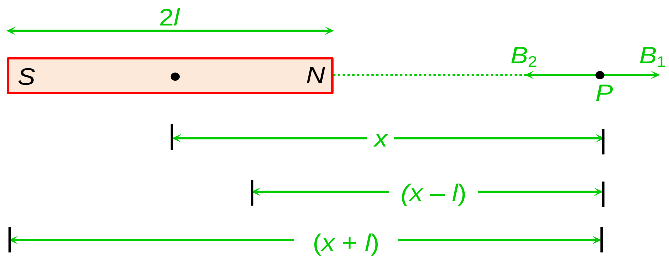
The magnetic field of a bar magnet when mapped using a small compass needle, a pattern of field lines is formed around the magnet

It should be clearly understood that the field lines exist in all the space around the magnet. Comparison of the two field patterns shows that current carrying solenoid from outside resembles a bar magnet. Inside the solenoid, there is a strong magnetic field which can magnetise a specimen. Solenoid is hollow from inside whereas the bar magnet is solid.

18 MAGNETIC FIELD DUE TO A BAR MAGNET (MAGNETIC DIPOLE)

18.1 AT A POINT LYING ON ITS AXIAL LINE

Consider a bar magnet of pole strength m and magnetic length ($2l$) (distance between poles). Let P be a point on its axial line at a distance x from its centre.



$\therefore$ Magnetic field at P due to N pole of the magnet is

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{(x-l)^2} \text{ (pointing towards right)}$$

Similarly, magnetic field at P due to the S pole

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{(x+l)^2} \text{ (pointing towards left)}$$

$\therefore$ Net magnetic field at P due to the magnet

$$B = B_1 - B_2 = \frac{\mu_0 m}{4\pi}\left(\frac{1}{(x-l)^2} - \frac{1}{(x+l)^2}\right)$$

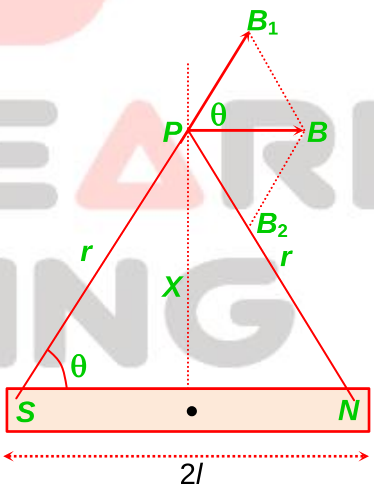
$$= \frac{\mu_0}{4\pi}\frac{m \times 4lx}{(x^2 - l^2)^2}$$

$$B = \frac{\mu_0}{4\pi}\frac{2Mx}{(x^2 - l^2)^2} \quad \text{... (27)}$$

As a far off point, i.e. $x >> 1$

$$B = \frac{\mu_0}{4\pi}\frac{2M}{x^3}$$

18.2 AT A POINT LYING ON ITS EQUATORIAL LINE



Let P be a point at a distance x from the centre of the magnet.

$\therefore$ Magnetic field at P due to the north (N) pole of the magnet is

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{r^2} \text{ (pointing along } NP\text{)}$$

and the field due to the south (S) pole is

$$B_2 = \frac{\mu_0}{4\pi}\frac{m}{r^2} \text{ (pointing along } PS\text{)}$$

$\therefore$ $|\vec{B}_1| = |\vec{B}_2| = B_1$

$\therefore$ Net field at P due to the bar magnet is

$$B = 2B_1\cos\theta = 2 \times \frac{\mu_0}{4\pi}\frac{m}{r^2}\frac{l}{r} = \frac{\mu_0}{4\pi}\frac{2ml}{r^3}$$

or $$B = \frac{\mu_0}{4\pi}\frac{M}{(x^2 + l^2)^{3/2}} \quad \textbf{(As } r = \sqrt{x^2 + l^2}\textbf{)} \quad \text{... (28)}$$

At a far-off point $x >> l$

$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}$$

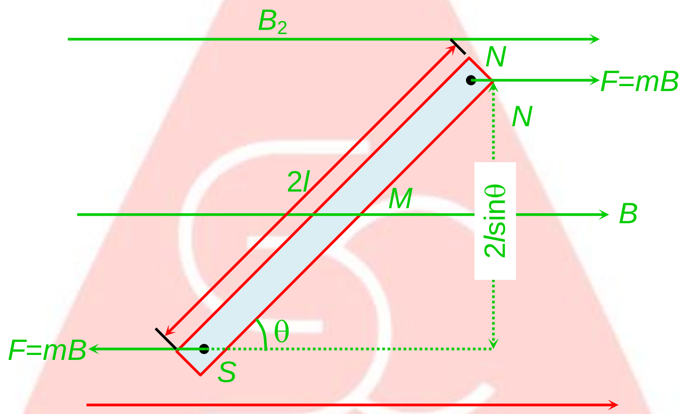
19 TORQUE ON A BAR MAGNET IN A UNIFORM MAGNETIC FIELD

Consider a bar magnet of pole strength *m* and length 2*l* kept in a uniform magnetic field $\vec{B}$ pointing horizontally from left to right, as shown in figure.

Let θ be the angle between the axis of the magnetic and the magnetic field (the angle between $\vec{M}$ of the magnetic and $\vec{B}$)

∴ Force acting on each pole of the magnetic is

$F = mB$



Since these equal forces act on the magnet in the opposite direction, their resultant force is zero. But these forces constitute a couple which tends to rotate the magnetic in the clock wise direction and brings it along the direction of $\vec{B}$.

∴ Torque due to the couple is

τ = force × perpendicular between the forces

$= mB \times 2l \sin\theta$

$= (m\,2l)\,B \sin\theta$

or τ $= MB \sin\theta$

$$\vec{\tau} = \vec{M} \times \vec{B}$$

The direction of the torque $\vec{\tau}$ is given by the right hand screw rule for the vector product.

If $\theta = 90^0$, $B = 1\,T$, then $\tau = M$

Therefore, magnetic dipole moment of a magnet is numerically equal to the torque acting on the magnet when held perpendicular to a uniform magnetic field of one Tesla.