



NUCLEI

5 NUCLEAR PHYSICS

So far the only knowledge of nucleus we have as a tiny positively charged object whose primary contributions are to provide the atom with most of its mass and to hold its electrons in captivity. The chief properties of atoms, molecules, solids and liquids can all be traced to the behaviour of atomic electrons not to the behaviour of nuclei. However, the nucleus turns out to be of paramount importance in the grand scheme of things. To start with the very existence of the various elements is due to the ability of nuclei to possess multiple electric charge. Furthermore, the energy involved in almost all natural processes can be traced to nuclear reactions and transformations. In the following sections we will study about the nucleus and phenomenon associated with nucleus.

5.1 NUCLEAR CHARACTERISTICS

- (i) **Nuclear mass:** It was observed in Rutherford's α -particle scattering experiment that mass of an atom is concentrated within a very small positively charged region at the centre called nucleus. The total mass of nucleons in the nucleus is called as nuclear mass.
Nuclear mass = mass of protons + mass of neutrons
- (ii) **Size and shape of the nucleus:** The nucleus is nearly spherical. Hence its size is usually given in terms of radius. The radius of nucleus was measured by Rutherford and it was found to have following relation
$$R = R_0 A^{1/3}$$
where $R_0 = 1.1 \text{ fm} = 1.1 \times 10^{-15} \text{ m}$ and A is mass number of particular element.
- (iii) **Nuclear charge:** Nucleus is made of protons and neutrons. Protons have positive charge of magnitude equal to that of electron and neutrons are uncharged. So, nuclear charge = Ze
- (iv) **Nuclear density:** The ratio of the mass of the nucleus to its volume is called nuclear density. As the masses of proton and neutron are roughly equal, the mass of a nucleus is roughly proportional to A .

As volume of a nucleus is

$$V = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi R_0^3 A$$

$$V \propto A$$

density within a nucleus is independent of A .

5.2 DIFFERENT TYPES OF NUCLEI

There are different types of nuclei depending upon the number of protons or the total number of nucleons in them.

(a) **Isotopes:** The atoms of an element having same atomic number but different mass number are called isotopes of that element i.e. different isotopes of the same element have same number of protons inside the nucleus but different number of neutrons inside the nucleus. Though isotopes have same chemical properties but their nuclear properties are highly different. Examples of isotopes are ${}_1\text{H}^1$, ${}_1\text{H}^2$, ${}_1\text{H}^3$ and ${}_8\text{O}^{16}$, ${}_8\text{O}^{17}$, ${}_8\text{O}^{18}$ etc.

(b) **Isotones:** Atoms whose nuclei have same number of neutrons are called isotones. For them, both the atomic number Z and atomic mass A are different but the value of difference ($A - Z$) is same. Examples of isotones are ${}_1\text{H}^3$, and ${}_2\text{He}^4$, ${}_1\text{H}^2$, and ${}_2\text{He}^3$ etc.

(c) **Isobars:** Atoms of same mass number but different atomic number are called isobars examples of isobars are ${}_1\text{H}^3$ and ${}_2\text{He}^3$, ${}_6\text{C}^{14}$ and ${}_7\text{N}^{14}$ etc.

5.3 NUCLEAR FORCES

The strong forces of attraction, which firmly hold the nucleons in the nucleus, are known as nuclear forces. Though the exact theory of nuclear forces is still to be understood completely, yet it is undoubtedly established that

these forces exist between the nucleons i.e. between a neutron and a proton, between two protons and between two neutrons. The stability of nucleus is due to the presence of these forces. Nuclear forces have following important characteristics.

- (i) They are attractive i.e. nucleons exert attractive force on each other hence they are also called **cohesive forces**.
- (ii) They are extremely strong. These forces are strongest possible force in nature.
- (iii) They are charge independent.
- (iv) They are short-range forces i.e. they act only over a short range of distances.
- (v) They are spin dependent i.e. nuclear forces acting between two nucleons depend on the mutual orientation of the spins of the nucleons.
- (vi) They are saturated i.e. their magnitude does not increase with the increase in the number of nucleons, beyond a certain number.

5.4 EINSTEIN'S MASS ENERGY EQUIVALENCE PRINCIPLE

Before the discovery of Einstein's mass energy equivalence principle, mass and energy were considered independent physical quantities. Einstein on the basis of theory of relativity showed that mass of a body is not independent of energy but they are inter convertible. According to Einstein if a substance loses an amount Δm of its mass, an equivalent amount ΔE of energy is produced, where $\Delta E = (\Delta m) c^2$

where c is the speed of light. This is called Einstein's mass-energy equivalence principle. In nuclear physics mass is usually represented in terms of energy according to the conversion formula $E = mc^2$. For example the mass of an electron is 9.1×10^{-31} kg and the equivalent energy is 511 KeV/c². Similarly, the mass of a proton is 938 MeV/c², and the mass of a neutron is 939 MeV/c². The energy corresponding to the mass of a particle when it is at rest is called its rest mass energy. Another useful unit of mass in nuclear physics is unified atomic mass unit, denoted by the symbol u . It is (1/12)th of the mass of a neutral carbon atom in its lowest energy state which has six protons, six neutrons and six electrons. We have

$$1u = 1.67 \times 10^{-27} \text{ kg} = 931.478 \text{ MeV}/c^2$$

5.5 BINDING ENERGY OF NUCLEUS

(i) The total energy required to liberate all the nucleons from the nucleus (i.e., the disintegrate the nucleus completely into its constituent particles) is called binding energy of the nucleus. Clearly, this is the same energy with which the nucleons are held together within the nucleus. The origin of binding energy results from strong nuclear exchange forces. In other words, we may think of existence of binding energy in other useful way also. A nucleus is made by the coming together of various nucleons. It has been observed experimentally that the mass of the nucleus is always less than the sum of the masses of its constituents when measured in free state. For example, deuteron (${}_1H^2$) is composed of one proton and 1 neutron. The question arises where the difference in mass has gone? The answer is that this decrease in mass has been converted into energy binding the nucleons together according to the following relation:

$$\Delta E = \Delta mc^2$$

where, ΔE = binding energy of nucleus, Δm = decrease in mass, called mass defect and c = velocity of light.

Hence in the formation of stable nucleus, the following equation holds good.

Mass of protons + Mass of neutrons = Mass of nucleus + Binding energy

Example: Consider a deuteron (${}_1H^2$) nucleus. It is the nucleus of heavy hydrogen or deuterium (${}_1H^2$). It contains 1 proton and 1 neutron. We shall compare the mass of one free proton and one free neutron with their mass when combined to form deuteron and thus find out mass defect and binding energy of deuteron.

Mass one free neutron	=	$1.675 \times 10^{-27} \text{ kg} = 1.008665 \text{ a.m.u}$
Mass of one free proton	=	$1.673 \times 10^{-27} \text{ kg} = 1.007825 \text{ a.m.u}$
Their total	=	$3.348 \times 10^{-27} = 2.01649 \text{ a.m.u}$
Mass of deuteron	=	$3.348 \times 10^{-27} = 2.014103 \text{ a.m.u}$
∴ Mass defect, Δm	=	0.002387 a.m.u

It shows that when a proton and a neutron come together to form a deuteron, a small mass of 0.002387 disappears. In fact, this mass is converted into binding energy according to the following relation:

$$\Delta E = \Delta mc^2 = \frac{0.002387 \times 1.66 \times 10^{-27} \times (3 \times 10^8)^2}{1.6 \times 10^{-19}} = 2.22 \times 10^6 \text{ eV} = 2.22 \text{ MeV}$$

Binding energy curve

(i) **Expression for binding energy per nucleon:** In order to compare the stability of various nuclei, we calculate binding energy per nucleon. Higher is the binding energy per nucleon more stable is the nucleus.

We have seen that the mass defect during the formation of a nucleus:

$\Delta m = Zm_p + (A - Z)m_n - m$, where m_p , m_n and m are masses of proton, neutron and nucleus respectively.

\therefore Total binding energy of nucleus

$$\Delta E = \Delta mc^2 = [Zm_p + (A - Z)m_n - m] \times c^2$$

\therefore Mean binding energy per nucleon

$$= \frac{\Delta E}{A} = \frac{\Delta mc^2}{A} = \left[\frac{Z}{A}(m_p - m_n) + m_n - \frac{m}{A} \right] \times c^2$$

If the mass m of the nucleus is found experimentally, we can find mean binding energy per nucleon since all other factors are known to us.

(ii) **Binding energy curve:** A graph between the binding energy per nucleon and the mass number of nuclei is called as the binding energy curve.



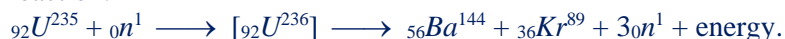
The following points may be noted from the binding energy curve:

- The binding energy per nucleon is maximum (≈ 8.8 MeV) for the nucleus having mass number 56. So, this nucleus is most stable i.e. iron is the most stable element of periodic table.
- The light nuclei with $A < 20$ are least stable.
- The curve has certain peaks indicating that certain nuclei like ${}^4_2\text{He}$, ${}^{12}_6\text{C}$ and ${}^{16}_8\text{O}$ are much more stable than the nuclei in their vicinity.
- For atomic number $Z > 56$, the curve takes a downside turn indicating lesser stability of these nuclei.
- Nuclei of intermediate mass are most stable. This means maximum energy is needed to break them into their nucleons.
- The binding energy per nucleon has a low value for both very light and very heavy nuclei. Hence, if we break a very heavy nucleus (like uranium) into comparatively lighter nuclei then the binding energy per nucleon will increase. Hence a large quantity of energy will be liberated in this process. This phenomenon is called nuclear fission.
- Similarly, if we combine two or more very light nuclei (e.g. nucleus of heavy hydrogen ${}^2_1\text{H}$) into a relatively heavier nucleus (e.g. ${}^4_2\text{He}$), then also the binding energy per nucleon will increase i.e., again energy will be liberated. This phenomenon is called nuclear fusion.

5.6 NUCLEAR FISSION

The phenomenon of breaking a heavy nucleus into two light nuclei of almost equal masses along with the release of huge amount of energy is called nuclear fission. The process of nuclear fission was first discovered by German Scientists Otto Hahn and Strassman in 1939. They bombarded uranium nucleus (${}_{92}\text{U}^{235}$) with slow neutrons

and found that ${}_{92}\text{U}^{236}$ was split into two medium weight parts with the release of enormous energy. These fragments has atomic numbers far less than the target nucleus (${}_{92}\text{U}^{235}$). The nuclear fission of ${}_{92}\text{U}^{235}$ is given by the following nuclear reaction:



The fission of ${}_{92}\text{U}^{235}$ nucleus when bombarded with a neutron takes place in following manner. When a neutron strikes ${}_{92}\text{U}^{235}$ nucleus, it is absorbed by it, producing a highly unstable ${}_{92}\text{U}^{236}$ nucleus. Instead of emitting α or β particles or γ rays, this unstable nucleus is split into two middle weight parts viz ${}_{56}\text{Ba}^{144}$ and krypton (${}_{36}\text{Kr}^{89}$). During this fission, three neutrons are given out and a small mass defect occurs which is converted into enormous amount of energy. The following points are worth noting about nuclear fission process:

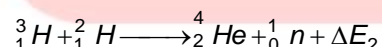
- The energy released in the fission of uranium is about 200 MeV per nucleus. This can be easily verified. If we obtain atomic mass unit values of reactants and products in the fission of ${}_{92}\text{U}^{235}$ nucleus, we find that there occurs a mass defect of 0.214 a.m.u. Which is converted into energy. Energy released per fission of ${}_{92}\text{U}^{235}$ nucleus = 0.214×931 .
 $\approx 200 \text{ MeV}$
- The products of uranium fission are not always barium and krypton. Sometimes, they are Strontium and Xenon. There are other pairs as well. However, in each case, neutrons are emitted and tremendous amount of energy is released.
- Energy is released in the form of kinetic energy of fission fragments. Some of the energy is also released in the form of γ -rays, heat energy sound energy and light energy.
- The pressure and temperature is very high in fission process.

5.7 NUCLEAR FUSION

The process of combining two light nuclei to form a heavy nucleus is known as nuclear fusion. An important feature of nuclear fusion is that there is a release of huge amount of energy in the process. This can be easily understood. When two light nuclei are combined to form a heavy nucleus there occurs a small mass defect. This small mass defect results in the release of huge amount of energy according to the relation $\Delta E = mc^2$. For example by the fusion of two nuclei of heavy hydrogen, the following reaction is possible.



The nucleus of tritium ${}_1^3\text{H}$ so formed can again fuse with a deuterium nucleus.



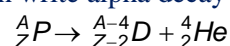
Nuclear fusion is a very difficult process to achieve. This is because when positively charged nuclei come close to each other for fusion they required very high energy to counter repulsive force between them. So a high temperature is required for fusion. Though the energy output in the process of nuclear fission is much more than in a nuclear fusion the energy liberated by the fusion of a certain mass of heavy hydrogen is much more than the energy liberated by the fission of equal mass of uranium.

6 RADIOACTIVITY

The phenomenon of **spontaneous emission of radiations** from radioactive substances is known as Radioactivity. This is exhibited naturally by certain heavy elements like uranium, radium, thorium, etc is called natural radioactivity. However it was later established that it can be induced in lighter elements as well using modern techniques and this is called induced radioactivity.

Rutherford analysed the radiations coming from radioactive sources and showed that this consists of three types of rays namely α , β and γ rays. The prime reason for the emission of these rays is that nucleus can have excited states; these excited states can decay by the emission of high-energy photons (γ rays) to the ground states, directly or via lower energy states. In addition nuclei in both excited and ground states can spontaneously emit other particles (α and β) to reach lower energy configuration.

α -decay: In alpha decay an α particle is ejected from a nucleus and the parent nucleus loses two protons and two neutrons. Therefore its atomic number z decreases by two units and its mass number A decreases by four units, so that the daughter D and parent P , are different chemical elements. Applying conservation of charge and nucleons we can write alpha decay symbolically as



β -decay: It is possible for a nuclear process to occur where the charge Ze of a nucleus changes but the

number of nucleons remains unchanged. This can happen with a nucleus emitting an electron (β -decay), emitting a positron (β -decay) or capturing an inner atomic electron (electron capture). In each of these processes either a proton is converted into a neutron or vice-versa. It is also found that in each of these processes an extra particle called a neutrino appears as one of the decay products. The properties of a neutrino are electric charge = 0, rest mass ≈ 0 , intrinsic spin, $\frac{1}{2}$, and, as with all massless particles it has speed C (speed of light)

γ -decay: a gamma ray emission does not affect either the charge or the mass number

The statistical radioactive law: In a typical radioactive decay an initially unstable nucleus called the parent, emits a particle and decays into a nucleus called the daughter, effectively, the birth of the daughter arises from the death of the parent. The daughter may be either the same nucleus in a lower energy state, as in the case of a γ -decay or an entirely new nucleus as arises from α and β decays. No matter what types of particles are emitted all nuclear decays follow the same radioactive decay law. If there are initially no unstable parents nuclei present, the number

N of parents that will be left after a time t is

$$N = N_0 e^{-\lambda t} \quad \dots (12)$$

The constant λ is called the decay constant or disintegration constant and depends on the particular decay process.

Equation (1) is statistical, not a deterministic, law, it gives the expected number N of parent that survive after a time t . However for a large number of unstable nuclei, the actual number and expected number of survivors will almost certainly differ by no more than an insignificant fraction. The rapidity of decay of a particular radioactive sample is usually measured by the half life $T_{1/2}$, defined as the time interval in which the number of parent nuclei at the beginning of the interval is reduced by a factor of one half. The half-life is readily obtained in terms of λ as :

$$T_{1/2} = \frac{\ln 2}{\lambda} \quad \dots (13)$$

Another quantity that measures the rapidity of decay is the average or mean lifetime of a nucleus, T_{av} , is given by

$$T_{av} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} \quad \dots (14)$$

\therefore Average life = 1.44 times the half-life.

Activity of a radioactive substance

The activity of a radioactive substance is the rate of decay or the number of disintegrations per second. It is denoted by A .

$$\begin{aligned} \therefore A &= \frac{dN}{dt} \quad \dots (15) \\ &= \frac{d}{dt} (N_0 e^{-\lambda t}) \\ &= -\lambda N_0 e^{-\lambda t} \end{aligned}$$

$$\therefore A = -\lambda N$$

$$\text{Also, } A = A_0 e^{-\lambda t} \quad \dots (16)$$

The activity of a sample is measured in unit called Curie. A Curie is defined as that quantity of radioactive substance in which the number of disintegrations per second is 3.7×10^{10} . This is also equal to the activity of one gram of radium.

The activity at time t is given by $A = \frac{A_0}{2^{t/T_{1/2}}}$.



MIND MAP

1. Radius of allowed Bohr's orbit

$$= \frac{\epsilon_0 h^2 n^2}{\pi m Z e^2} = 0.53 \frac{n^2}{Z} \text{ \AA}$$

Energy of hydrogen atom in n^{th} energy state

$$= -\frac{mZ^2 e^4}{8\epsilon_0^2 h^2 n^2} = -13.6 \frac{Z^2}{n^2}$$

Velocity of electron Bohr orbit

$$V = \frac{Ze^2}{2\epsilon_0 hn} = 2.191 \cdot 10^6 \frac{Z}{n}$$

3. de-Broglie wavelength of a particle of mass m and moving with velocity v is

$$\lambda = \frac{h}{mv}$$

Einstein photoelectric equation is

$$h\nu = W_0 + (\text{K.E.})_{\text{max}}$$

Stopping potential

$$V_0 = \left(\frac{h}{e}\right)v - \frac{W_0}{e}$$

2. The wave number of a spectral line is given by

$$\bar{\nu} = RZ^2 \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

$$m = 1, n > 2$$

Corresponds to Lyman series,

$$m = 2, n > 2$$

Corresponds to Balmer series,

$$m = 3, n > 3$$

Corresponds to Paschen series,

$$m = 4, n > 4$$

Corresponds to Brackett series,

$$m = 5, n > 5$$

Corresponds to Pfund series,

(n is a natural number)

4. For continuous X-rays

$$\lambda_{\text{min}} = \frac{hc}{eV}$$

Moseley law for characteristic X-rays

$$\sqrt{\nu} = a(Z - b)$$

MODERN

5. Einstein mass energy equivalence principle $\Delta E = (\Delta m)c^2$

Binding energy per nucleon

$$= \left[\frac{Z}{A}(m_p - m_n) + m_n - \frac{m}{A} \right] c^2$$

The statistical radioactive law

$$N = N_0 e^{-\lambda t}$$

Half life

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

Mean life,

$$T_{av} = \frac{1}{\lambda}$$

$$\text{Activity } A = A_0 e^{-\lambda t}$$