



RAY OPTICS

1 RAY OPTICS

Ray optics treats propagation of light in terms of rays and is valid only if the size of the obstacle is much greater than the wavelength of light. It concerns with the image formation and deals with the study of the simple facts such as rectilinear propagation, laws of reflection and refraction by geometrical methods.

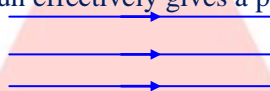
1.1 RAY

A ray can be defined as an imaginary line drawn in the direction in which light is travelling. Light behaves as a stream of energy propagated along the direction of rays. The rays are directed outward from the source of light in straight lines.

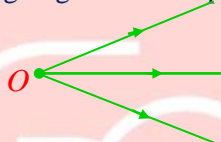
1.2 BEAM OF LIGHT

A beam of light is a collection of these rays. There are mainly three types of beams.

(i) **Parallel beam of light:** A search light and the headlight of a vehicle emit a parallel beam of light. The source of light at a very large distance like sun effectively gives a parallel beam.



(ii) **Divergent beam of light:** The rays going out from a point source generally form a divergent beam.



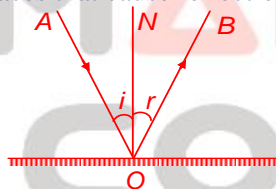
(iii) **Convergent beam of light:** A beam of light that is going to meet (or converge) at a point is known as a convergent beam. A parallel beam of light after passing through a convex lens becomes a convergent beam.



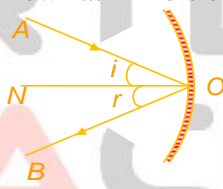
2 REFLECTION

When a ray of light is incident at a point on the surface, the surface throws partly or wholly the incident energy back into the medium of incidence. This phenomenon is called reflection.

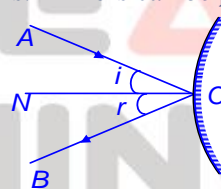
Surfaces that cause reflection are known as mirrors or reflectors. Mirrors can be plane or curved.



Plane mirror



Concave mirror



Convex mirror

In the above figures,
 O is the point of incidence
 AO is the incident ray
 OB is the reflected ray
 ON is the normal at the incidence

Angle of incidence: The angle which the incident ray makes with the normal at the point of incidence is called the angle of incidence. It is generally denoted by ' i '.

Angle of reflection: The angle which the reflected ray makes with the normal at the point of incidence is called the angle of reflection. It is generally denoted by ' r '.

Glancing angle: The angle which the incident ray makes with the plane reflecting surface is called glancing angle. It is generally denoted by 'g'.

$$g = 90^\circ - i \quad \dots (1)$$

2.1 LAWS OF REFLECTION

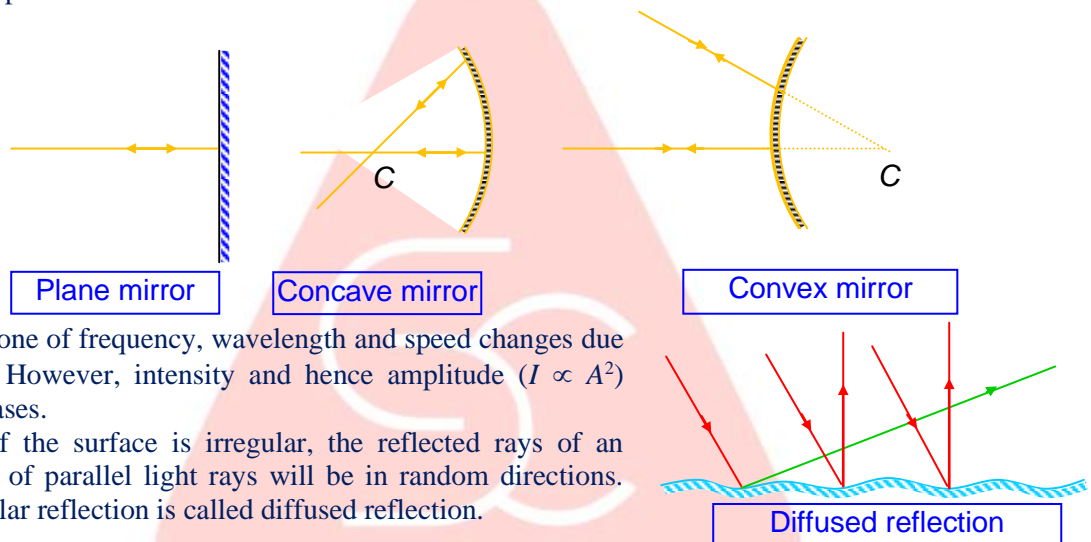
(i) The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence, all lie in the same plane.

(ii) The angle of incidence is equal to the angle of reflection, i.e., $\angle i = \angle r$

These laws hold good for all reflecting surfaces either plane or curved.

Some important points

(i) If $\angle i = 0$, $\angle r = 0$, i.e., if a ray is incident normally on a boundary, after reflection it retraces its path.



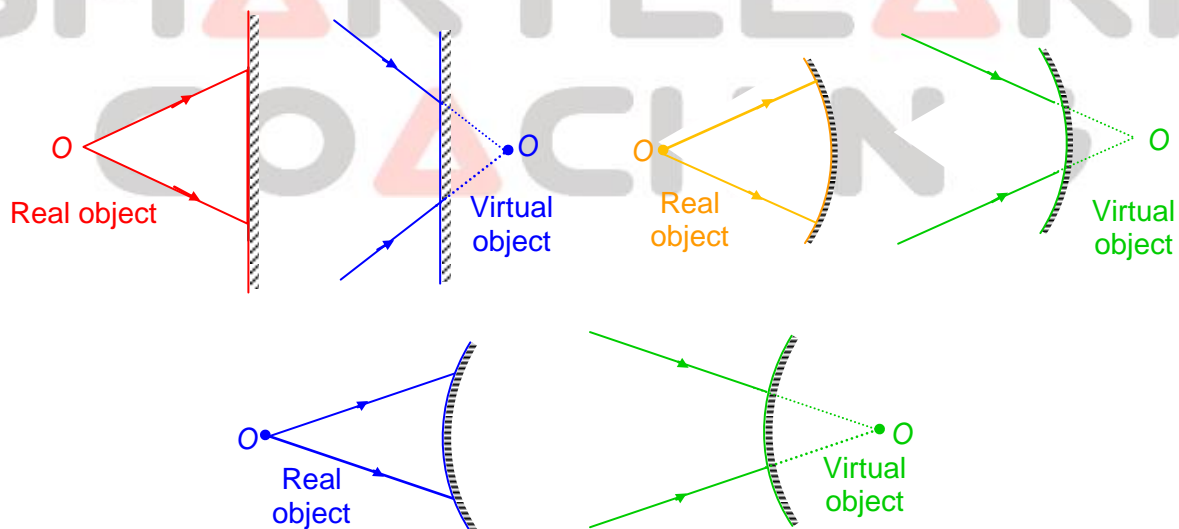
(ii) None of frequency, wavelength and speed changes due to reflection. However, intensity and hence amplitude ($I \propto A^2$) usually decreases.

(iii) If the surface is irregular, the reflected rays of an incident beam of parallel light rays will be in random directions. Such an irregular reflection is called diffused reflection.

2.2 REAL AND VIRTUAL OBJECTS

If the rays from a point on an object actually diverge from it, then the object is said to be real.

If the rays incident on the mirror do not start from a point but appear to converge at a point, then that point is the virtual object for the mirror.



2.3 REAL AND VIRTUAL IMAGES

If the rays from a point object after reflection (or refraction) actually meet at or appear to diverge from a point I , then I is said to be the image of the object O .

Images can be real or virtual. If the reflected or refracted rays actually meet at the point I , then I is said to be a real image of the object O . But if the reflected or a refracted ray do not actually meet but only appear to diverge from the point I , then I is said to be the virtual image of the object O .

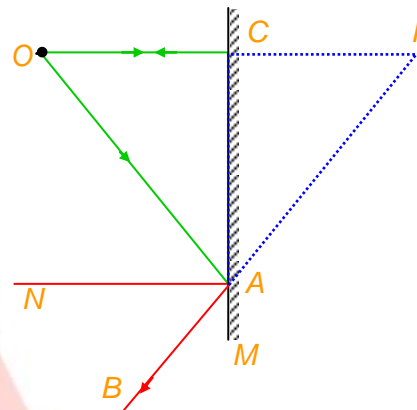
3 IMAGE FORMATION BY A PLANE MIRROR

Let us consider a point object O placed in front of a mirror M . A ray of light OA from O incident on M at A , is reflected along AB so that $\angle OAN = \angle NAB$. A ray OC incident on the mirror at C , is reflected back along CO . Thus, the ray reflected by the plane mirror M appears to come from a point I behind the mirror where I is the point of intersection of BA and OC produced. Thus, I is the image of the point O .

Now,

$$\begin{aligned}\angle AOC &= \angle OAN \\ \angle NAB &= \angle CIA\end{aligned}$$

$$\begin{aligned}\text{Also, } \angle OAN &= \angle NAB \quad (\because \angle i = \angle r) \\ \therefore \angle AOC &= \angle CIA \\ \therefore \triangle OCA &\cong \triangle ACI \\ \Rightarrow OC &= CI\end{aligned}$$



3.1 CHARACTERISTICS OF THE IMAGE FORMED BY A PLANE MIRROR

- (i) The image formed is at the same distance behind the reflecting surface as the object is in front of it.
- (ii) The size of the image is the same as that of the object.
- (iii) The image is virtual and erect which means no light actually passes through it
- (iv) The image is laterally inverted i.e., side-wise inverted. Example:- If a right handed batsman observes his stance in a plane mirror. He appears left handed. The left hand side of the image thus corresponds to the right hand side of the object and vice-versa. Thus, the image is said to be laterally inverted with respect to the object.

3.2 DEVIATION PRODUCED BY A PLANE MIRROR

Deviation is defined as the angle between directions of the incident ray and the reflected ray (or, the emergent ray). It is generally denoted by δ .

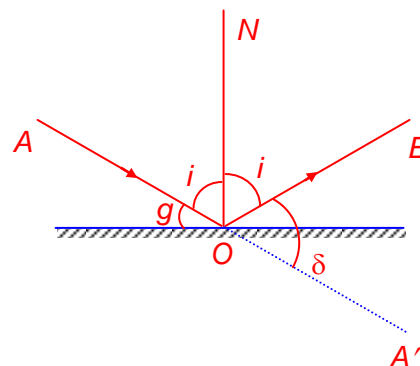
$$\text{Here, } \angle A'OB = \delta = \angle AOA' - \angle AOB = 180^\circ - 2i$$

$$\text{Or, } \delta = 180^\circ - 2i \quad \dots (2)$$

$$\text{We know, } g = 90^\circ - i$$

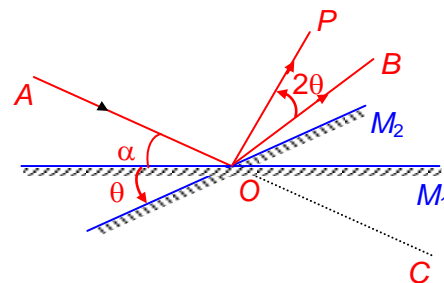
$$\therefore \delta = 180^\circ - 2(90^\circ - g)$$

$$\text{or } \delta = 2g \quad \dots (3)$$



3.3 ROTATION OF THE REFLECTED RAY BY A PLANE MIRROR

Let a ray AO be incident at O on a plane mirror M_1 . Let α be the glancing angle with M_1 . If OB is the reflected ray, then the angle of deviation ($\angle COB$) = 2α . Let the mirror be rotated through an angle θ to a position M_2 , keeping the direction of the incident ray constant. The ray is now reflected from M_2 along OP .



The glancing angle with $M_2 = (\alpha + \theta)$.
Hence, the new angle of deviation (i.e., $\angle COP$)

$$= 2(\alpha + \theta)$$

The reflected ray has thus been rotated through $\angle BOP$ when the mirror is rotated through an angle θ and since

$$\angle BOP = \angle COP - \angle COB$$

$$\text{or, } \angle BOP = 2(\alpha + \theta) - 2\alpha = 2\theta$$

Thus, keeping the incident ray fixed, if the plane mirror is rotated through an angle θ about an axis in the plane of mirror, then the reflected ray is rotated through an angle 2θ .

3.4 IMAGES OF AN OBJECT FORMED BY MIRRORS INCLINED TO EACH OTHER

If two plane mirrors are kept inclined to each-other at angle θ with their reflecting surfaces facing each-other, then multiple reflections take place and more than one images are formed.

NUMBER OF IMAGES FORMED

(i) if $\left(\frac{360}{\theta}\right)$ is an even integer, the number of images formed

$$n = \frac{360^\circ}{\theta} - 1 \quad \dots (4)$$

(ii) If $\left(\frac{360^\circ}{\theta}\right)$ is an odd integer, the number of images formed

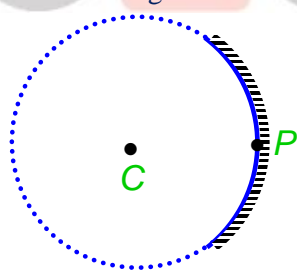
$$n = \frac{360^\circ}{\theta} \quad \text{when the object is placed unsymmetrical to the mirrors... (5 A)}$$

$$n = \frac{360^\circ}{\theta} - 1 \quad \text{when the object is placed symmetrical to the mirrors... (5 B)}$$

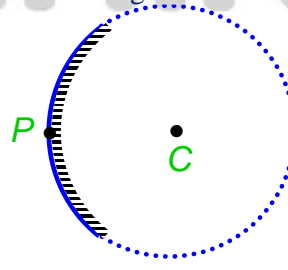
4 REFLECTION AT SPHERICAL MIRROR

4.1 SOME IMPORTANT DEFINITIONS

(i) **Spherical Mirrors:** A spherical mirror is a part of a hollow sphere or a spherical surface. They are classified as concave or convex according to the reflecting surface being concave or convex respectively.

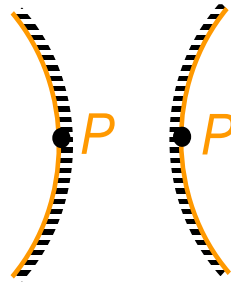


Concave mirror



Convex mirror

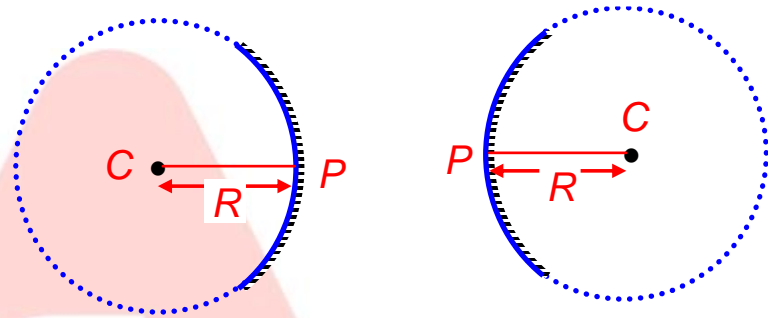
(ii) **Pole or Vertex:** The geometrical centre of the spherical mirror is called the pole or vertex of the mirror.



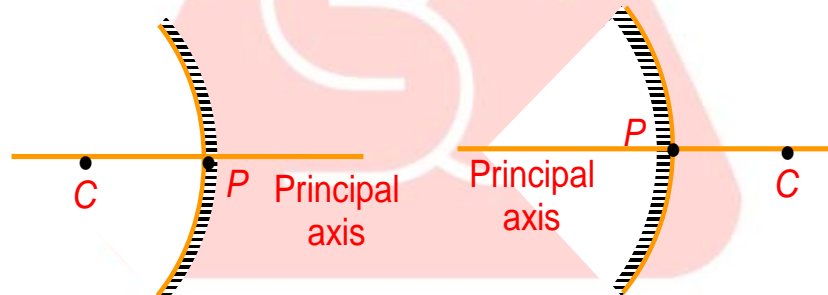
In the above figures, the point P is the pole.

(iii) Centre of curvature: The centre C of the sphere of which the spherical mirror is a part, is the centre of curvature of the mirror.

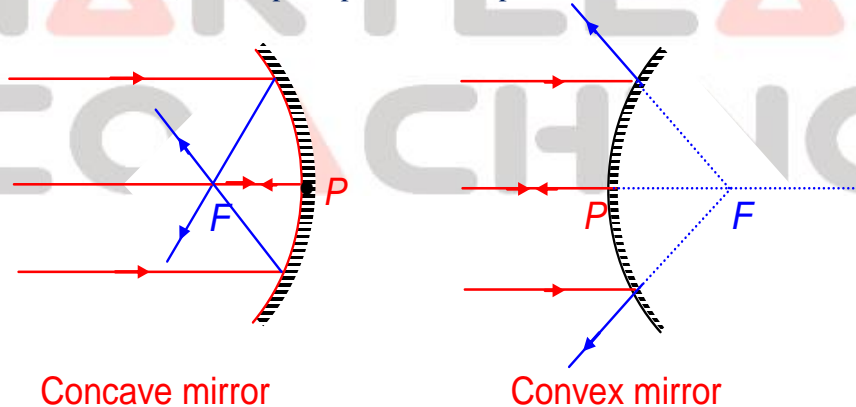
(iv) Radius of curvature (R):
Radius of curvature is the radius R of the sphere of which the mirror forms a part.



(v) Principal axis: The line CP joining the pole and the centre of curvature of the spherical mirror is called the principal axis.



(v) Focus (F): If a parallel beam of rays, parallel to the principal axis and close to it, is incident on a spherical mirror; the reflected rays converge to a point F (in case of a concave mirror) or appear to diverge from a point F (in case of a convex mirror) on the principal axis. The point F is called the focus of the spherical mirror.

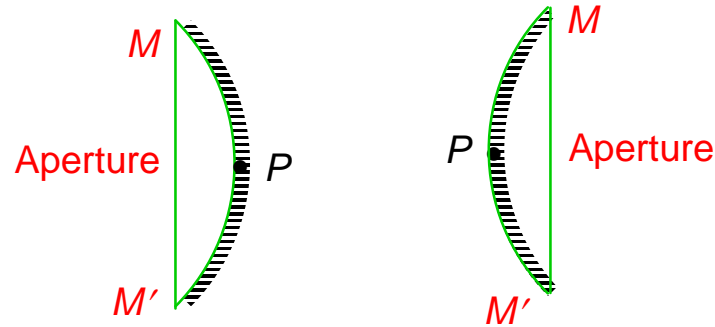


Concave mirror

Convex mirror

(vi) Focal Length (f): Focal length is the distance PF between the pole P and focus F along the principle axis.

(vii) Aperture: The line joining the end points of a spherical mirror is called the aperture or linear aperture.



4.2 RELATION BETWEEN f AND R

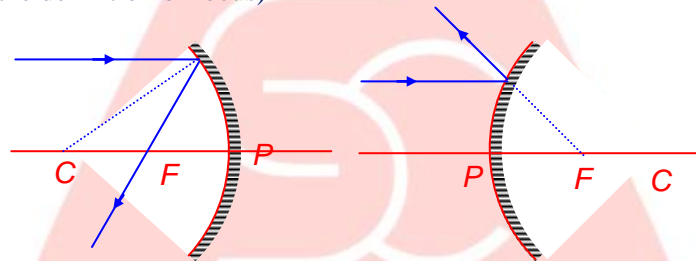
The magnitude of focal length in spherical mirrors is half the radius of curvature, i.e.,

$$f = \frac{R}{2} \quad \dots (6)$$

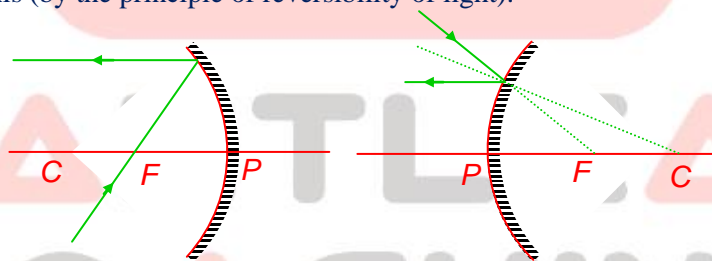
4.3 RULES FOR IMAGE FORMATION

The reflection of light rays and formation of images are shown with the help of ray diagrams. Some typical incident rays and the corresponding reflected rays are shown below.

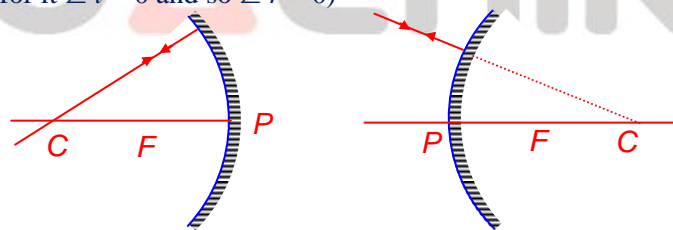
(i) A ray passing parallel to the principal axis after reflection from the spherical mirror passes or appears to pass through its focus (by the definition of focus)



(ii) A ray passing through or directed towards focus after reflection from the spherical mirror becomes parallel to the principal axis (by the principle of reversibility of light).

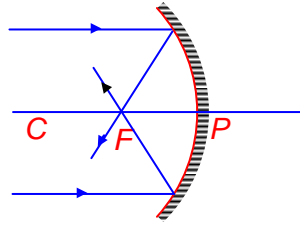


(iii) A ray passing through or directed towards the centre of curvature, after reflection from the spherical mirror, retraces its path (as for it $\angle i = 0$ and so $\angle r = 0$)



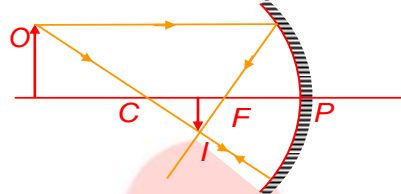
4.4 IMAGE FORMATION BY A CONCAVE MIRROR FOR A REAL LINEAR OBJECT

(i) When the object is at infinity:



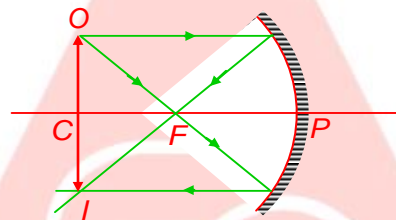
The image is formed at F . It is real, inverted and highly diminished.

(ii) When the object lies beyond C (i.e., between infinity and C):



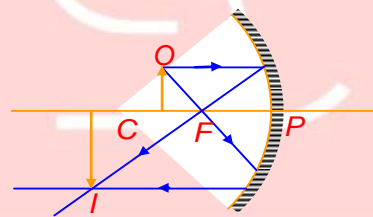
The image is formed between F and C . It is real, inverted and diminished.

(iii) When the object lies at C :



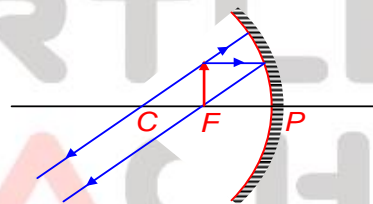
The image is formed at C itself. It is real, inverted and of the same size as the object is.

(iv) When the object lies between F and C :



The image is formed beyond C (i.e., between C and infinity). It is real, inverted and enlarged.

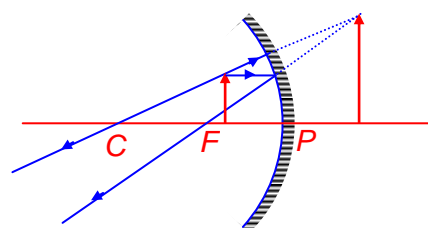
(v) When the object is at F :



The image is formed at infinity. It is real, inverted and highly enlarged.

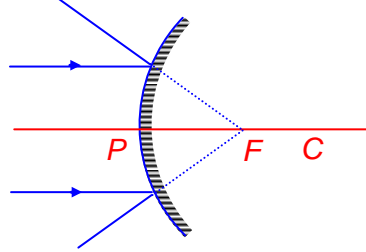
(vi) When the object lies between P and F :

The image is formed behind the concave mirror. It is virtual, erect and enlarged.



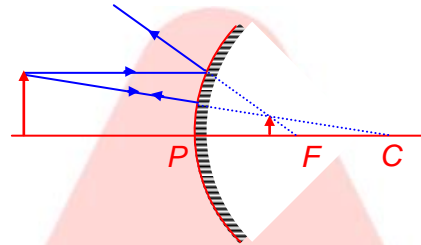
4.5 IMAGE FORMATION BY A CONVEX MIRROR FOR A REAL LINEAR OBJECT

(i) When the object is at infinity:



The image is formed at F . It is virtual, erect and highly diminished.

(ii) When the object lies in between infinity and P :



The image is formed between P and F . It is virtual, erect and diminished.

In case of image formation unless stated otherwise, object is taken to be real and we consider only rays that are close to the principal axis and that make small angles with it. Such rays are called paraxial rays. In practice this condition may be achieved by using a mirror whose size is much smaller than the radius of curvature of the surface. Otherwise the image will be distorted.

4.6 NOTATION USED

u : Distance of the object from the pole of spherical mirror.

v : Distance of the image from the pole of the spherical mirror.

f : Focal length of the spherical mirror.

R : Radius of curvature of the spherical mirror.

4.7 SIGN CONVENTION

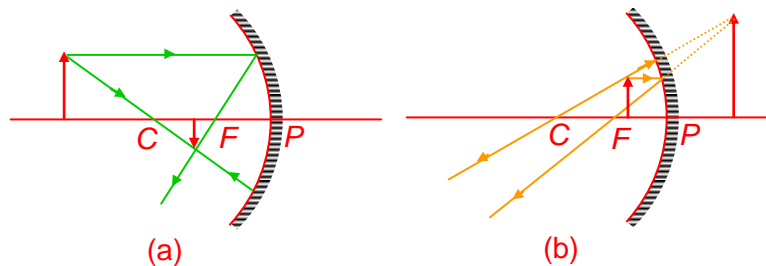
(i) Whenever and wherever possible, the ray of light is taken to travel from left to right.

(ii) All distances are measured from the pole of the spherical mirror along the principal axis.

(iii) Distances measured along the principal axis in the direction of the incident ray are taken to be positive while the distances measured along the principal axis against the direction of the incident rays are taken to be negative.

(iv) Distances measured above the principal axis are taken to be positive while distances measured below the principal axis are taken to be negative.

Example:



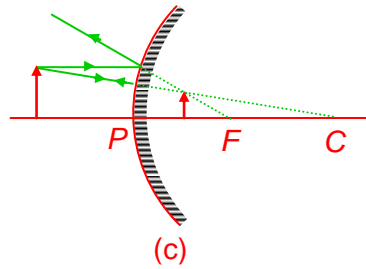


Figure	U	v	R	f
(a)	- ve	- ve	- ve	- ve
(b)	- ve	+ ve	- ve	- ve
(c)	- ve	+ ve	+ ve	+ ve

Important Points Regarding Sign Convection:

(i) If the point (i) is valid, our convention coincides with right hand co-ordinate (or new Cartesian co-ordinate system). If the point (i) is not valid, convention is still valid but does not remain co-ordinate convention.

(ii) In this sign convention, focal length of a concave mirror is always negative while the focal length of a convex mirror is always positive.

4.8 MIRROR FORMULA

Object distance, image distance and focal length in spherical mirrors are related by the equation:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R} \quad \dots (7)$$

4.9 LINEAR MAGNIFICATION

For linear object, the ratio of the image size (I) to the object size (O) is called linear magnification or transverse magnification or lateral magnification. It is generally denoted by m .

$$m = \frac{\text{Height of the image}}{\text{Height of the object}} = \frac{I}{O} \quad \dots (8)$$

$$m = -\frac{v}{u} = \frac{f-v}{f} = \frac{f}{f-u} \quad \dots (9)$$

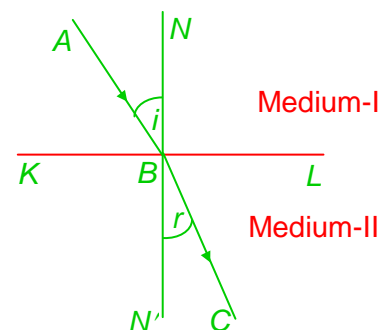
5 REFRACTION OF LIGHT AT PLANE SURFACE

5.1 REFRACTION

In a homogeneous medium, light rays travel in a straight line. Whenever a ray of light passes from one transparent medium to another, it gets deviated from its original path while crossing the interface of the two media (except in case of normal incidence). This phenomenon of deviation or bending of light rays from their original path while passing from one medium to another is called refraction.

$AB \rightarrow$ Incident ray.

$BC \rightarrow$ Refracted ray



NBN' → The normal to the refracting surface at the point of incident

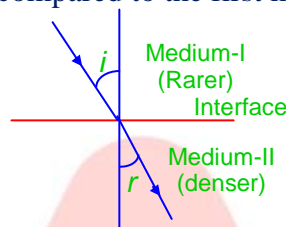
$\angle i$ → The angle of incidence.

$\angle r$ → The angle of refraction.

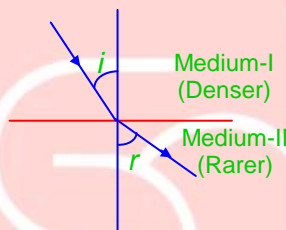
KL → Interface.

In the second medium, the ray either bends towards the normal or away from the normal with respect to its path in the first medium.

If the refracted ray bends towards the normal with respect to the incident ray, then the second medium is said to be optically denser as compared to the first medium.



If the refracted ray bends away from the normal, then the second medium is said to be (optically) rarer as compared to the first medium.



5.2 LAWS OF REFRACTION

The phenomenon of refraction takes place according to the following two laws:

(i) The incident ray, the refracted ray and the normal to the refracting surface at the point of incidence, all lie in the same plane.

(ii) The ratio of sine of the angle of incidence to the sine of the angle of refraction is a constant for any two given media and for light of given colour.

If the angle of incidence and the angle of refraction be i and r respectively, then

$$\frac{\sin i}{\sin r} = \text{constant} \quad \dots (10)$$

This law is called Snell's law

Some Important Points

(i) According to Snell's law, $\frac{\sin i}{\sin r} = \text{constant}$.

This constant is known as refractive index (R. I.) of the second medium with respect to the first medium. It is denoted by ${}_1\mu_2$ (or ${}_1n_2$).

(ii) Refractive index is the relative property of the two media. If the first medium carrying the incident ray is air (strictly vacuum), then the ratio $\frac{\sin i}{\sin r}$ is called the absolute refractive index of the

second medium. It is denoted by μ or n .

(iii) According to the wave theory of light.

$${}_1\mu_2 = \frac{\sin i}{\sin r} = \frac{\text{velocity of light in medium 1}}{\text{velocity of light in medium 2}} = \frac{v_1}{v_2}$$

Hence, the absolute refractive index of the medium is given by

$$\mu = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in medium}} = \frac{C}{V}$$

(iv) If the absolute refractive indices of the media 1 and 2 are μ_1 and μ_2 respectively, then

$${}_1\mu_2 = \frac{V_1}{V_2} = \frac{C/V_1}{C/V_2} = \frac{\mu_2}{\mu_1}$$

(v) We know ${}_1\mu_2 = \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

$$\text{or, } \mu_1 \sin i = \mu_2 \sin r \quad \dots (11)$$

This is the most general formula for the refraction at a surface.

(vi) We have

$$\frac{\mu_2}{\mu_1} = \frac{C/V_2}{C/V_1} = \frac{V_1}{V_2}$$

$$\text{or, } V \propto \frac{1}{\mu}$$

This shows that the higher is the refractive index of a medium, lesser is the velocity of light in that medium. Thus, a medium having higher absolute refractive index is called optically denser; while the one having smaller absolute refractive index is called to be optically rarer.

(vii) The frequency of light remains unchanged while passing from one medium to another. Hence

$$\frac{\mu_2}{\mu_1} = \frac{V_1}{V_2} = \frac{v\lambda_1}{v\lambda_2} = \frac{\lambda_1}{\lambda_2}$$

(viii) If the ray of light is incident normally on a boundary, i.e., $\angle i = 0$, then from Snell's law

$$\mu_1 \sin \theta = \mu_2 \sin r$$

$$\text{or, } \sin r = 0$$

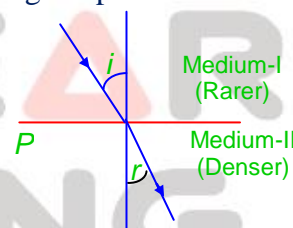
$$\Rightarrow \angle r = 0$$

Thus, light in the second medium will pass undeviated.

5.3 PRINCIPLE OF REVERSIBILITY OF LIGHT RAYS

A ray travelling along the path of the refracted ray is reflected along the path of the incident ray.

In the same way, a refracted ray reversed to travel back along its path will get refracted along the path of the incident ray. Thus, the incident ray and the refracted ray are mutually reversible. This is called the principle of reversibility of light.



When a ray of light travels from first medium to the second medium, we have

$${}_1\mu_2 = \frac{\sin i}{\sin r} \quad \dots (i)$$

When the path of the ray is reversed, it travels from medium-II to medium-I.

Using Snell's Law. We have

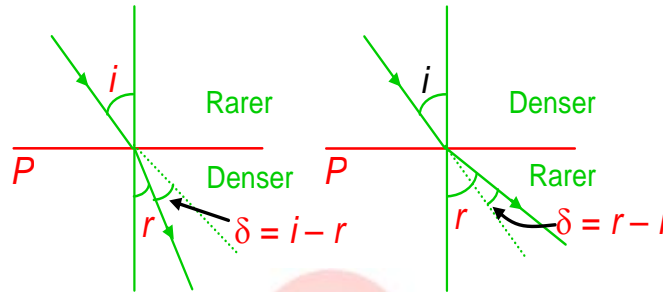
$${}_2\mu_1 = \frac{\sin r}{\sin i} \quad \dots (ii)$$

Multiplying (i) and (ii), we get,

$${}_1\mu_2 \times {}_2\mu_1 = 1$$

or, ${}_1\mu_2 = \frac{1}{{}_2\mu_1}$... (12)

5.4 DEVIATION OF A RAY DUE TO REFRACTION



When a light ray goes from the rarer medium to the denser medium. It bends towards the normal. If a light ray goes from the denser medium to the rarer medium, it bends away from the normal.

In both cases, the magnitude of the angle of the deviation for the light ray is $= |i - r|$.

5.5 REFRACTION THROUGH A GLASS SLAB

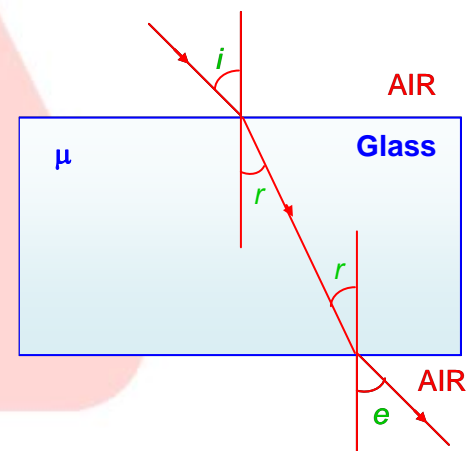
When a light ray passes through a glass slab having parallel faces, it gets refracted twice before finally emerging out of it.

First refraction takes place from air to glass.

$$\text{So, } \mu = \frac{\sin i}{\sin r} \quad \dots (i)$$

The second refraction takes place from glass to air.

$$\text{So, } \frac{1}{\mu} = \frac{\sin r}{\sin e} \quad \dots (ii)$$



From equation (i) and equation (ii), we get

$$\frac{\sin i}{\sin r} = \frac{\sin e}{\sin r} \Rightarrow i = e$$

Thus, the emergent ray is parallel to the incident ray.

5.6 LATERAL SHIFT

The perpendicular distance between the incident ray and the emergent ray, when the light is incident obliquely on a parallel sided refracting glass slab is called 'lateral shift'.

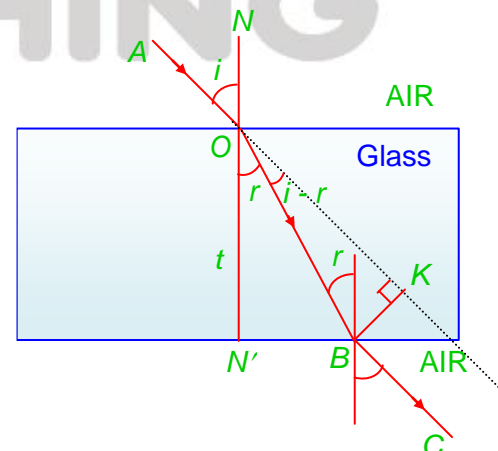
In right-angled triangle OBK , we have

$$\angle BOK = i - r$$

$$\therefore \sin(i - r) = \frac{d}{OB}$$

$$\text{or, } d = OB \sin(i - r) \quad \dots (i)$$

In right angled triangle $ON'B$, we have



$$\cos r = \frac{ON'}{OB} \quad \text{or,} \quad OB = \frac{t}{\cos r}$$

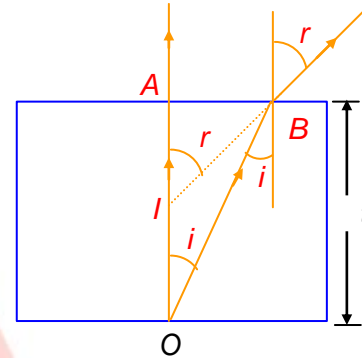
Substituting the above value of OB in equation (i), we get

$$d = \frac{t}{\cos r} \sin(i - r) \quad \dots (13)$$

5.7 APPARENT DEPTH AND NORMAL SHIFT

Case I: When the object is in denser medium and the observer is in rarer medium (near normal incidence)

When an object O is in denser medium of depth ' t ' and absolute refractive index μ and is viewed almost normally to the surface from the outside rarer medium (say air), its image is seen at I . AO is the real depth of the object. AI is the apparent depth of the object. OI is called apparent shift.



According to Snell's law. $\frac{1}{\mu} = \frac{\sin i}{\sin r}$

or, $\frac{1}{\mu} = \frac{\tan i}{\tan r}$ (\because i and r are small angles)

or, $\mu = \frac{\tan r}{\tan i}$ or, $\mu = \frac{AB}{AI} \times \frac{AO}{AB}$

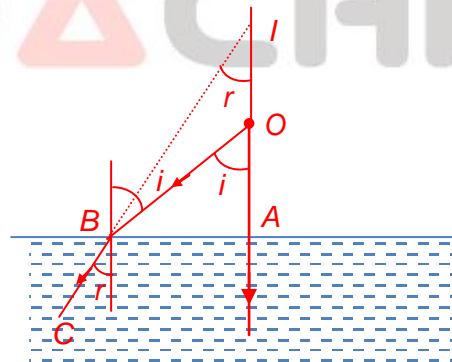
or, $\mu = \frac{AO}{AI}$

$\Rightarrow \mu = \frac{\text{Real depth}}{\text{Apparent depth}} \quad \dots (14)$

Also,

Apparent depth = $\frac{\text{Real depth}}{\mu} = \frac{t}{\mu}$ and, apparent shift (OI) = $t - t/\mu = t[1 - \frac{1}{\mu}]$

Case II: When the object is in rarer medium and the observer is in denser medium (Near normal incidence).



When an object O is in rarer medium (say air) is seen from within a denser medium (say water), the image of O appears to be raised up to I .

The real height = $AO = h$

The apparent height = AI ; and the

Apparent shift = OI

The refraction is taking place from the rarer medium to the denser medium. So, according to

Snell's law $\mu = \frac{\sin i}{\sin r}$

or, $\mu = \frac{\tan i}{\tan r}$ ($\because i$ and r are small angles)

or, $\mu = \frac{AB}{AO} \times \frac{AI}{AB}$ or, $\mu = \frac{AI}{AO}$

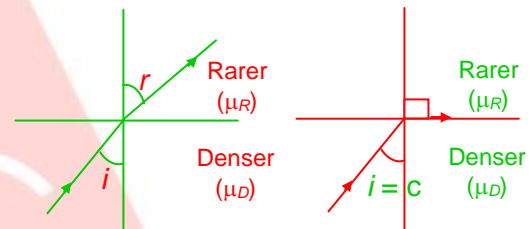
$\Rightarrow \mu = \frac{\text{Apparent height}}{\text{Real height}} \Rightarrow AI = \mu AO$... (15)

\therefore Apparent shift (OI) = $AI - AO = (\mu - 1)h$

5.8 CRITICAL ANGLE

When a ray of light goes from a denser medium to a rarer medium, the angle of refraction is greater than the angle of incidence. If the angle of incidence is increased, the angle of refraction may eventually become 90° .

The angle of incidence for which the angle of refraction is 90° is called the critical angle for that interface. It is generally denoted by C .



5.9 EXPRESSION FOR CRITICAL ANGLE

Let μ_R be the refractive index of the rarer medium and μ_D be the refractive index of the denser medium. Obviously, $\mu_R < \mu_D$

From Snell's law

$$\frac{\sin C}{\sin 90^\circ} = \frac{\mu_R}{\mu_D}$$

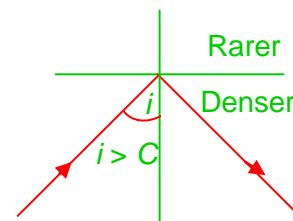
or, $\frac{\sin C}{1} = \frac{\mu_R}{\mu_D}$

or, $\sin C = \frac{1}{\mu}$... (16)

where $\mu = \frac{\mu_D}{\mu_R}$ is refractive index of the denser medium w.r. t the rarer medium.

5.10 TOTAL INTERNAL REFLECTION

If a ray of light travelling in a denser medium strikes a rarer medium at an angle of incidence i which is greater than the critical angle C , it gets totally reflected back into the same medium. This phenomenon is called as 'total internal reflection'.



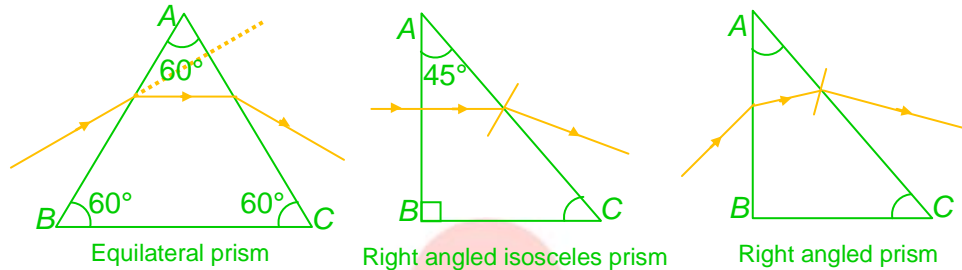


6 REFRACTION THROUGH A PRISM

6.1 PRISM

Prism is a transparent medium bounded by any number of surfaces in such a way that the surface on which light is incident and the surface from which light emerges are plane and non-parallel.

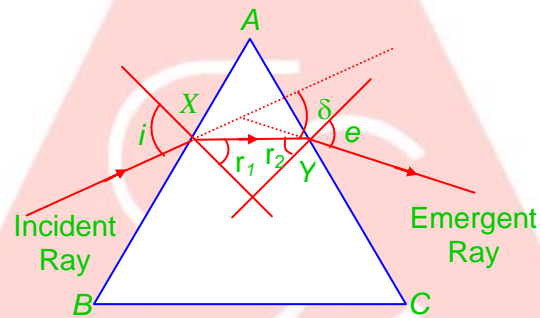
Generally equilateral, right-angled isosceles or right-angled prism are used.



6.2 ANGLE OF THE PRISM OR REFRACTING ANGLE OF THE PRISM

The angle between the two refracting faces involved is called the refracting angle or the angle (A) of the prism.

6.3 DEVIATION PRODUCED BY A PRISM:



A ray of light striking at one face of a triangular glass prism gets refracted twice and emerges out from the other face as shown above.

The angle between the emergent and the incident rays is called the angle of deviation (δ).

From ΔAXY , we have

$$A + (90^\circ - r_1) + (90^\circ - r_2) = 180^\circ$$

$$\Rightarrow r_1 + r_2 = A \quad \dots (17)$$

Now, deviation $\delta = (i - r_1) + (e - r_2)$

$$= (i + e) - (r_1 + r_2)$$

$$\Rightarrow \delta = (i + e) - A \quad \dots (18)$$

$$\text{or, } \delta + A = i + e$$

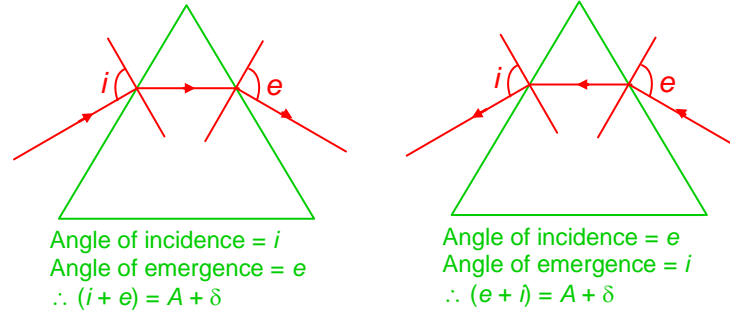
Important Points

(i) We have two equations from Snell's law at X and Y

$$\frac{\sin i}{\sin r_1} = \mu \quad \text{and} \quad \frac{\sin r_2}{\sin e} = \frac{1}{\mu}$$

(ii) It can be easily seen that if we reverse the emergent ray, it goes back along the same path.

The angles of incidence and emergence get interchanged but the angle of deviation remains the same.



Thus the same angle of deviation δ is possible for two different angles of incidence: i and e such that

$$i + e = A + \delta$$

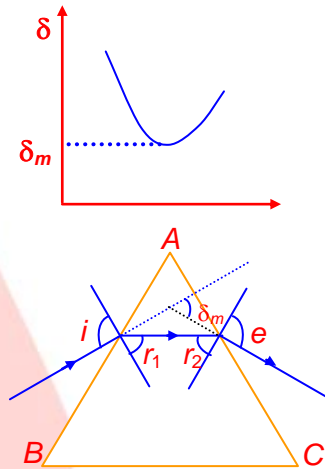
6.4 MINIMUM DEVIATION AND CONDITION FOR MINIMUM DEVIATION

The angle of deviation depends on the angle of incidence in a peculiar way. When the angle of incidence is small, the deviation is large. As i increase, δ decreases rapidly and attains a minimum value and then increases slowly with increase of i . The minimum value of δ so attained is called the minimum deviation (δ_m).

Theory and experiment shows that δ will be minimum when the path of the light ray through the prism is symmetrical, i.e.,

Angle of incidence = angle of emergence

or, $\angle i = \angle e$



For the refraction at the face AB , we have

$$\frac{\sin i}{\sin r_1} = \mu \text{ (Snell's Law) or, } \sin i = \mu \sin r_1$$

and, $\frac{\sin e}{\sin r_2} = \mu$

or, $\sin e = \mu \sin r_2$

$\therefore \mu \sin r_1 = \mu \sin r_2$

or, $r_1 = r_2$

Hence, the condition for **minimum deviation is $i = e$ and $r_1 = r_2$... (19)**

6.5 RELATION BETWEEN REFRACTIVE INDEX AND THE ANGLE OF MINIMUM DEVIATION

When $\delta = \delta_m$, we have

$$e = i \text{ and } r_1 = r_2 = r \text{ (say)}$$

We know

$$A = r_1 + r_2 = r + r = 2r$$

or, $r = \frac{A}{2}$

Also, $A + \delta = i + e$

or, $A + \delta_m = i + e$

or, $i = \frac{A + \delta_m}{2}$

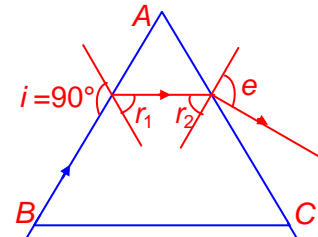
The refractive index of the material of the prism is given by

$$\mu = \frac{\sin i}{\sin r} \quad (\text{Snell's law})$$

or,
$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}} \quad \dots (20)$$

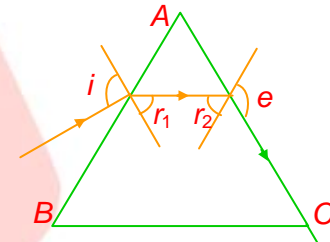
6.6 GRAZING INCIDENCE

When $i = 90^\circ$, the incident ray grazes along the surface of the prism and the angle of refraction (r_1) inside the prism becomes equal to the critical angle for glass-air pair. This is known as **grazing incidence**.



6.7 GRAZING EMERGENCE

When $e = 90^\circ$, the emergent ray grazes along the prism surface. This happens when the light ray strikes the second face of the prism at the critical angle for glass-air pair. This is known as **grazing emergence**.



6.8 MAXIMUM DEVIATION

We know $\delta = (i + e) - A$

Deviation will be maximum when the angle of incidence i is maximum, i.e., $i = 90^\circ$. Hence,

$$(\delta)_{\max} = (90^\circ + e) - A$$

This is the expression for the maximum deviation.

6.9 DEVIATION THROUGH A PRISM OF SMALL ANGLE

If the angle of the prism A is small, r_1 and r_2 (as $r_1 + r_2 = A$) and i and e will be small.

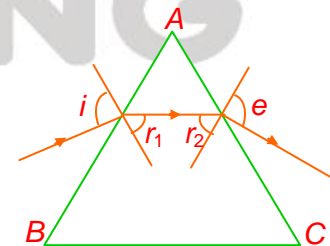
For the refraction at the face AB , we have $\mu = \frac{\sin i}{\sin r_1}$

or,
$$\mu = \frac{i}{r_1} \quad (\text{since } i \text{ and } r_1 \text{ are small angles, } \sin i \approx i \text{ and } \sin r_1 \approx r_1)$$

\Rightarrow For refraction at the face AC , we have

$$\mu = \frac{\sin e}{\sin r_2}$$

or,
$$\mu = \frac{e}{r_2} \quad (\because e \text{ and } r_2 \text{ are small angles, so } \sin e \approx e \text{ and } \sin r_2 \approx r_2)$$



$$\Rightarrow e = \mu r_2 \quad \dots (ii)$$

Now, deviation produced by a prism

$$\delta = (i + e) - A$$

$$\text{or, } \delta = (\mu r_1 + \mu r_2) - A \quad \text{or, } \delta = \mu (r_1 + r_2) - A$$

$$\text{or } \delta = \mu A - A \quad [\because r_1 + r_2 = A]$$

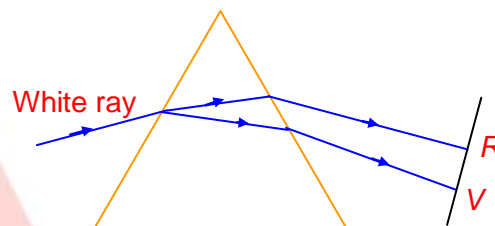
$$\text{or, } \delta = (\mu - 1) A \quad \dots (21)$$

The above formula is valid for all positions of the prism provided the angle of the prism A is small (say $\leq 10^\circ$).

7 DISPERSION OF LIGHT THROUGH A PRISM

Dispersion: When a ray of white light is passed through a prism, it splits up into its constituent colours. This phenomenon of splitting up of white light into constituent colours is called dispersion. The band of seven colours produced on the screen is called the spectrum of the source emitting the incident white light.

Rainbow the most colourful phenomenon in nature is primarily due to the dispersion of sunlight by raindrops suspended in air.



7.1 CAUSE OF DISPERSION

The different colour of light are due to different wavelengths. The wavelengths of violet colour is smaller than that of the red colour.

Cauchy obtained expression for the refractive index of a material in terms of the wavelength of the light. It is given by

$$\mu = a + \frac{b}{\lambda^2}, \quad \dots (22)$$

where a & b are constants for the material

Since the wavelength of violet colour is smaller than that of red colour, from the above formula, it follows that the refractive index of the material for violet colour is more than that for red colour.

$$\text{i.e., } \mu_v > \mu_R$$

for a small angled prism, we have

$$\delta = (\mu - 1) A$$

since $\mu_v > \mu_R$, we have

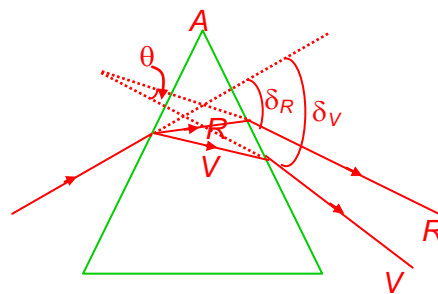
$$\delta_v > \delta_R$$

7.2 ANGULAR DISPERSION

In case of dispersion of light, the angle between the extreme rays (i.e., violet and red) is called angular dispersion or simply dispersion

Angular dispersion = $\theta = \delta_V - \delta_R$... (23)

Let μ_V and μ_R be the refractive indices of the material of the prism for violet and red colours respectively. Let A be the angle of the prism.



We have ,

$$\delta_V = (\mu_V - 1) A;$$

$$\delta_R = (\mu_R - 1) A$$

$$\therefore \delta_V - \delta_R = (\mu_V - \mu_R)A$$

7.3 DISPERSIVE POWER (ω)

The ratio of (angular) dispersion to the deviation of the mean ray (yellow) is called the **dispersive power of the prism**. It is denoted by ω .

$\omega = \frac{\theta}{\delta} = \frac{\delta_V - \delta_R}{\delta}$, where δ is the deviation of the mean ray; δ_V and δ_R are the deviations of the violet and the red rays respectively.

7.4 DISPERSIVE POWER IN TERMS OF REFRACTIVE INDEX

In a thin prism,

$$\delta = (\mu - 1) A$$

$$\therefore \delta_V = (\mu_V - 1) A; \delta_R = (\mu_R - 1) A$$

Hence, $\delta_V - \delta_R = (\mu_V - \mu_R) A$

$$\therefore \omega = \frac{\theta}{\delta} = \frac{\delta_V - \delta_R}{\delta} \Rightarrow \omega = \frac{\mu_V - \mu_R}{\mu - 1} \quad \dots (24)$$

7.5 DISPERSION WITHOUT DEVIATION (DIRECT VISION SPECTROSCOPE)

Let two prisms of different material of dispersive powers ω and ω' be placed in contact, with their bases turned opposite to each-other. Let A and A' be the angles of the first and the second prism respectively.

Let μ_V , μ_R and μ be the refractive indices of the material of the prism for violet, red and yellow colours respectively.

Let μ'_V , μ'_R , μ' be the corresponding values of the material of the second prism.

Deviation of the mean ray by the first prism is

$$\delta = (\mu - 1) A$$

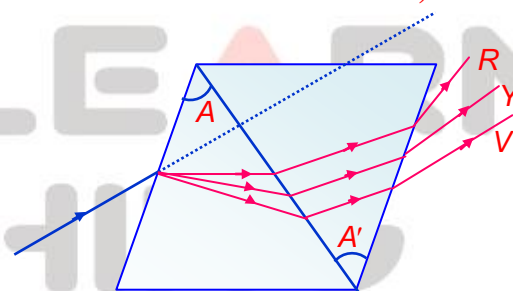
Deviation of the mean ray by the second prism is

$$\delta' = (\mu' - 1) A'$$

Since the combination does not produce any deviation

$$\delta + \delta' = 0 \quad \text{or,} \quad (\mu - 1) A + (\mu' - 1) A' = 0$$

$$\text{or,} \quad \frac{A'}{A} = -\frac{(\mu - 1)}{\mu' - 1} \quad \dots (25)$$



The negative sign indicates that the two prisms are placed with their bases opposite to each-other.

NET ANGULAR DISPERSION

The angular dispersion produced by the first prism,

$$\delta_V - \delta_R = (\mu_V - \mu_R) A$$

The angular dispersion produced by the second prism,

$$\delta_{V'} - \delta_{R'} = (\mu_{V'} - \mu_{R'}) A'$$

\therefore Net angular dispersion

$$= (\delta_V - \delta_R) + (\delta_{V'} - \delta_{R'})$$

$$= (\mu_V - \mu_R) A + (\mu_{V'} - \mu_{R'}) A'$$

$$= A [(\mu_V - \mu_R) + (\mu_{V'} - \mu_{R'}) \frac{A'}{A}]$$

Substituting $\frac{A'}{A} = -\left(\frac{\mu - 1}{\mu' - 1}\right)$ in the above equation,

$$\text{Net angular dispersion} = A \left[\mu_V - \mu_R - \frac{(\mu - 1)}{(\mu' - 1)} (\mu_{V'} - \mu_{R'}) \right]$$

$$= (\mu - 1) A \left[\left(\frac{\mu_V - \mu_R}{\mu - 1} \right) - \left(\frac{\mu_{V'} - \mu_{R'}}{\mu' - 1} \right) \right]$$

or, **net angular dispersion = $(\mu - 1) A [\omega - \omega']$... (26)**

7.6 DEVIATION WITHOUT DISPERSION (ACHROMATIC COMBINATION OF PRISM)

Let two thin prisms of dispersive powers ω and ω' and angles A and A' placed in contact with their bases turned opposite to each-other.

Let μ_V , μ_R and μ be the refractive indices of the material of the first prism for violet, red and mean light respectively.

Let $\mu_{V'}$, $\mu_{R'}$ and μ' be the corresponding values for the material of the second prism.

Angular dispersion produced by the first prism

$$(\delta_V - \delta_R) = (\mu_V - \mu_R) A$$

The angular dispersion produced by the second prism.

$$(\delta_{V'} - \delta_{R'}) = (\mu_{V'} - \mu_{R'}) A'$$

Since the combination does not produce any dispersion

$$(\delta_V - \delta_R) + (\delta_{V'} - \delta_{R'}) = 0$$

or, $A(\mu_V - \mu_R) + A'(\mu_{V'} - \mu_{R'}) = 0$

or, $\frac{A'}{A} = -\left(\frac{\mu_V - \mu_R}{\mu_{V'} - \mu_{R'}}\right)$... (27)

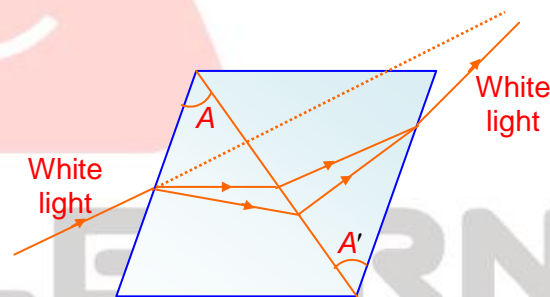
This is the condition for no dispersion or condition for achromatism.

The negative sign indicates that two prisms should be placed in opposite manner.

Another form:

We have

$$A(\mu_V - \mu_R) + A'(\mu_{V'} - \mu_{R'}) = 0$$



$$\text{or, } \left(\frac{\mu_V - \mu_R}{\mu - 1} \right) (\mu - 1)A + \left(\frac{\mu_V' - \mu_R'}{\mu' - 1} \right) \times (\mu' - 1)A' = 0$$

But $(\mu - 1)A = \delta$ and $(\mu' - 1)A' = \delta'$, the deviations for mean light by the first and the second prism respectively.

Also, $\omega = \frac{\mu_V - \mu_R}{\mu - 1}$, the dispersive power of the first prism.

$$\omega' = \left(\frac{\mu_V' - \mu_R'}{\mu' - 1} \right), \text{ the dispersive power of the second prism.}$$

$$\therefore \delta \omega + \delta' \omega' = 0$$

$$\Rightarrow \frac{\omega}{\omega'} = -\frac{\delta'}{\delta} \quad \dots (28)$$

NET DEVIATION

The deviation suffered by the mean light through the first prism.

$$\delta = (\mu - 1)A$$

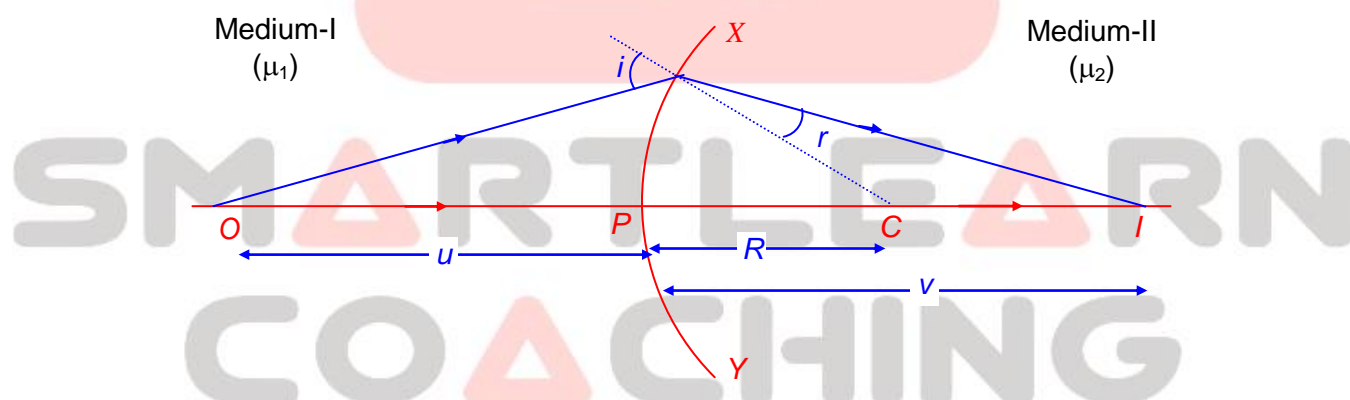
The deviation suffered by the mean light through the second prism

$$\delta' = (\mu' - 1)A'$$

$$\text{Net deviation} = \delta + \delta' = (\mu - 1)A + (\mu' - 1)A' = (\mu - 1)A \left[1 + \frac{(\mu' - 1)A'}{(\mu - 1)A} \right]$$

$$= (\mu - 1)A \left[1 + \frac{(\mu' - 1)}{(\mu - 1)} \left(\frac{\mu_V - \mu_R}{\mu_V' - \mu_R'} \right) \right]$$

$$\text{or, Net deviation} = (\mu - 1)A \left[1 - \frac{\omega}{\omega'} \right] \quad \dots (29)$$



Consider a spherical surface of radius R separating two media with refractive indices μ_1 and μ_2 ($\mu_2 > \mu_1$).

If u is the object distance and v is the image distance, for light rays going from medium I to the medium II,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \dots (30)$$

Also, the transverse magnification in this case is

$$m = \frac{I}{O} = \frac{\mu_1}{\mu_2} \frac{v}{u} \quad \dots (31)$$

IMPORTANT POINTS

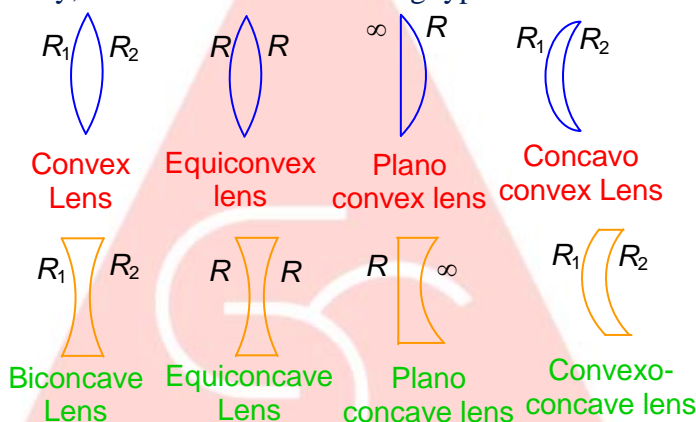
- (i) These equations are valid for all refracting surfaces convex, concave or plane. In case of plane refracting surface, $R \rightarrow \infty$.
- (ii) The rules for signs for a single refracting surface are same as for spherical mirrors.

9 REFRACTION THROUGH THIN LENSES

Lens : A lens is a transparent medium bounded by two refracting surfaces such that at least one of the refracting surfaces is curved.

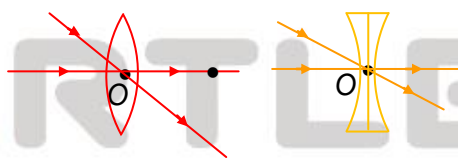
If the thickness of the lens is negligibly small in comparison to the object distance or the image distance, the lens is called thin. Here we shall limit our self to thin lenses.

Types of lenses: Broadly, lenses are of the following types:

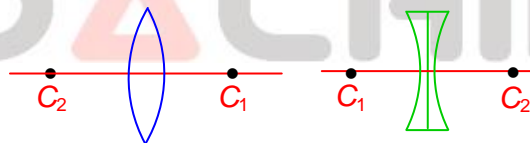


9.1 SOME IMPORTANT DEFINITIONS

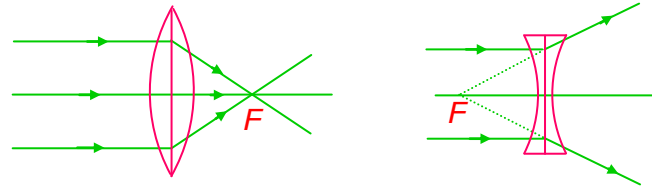
(i) Optical centre: Optical centre is a point for a given lens through which any ray passes undeviated.



(ii) Principal axis: The line joining the centres of curvature of the two bounding surfaces is called the principal axis.



(iii) Focus (F): A parallel beam of light, parallel and close to the principal axis of the lens after refraction passes through a fixed point on the principal axis (in case of convex lens) or appear to diverge from a point on the principal axis (in case of concave lens). This point is called the principal focus or focus of the lens. It is generally denoted by F .



Lenses have two foci because light can strike the lens from both the sides.

(iv) **Focal length (f)** : The distance between the optical centre and the focus of a lens is called its focal length. It is denoted by f .

9.2 SIGN CONVENTION

- (i) Whenever and wherever possible, rays of light are taken to travel from left to right.
- (ii) Distances are measured along the principal axis from the optical centre of the lens.
- (iii) Distances measured along the principal axis in the direction of the incident rays are taken as positive while those measured against the direction of the incident rays are taken negative.
- (iv) Distances measured above the principal axis are taken as positive and those measured below the principal axis are taken as negative.

Example

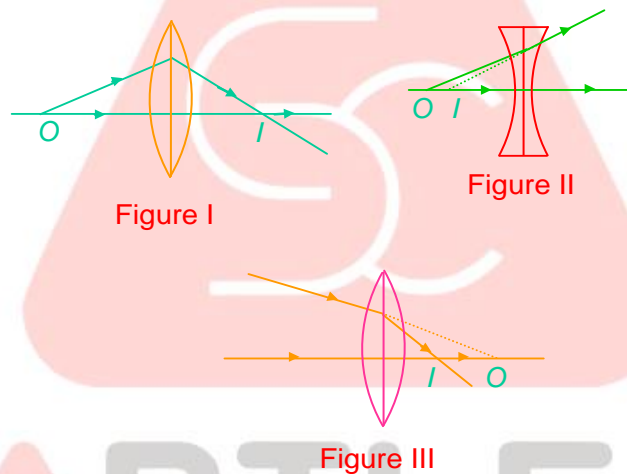
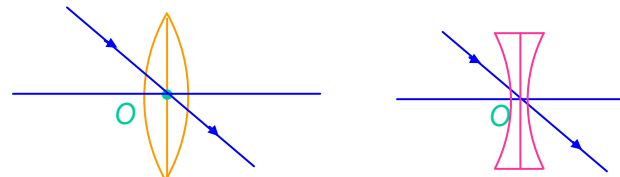


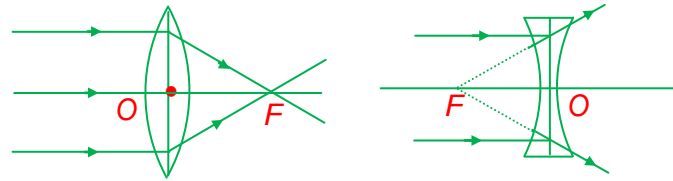
Figure	u	v	f	R_1	R_2
(i)	- ve	+ ve	+ ve	+ ve	- ve
(ii)	- ve	- ve	- ve	- ve	+ ve
(iii)	+ ve	+ ve	+ ve	+ ve	- ve

9.3 RULES FOR IMAGE FORMATION

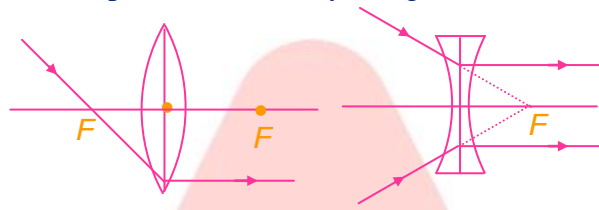
(i) A ray passing through the optical centre of the lens proceeds undeviated through the lens. (By the definition of optical centre)



(ii) A ray passing parallel to the principal axis after refraction through the lens passes or appear to pass through the focus. (By the definition of the focus)



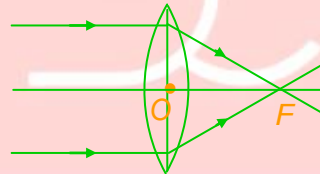
(iii) A ray through the focus or directed towards the focus, after refraction from the lens, becomes parallel to the principal axis. (Principle of reversibility of light)



Only two rays from the same point of an object are needed for image formation and the point where the rays after refraction through the lens intersect or appear to intersect, is the image of the object. If they actually intersect each-other, the image is real and if they appear to intersect the image is said to be virtual.

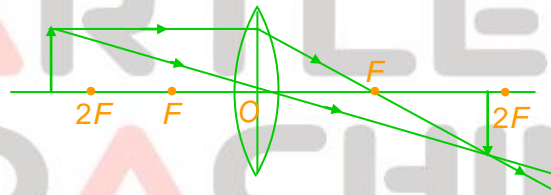
9.4 IMAGE FORMATION BY A CONVEX LENS OF THE LINEAR OBJECT

(i) When the object is at infinity:



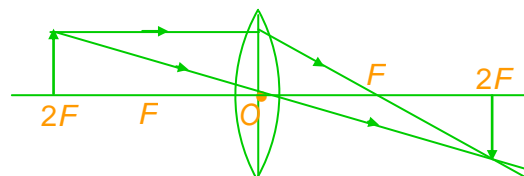
The image is formed at F . It is real, inverted and highly diminished.

(ii) When the object is beyond $2F$:



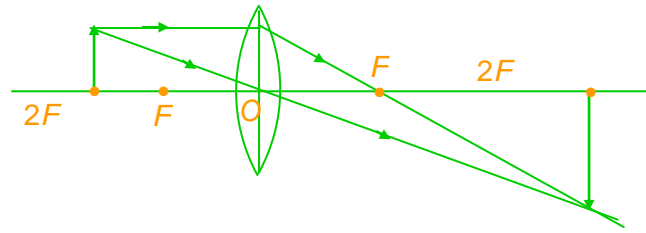
The image is formed between F and $2F$. It is real, inverted and diminished.

(iii) When the object is at $2F$:



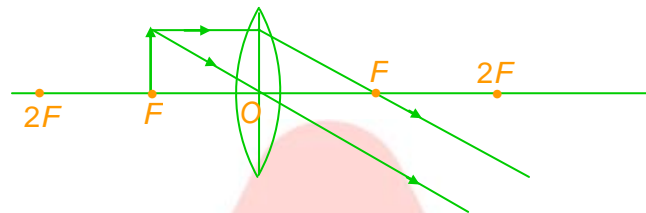
The image is formed at $2F$. It is real, inverted and of the same size.

(iv) When the object is between F and $2F$:



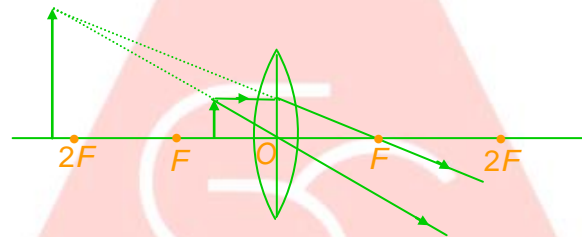
The image is formed beyond $2F$ (i.e., between $2F$ and ∞). It is real, inverted and enlarged.

(v) When the object is at F :



The image is formed at infinity. It is real, inverted and highly diminished.

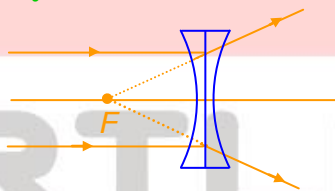
(vi) When the object is between F and O :



The image is on the same side as the object is. It is virtual, erect and magnified.

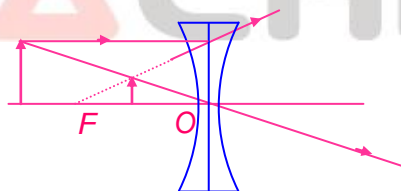
9.5 IMAGE FORMATION BY A CONCAVE LENS OF A LINEAR OBJECT

(i) When the object is at infinity:



The image is formed at the focus. It is virtual, erect and highly diminished.

(ii) When the object is in front of the lens:



The image is formed between F and the optical centre. It is virtual, erect and diminished.

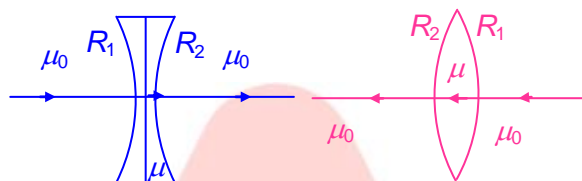
9.6 LENS MAKERS FORMULA

The focal length (f) of a lens depends upon the refractive indices of the material of the lens and the medium in which the lens is present and the radii of curvature of both sides. The following relation giving focal length (f) is called as 'lens maker's formula.

$$\frac{1}{f} = \left(\frac{\mu}{\mu_0} - 1 \right) \left[\frac{1}{R_1} - \frac{1}{R_2} \right], \quad \dots (32)$$

where μ = refractive index of the material of the lens

μ_0 = Refractive index of the medium.



9.7 LENS FORMULA

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \dots (33)$$

9.8 LINEAR MAGNIFICATION (m)

The ratio of the size of the image formed by a lens to the size of the object is called linear magnification produced by the lens. It is denoted by m .

If O and I are the sizes of the object and image respectively, then

$$m = \frac{I}{O} = \frac{v}{u} \quad \dots (34)$$

9.9 POWER OF A LENS

The power of a lens is defined as

$$P = \frac{1}{f} \quad \dots (35)$$

where f must be expressed in metres. The SI unit of power of a lens is m^{-1} . It is also known as dioptre (D). The focal length of a convex lens is positive. So, the power of a convex lens is positive. The focal length of a concave lens is negative. So, the power of a concave lens is negative.

9.10 COMBINATION OF LENSES AND MIRRORS

When several lenses or mirrors are used co-axially, the image formation is considered one after another in steps. The image formed by the lens facing the object serves as object for the next lens or mirror; the image formed by the second lens (or mirror) acts as object for the third and so on. The total magnification in such situations will be given by

$$m = \frac{I}{O} = \frac{I_1}{O} \times \frac{I_2}{I_1} \times \dots$$

$$\text{or, } m = m_1 \times m_2 \times m_3 \times \dots \quad \dots (36)$$

9.11 TWO THIN LENSES IN CONTACT

If two or more lenses of focal lengths f_1 , f_2 and so on are placed in contact, then this combination can be replaced by a lens called 'equivalent lens'. The focal length (F) of equivalent lens is given by

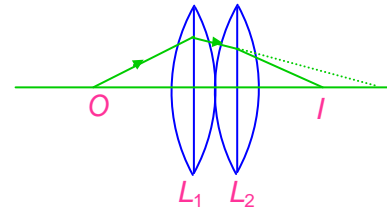
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \dots \quad \dots (37)$$

$$\text{Also, } P = P_1 + P_2 + \dots \quad \dots (38)$$

If two thin lenses of focal lengths f_1 and f_2 are separated by a distance 'd', then the focal length of the equivalent lens is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad \dots (39)$$

$$\text{Also, } P = P_1 + P_2 - dP_1P_2$$



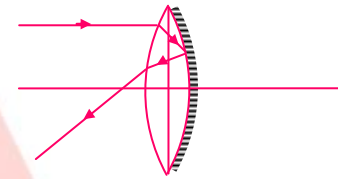
9.12 LENS WITH ONE SURFACE SILVERED

When one surface of a thin lens is silvered, the rays are reflected back at this silvered surface. The set up acts as a spherical mirror.

The focal length of the equivalent spherical mirror is given by

$$\frac{1}{f} = \sum \frac{1}{f_i},$$

where f_i = focal length of the lens or mirror, to be repeated as many times as the refraction or reflection takes place.



$$\dots (40)$$

SIGN CONVENTION

In the above formula, the focal length of the converging lens or mirror is positive; and that of diverging lens or mirror is negative.

9.13 DISPLACEMENT METHOD

Let us consider a real object and a screen fixed at a distance D apart. A convex lens is placed between them.

Let x be the distance of the object from the lens when its real image is obtained on the screen.

We have, $u = -x$

$$v = +(D - x)$$

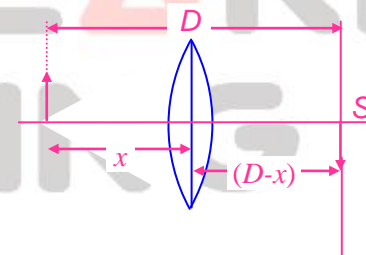
$$f = +f$$

Using lens formula, we have

$$\frac{1}{(D-x)} - \frac{1}{-x} = \frac{1}{f}$$

$$\text{or, } \frac{1}{(D-x)} + \frac{1}{x} = \frac{1}{f}$$

$$\text{or, } \frac{D}{(D-x)x} = \frac{1}{f}$$



or, $x^2 - Dx + Df = 0$

or, $x = \frac{D \pm \sqrt{D^2 - 4Df}}{2} = \frac{D \pm \sqrt{D(D-4f)}}{2}$

Now, there are three possibilities.

(i) If $D < 4f$; then in this case, x will be imaginary and so physically no position of lens is possible.

(ii) If $D = 4f$; then in this case, $x = \frac{D}{2} = 2f$. So only one position of the lens will be possible and in this situation.

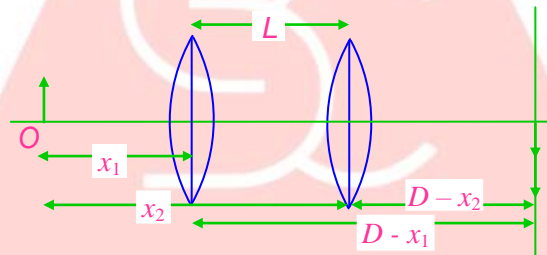
$$v = D - x = 4f - 2f = 2f (= x)$$

(iii) If $D > 4f$; in this situation both the roots of x will be real. Thus

$$x_1 = \frac{D - \sqrt{D(D-4f)}}{2}$$

$$\text{and } x_2 = \frac{D + \sqrt{D(D-4f)}}{2}$$

So if $D > 4f$, these are two positions of lens at distances x_1 and x_2 from the object for which real image is obtained on the screen. This method is called 'Displacement method' and is used in laboratory to determine the focal length of the convex lens.



In case of displacement method, if the distance between two positions of the lens is L , then

$$L = x_2 - x_1 = \sqrt{D(D-4f)}$$

or, $L^2 = D^2 - 4Df$

$$\Rightarrow f = \frac{D^2 - L^2}{4D} \quad \dots (41)$$

We have

$$L = (x_2 - x_1)$$

and $D = x_1 + x_2$

$$\therefore x_1 = \frac{D-L}{2}; x_2 = \frac{D+L}{2}$$

Now, $x_1 + x_2 = \frac{D-L}{2} + \frac{D+L}{2} = D$

$$\Rightarrow x_1 = D - x_2 \text{ \& } x_2 = D - x_1$$

Hence object and image distances are interchanged in the two positions of the lens. Let the magnification be m_1 and m_2 .

$$\therefore m_1 = \frac{(D-x_1)}{-x_1} = \frac{x_2}{x_1}$$

$$\text{and } m_2 = \frac{(D-x_2)}{-x_2} = -\frac{x_1}{x_2}$$

Now,

$$m_1 m_2 = \frac{x_2}{-x_1} \times \frac{-x_1}{x_2} = 1$$

$$\text{or, } \frac{l_1}{O} \frac{l_2}{O} = 1$$

$$\Rightarrow O = \text{object size} = \sqrt{l_1 l_2} \quad \dots (42)$$

i.e., the size of the object is the geometric mean of the sizes of the two images.

10 THE COMPOUND MICROSCOPE

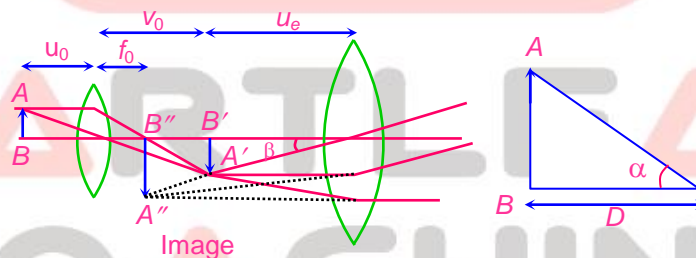
When a greater magnification than is obtained from a simple magnifier is needed, we usually use compound microscope.

10.1 CONSTRUCTION

A compound microscope consists of two convex lenses co-axially separated by some distance. The one facing the object is called the objective and the one close to the eye is called the eye piece or ocular. The objective has a smaller aperture and smaller focal length than those of the eye piece.

10.2 FORMATION OF IMAGE

The object AB to be viewed is placed slightly beyond the principal focus F_0 of the objective. The objective forms an enlarged, real inverted image $A'B'$. This image works as the object for the eyepiece. A very much enlarged virtual image $A''B''$ is formed by the eyepiece.



10.3. ANGULAR MAGNIFICATION OR MAGNIFYING POWER

Angular magnification of a compound microscope is defined as the ratio of the angle β subtended by the final image at the eye to the angle α subtended by the object seen directly, when it is placed at the least distance of distinct vision (D)

$$\begin{aligned} \text{Angular magnification } m &= \frac{\beta}{\alpha} = \frac{A'B'}{u_e} \times \frac{D}{AB} \\ &= \left(\frac{A'B'}{AB} \right) \times \left(\frac{D}{u_e} \right) \end{aligned}$$

$$\text{Also from figure } \frac{A'B'}{AB} = \frac{-v_0}{u_0}$$

Now $\frac{D}{u_e}$ is the magnifying power of the eye piece treated as a simple microscope. This is equal to $\frac{D}{f_e}$ in normal adjustment (image at infinity) and $1 + \frac{D}{f_e}$ for the adjustment when the image is formed at the least distance for clear vision i.e. at D .

Thus, the magnifying power of the compound microscope is

$$m = \frac{v}{u} \left(\frac{D}{f_e} \right) \text{ for normal adjustment and}$$

$$m = \frac{v}{u} \left(1 + \frac{D}{f_e} \right) \text{ for the adjustment when the final image is formed at the least distance of}$$

clear vision. Since the object is placed very close to the principal focus of the objective, therefore u_0 is nearly equal to f_0 . Moreover, the focal length of the eyepiece is small. So, the image $A' B'$ is formed very close to the eye piece. So v_0 is nearly equal to the length L of the microscope tube (separation between the two lenses)

$$m = -\frac{L}{f_0} \left(1 + \frac{D}{f_e} \right) \text{ for image at } D \text{ and}$$

$$m = -\frac{L}{f_0} \times \frac{D}{f_e} \text{ for normal adjustment}$$

10.4 TELESCOPE

To look at distant objects such as a star, a planet or a distant tree etc. we use another instrument called a telescope.

(A) Astronomical refracting telescope: It consists of an objective lens system and an eyepiece. The one facing the distant object is called the objective and has a large aperture and a large focal length. The other is called the eyepiece, as the eye is placed close to it. It has a smaller aperture and a smaller focal length. The lenses are fixed in tube.

Light from the distant object enters the objective and a real image is formed within the tube. The eye piece used again as a simple magnifier, leaves the final image inverted.

(i) When the final image is formed at infinity (normal adjustment)

From the figure, we see that the objective forms image $A' B'$ of an object AB placed at a distant place. If the point P is on the principal axis, the image point P is at the second focus of the objective. For normal adjustment image is formed at infinity.

Hence $u_e = f_e$

Angular magnification $m = \frac{\beta}{\alpha} = -\frac{A' B' / u_e}{A' B' / f_0} = -\frac{f_0}{u_e} = -\frac{f_0}{f_e}$

Where f_0 = focal length of objective

f_e = focal length of eye piece

(ii) When the final image is formed at the least distance of distinct vision. (near point)

If the telescope is adjusted, so that the final image is formed at the near point of the eye, the angular magnification is further increased.

Angular magnification $\Rightarrow -\frac{f_0}{v_e} = m$

But, for near point adjustment $v_e = -D$

So by lens formula

$$-\frac{1}{D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{f_e + D}{f_e D}$$

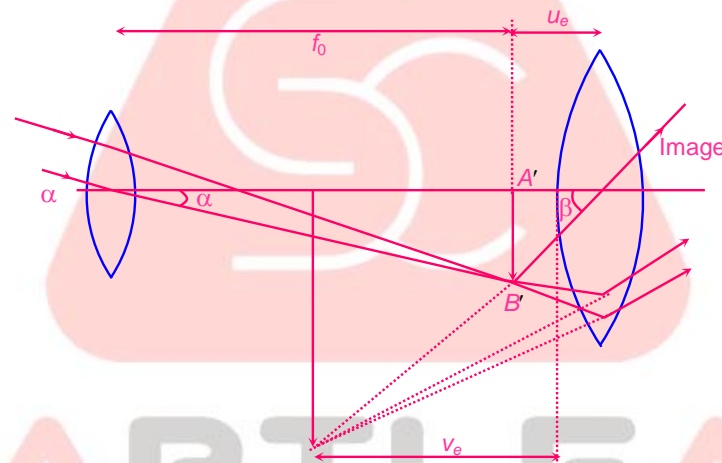
$$m = -\frac{f_0(f_e + D)}{f_e D} = -\frac{f_0}{f_e} \left(1 + \frac{f_e}{D}\right)$$

Length of telescope:

From the given figure, we see that the length of the telescope is $L = f_0 + u_e$

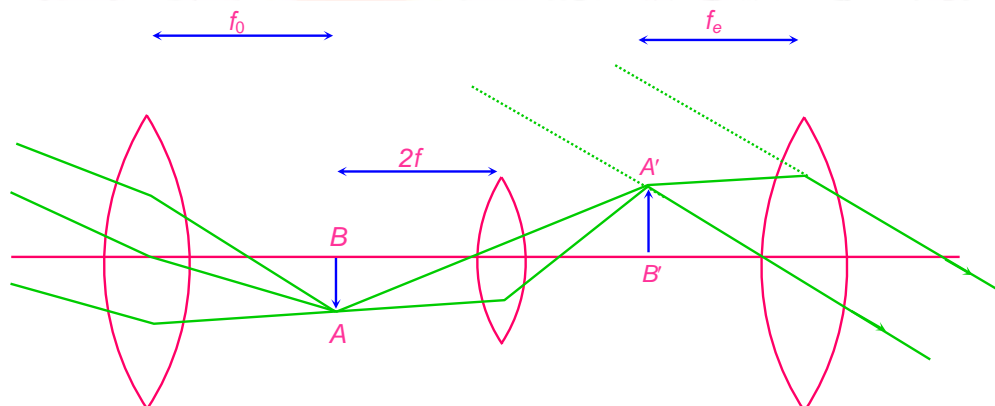
For normal adjustment $L = f_0 + f_e$

For near point adjustment $L = f_0 + u_e = f_0 + \frac{f_e D}{f_e + D}$



10.5 TERRESTRIAL TELESCOPE

The astronomical telescope produces an inverted image of an object viewed through it. So it is unsuitable for terrestrial use. The astronomical telescope can be converted into terrestrial telescope by introducing a converging lens between the objective and the eyepiece. This additional lens is called an erecting lens.



The erecting lens of focal length f is introduced in such a manner that the focal plane of the objective is a distance $2f$ away from this lens. An erect image $A' B'$ of same size is made by the new lens introduced. The length of the tube for normal adjustment is

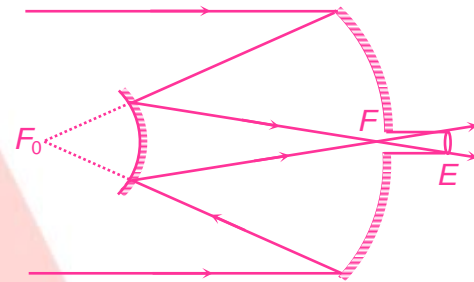
$$L = f_0 + 4f + f_e$$

Magnifying power $m = \frac{f_0}{f_e}$ for normal adjustment

$$= \frac{f_0}{f_e} \left(1 + \frac{v_e}{D} \right)$$

10.6 REFLECTING TELESCOPE (CASSEGRAIN)

Telescope with mirror objectives are called reflecting telescopes. The objective O of the telescope is a large parabolic mirror with a hole (small circular aperture) in the centre. A small convex parabolic mirror is placed in front of the objective. A convex lens acts as the eye piece. The eyepiece has a large focal length in a short telescope.



A beam of light parallel to the axis of the telescope from some distant light source is incident on the objective. The objective converges this light towards its principle focus F_0 . The reflected beam is intercepted by the convex mirror. The convex mirror forms an inverted image at F . This inverted image is seen through the eye piece E .

$$\text{Magnification of reflecting telescope} = \frac{f_0}{f_e}$$

Where f_0 = focal length of objective mirrors
 f_e = focal length of objective eye lens.

Advantages of reflecting telescope

- (1) There is no chromatic aberration, because it is based on reflection.
- (2) Spherical aberration can be removed by choosing parabolic reflector.
- (3) Mechanical support required is much less of a problem since the mirror weighs much less than a lens of equivalent optical quality and can be supported over its entire back surface.